Stability Approaches For Takagi-Sugeno Systems

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***Abstract*—This work concerns different stability approaches for Takagi-Sugeno (T-S) fuzzy systems. We present some recent approaches based on the idea of multiple Lyapunov functions and slack matrices to reduce the conservatism of stability analysis conditions. Then, new stability conditions are proposed in order to more reduce the conservatism using another upper bound of time-derivative of membership functions, fuzzy Lyapunov functions and slack matrices. Consequently, a large stability domain is obtained. Finally, examples are provided to illustrate the effectiveness of the proposed approaches in stability analysis**

Keywords*—Takagi-Sugeno Systems, Stability Analysis, Quadratic Lyapunov functions, Fuzzy Lyapunov functions, Upper bound of time-derivative of membership functions.

I. INTRODUCTION

During the last decades, the so-called Takagi-Sugeno models [1] have reached a great attention in the control community. The advantage of the T-S fuzzy model lies in the fact that the stability analysis and controller design of the system represented by a T-S fuzzy model can be obtained by using Lyapunov function approach. Tanaka and Sugeno proved in [2] that the stability of a T-S fuzzy model could be shown by finding a common positive definite symmetric matrix P for all the subsystems via linear matrix inequalities (LMIs) which can be efficiently solved by convex programming techniques already implemented in commercially available software [3]. Nonetheless, there are models which do not have a solution. It is well-known that quadratic Lyapunov function may be very restrictive, particularly for a large number of rules [2], [4] and [5]. Moreover, Common quadratic Lyapunov functions might not exist for some stable T-S systems, as demonstrated in [6]. Researchers have proposed several Lyapunov functions to deal with these drawback. In [7], [8] Fuzzy Lyapunov functions (FLFs) were proposed, thus constituting the first non-quadratic framework for T-S models. In [6], [9] and [10], researchers proved that the use of piecewise Lyapunov functions (PWLFs) have effectively relaxed the referred pessimism, though they require the MFs to induce a polyhedral partition of the state space. Unfortunately, this condition on the MFs of those T-S models obtained by sector nonlinearity approach is not fulfilled; moreover, the piecewise approach leads to bilinear matrix inequalities in the continuous-time context which cannot be optimally solved [10],[11]. In [12], a line-integral Lyapunov function is proposed to circumvent the MF time-derivative obstacle, though the line integral is asked to be path-independent thus significantly reducing its applicability. In this work, stability analysis of T-S fuzzy models is studied and less conservatism LMI conditions are proposed.

This rest of the paper is organized as follows: in section 2 T-S fuzzy models are given; in section 3, existing quadratic stability approaches are presented ; after that, we present non-quadratic existing approaches to improve the stability domain. In section 5 , by considering a new time-derivative bound of the membership functions, new stability conditions are presented and a comparison has been made between different stability methods to show the effectiveness of the proposed methods. Finally, the conclusion is given. Throughout the text, the following notations are adopted; diag$(.)$: Diagonal matrix, $(*)$: Symmetric block, $(A)^T$: Transpose of matrix A.

Also, we adopt the following notation:

$h_i \equiv h_i\left(z\left(t\right)\right)$

II. TAKAGI-SUGENO FUZZY MODEL

Fuzzy sets and systems have gone through much development and criticism over the last three decades [12-16], and nowadays they can be found in a wide range of applications [17]. In particular, model-based fuzzy control is a widespread approach for complex nonlinear dynamics. Within this context, a major role is played by the Takagi-Sugeno fuzzy model [1]. It is proved that the T-S fuzzy model is an universal approximation tool [18],[19]. Takagi-Sugeno fuzzy model is described by the following fuzzy rules :

$$\begin{array}{l}\text{If}\, z_1\left(t\right)\ \ is\, F_i^1\, and,\, \ldots,\, z_p\left(t\right)\ \ is\, F_i^p\\ Then\ \dot{x}\left(t\right) = A_i x\left(t\right) + B_i u\left(t\right)\end{array} \tag{1}$$

Where $i = \{1, .., r\}$ denotes the rules number; F_i^j denote the fuzzy sets; $z_1\left(t\right)\ ,\, \ldots,\, z_p\left(t\right)$denotes the premise variables; $x\left(t\right)$ is the state vector; the control signal is represented by $u\left(t\right) \in \mathbf{R^m}$; $A_i \in \mathbf{R^{n\times n}}\ and\ B_i \in \mathbf{R^{n\times q}}$ are the local system matrices. The set of rules (1) yields the following inferred model:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i \left(A_i x(t) + B_i u(t)\right) \tag{2}$$

where $h_i(z(t))$ is the averaging weight for each rule, representing the normalized grade of membership, and

$$\begin{cases} \sum_{i=1}^{r} h_i = 1 \\ 0 \leq h_i \leq 1 \end{cases}$$

III. Quadratic stability conditions

Stability analysis are usually based on Lyapunov theory [20], in this section, we will give some theories based on quadratic Lyapunov functions used in literature. In fact, the theory of quadratic stability was developed in 1980 by Hollot and Barmish [21]. It was the base for numerous works so far. This concept implies the existence of a common Lyapunov function, independent from parameters and insuring stability of global system. It establishes one of the most important results in the context of control [2], [4] and [5].

A. Theorem 1 [2]

The T-S fuzzy system is asymptotically stable if there exists symmetric matrix P satisfying:

$$A_i^T P + P A_i < 0, \quad i = 1..r \tag{3}$$

Unfortunately, common quadratic Lyapunov functions might not exist for some stable T-S systems. In the next section, we will present another type of Lyapunov functions to deal with these drawback.

IV. Non Quadratic stability conditions

Fuzzy Lyapunov functions are generally given by the following expression:

$$V(x(t)) = x^T(t) \left(\sum_{i=1}^{r} h_i P_i\right) x(t), \quad P_i \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}},\ P_i = P_i^T > 0 \tag{4}$$

This function allows relaxing the constraints imposed by the quadratic approach. Indeed, finding a Lyapunov matrix for each local model is easier than find a common Lyapunov matrix for all local models. To find the matrices P_i, a convex optimization procedure was proposed by Johansson [6] in the case of nonlinear systems continuously differentiable. Several studies using this type of functions in the continuous case, we can mention the works in [13],[14].

A. Theorem 2 [13]

Assume that $\left|\dot{h}_\rho\right| \leq \varphi_\rho,\ \ \varphi_\rho \geq 0,\ \rho = 1, ..., r$ The T-S fuzzy system is asymptotically stable if there exists symmetric matrices P_i and X satisfying:

$$P_i = P_i^T \geq 0 \tag{5}$$

$$P_i + X \geq 0,\ \ i = 1..r \tag{6}$$

$$\sum_{\rho=1}^{r} \varphi_\rho (P_\rho + X) + \frac{1}{2}\left\{A_i^T P_j + P_j A_i + A_j^T P_i + P_i A_j\right\} < 0, i \leq j \tag{7}$$

B. Theorem 3 [14]

Assume that $\left|\dot{h}_\rho\right| \leq \varphi_\rho,\ \ \varphi_\rho \geq 0,\ \rho = 1, ..., r$, the TS fuzzy system is asymptotically stable if there exists symmetric matrices P_i and X , M_1 and M_2 any matrices satisfying:

$$P_i = P_i^T \geq 0 \tag{8}$$

$$P_i + X > 0,\ \ i = 1..r \tag{9}$$

$$\Xi_i < 0, \quad i = 1..r \tag{10}$$

Where

$$\Xi_i = \begin{bmatrix} P_\phi - M_1 A_i - A_i^T M_1^T & (*)^T \\ P_i - M_2 A_i + M_1^T & M_2 + M_2^T \end{bmatrix}$$

$$P_\phi = \sum_{\rho=1}^{r} \varphi_\rho (P_\rho + X)$$

Another way to overcome the disadvantages of the quadratic approach is to consider the following line integral Lyapunov function [12]:

$$V(x) = 2 \int_{\Gamma(0,x)} f(\psi) \cdot d(\psi) \tag{11}$$

Where $\Gamma(0,x)$ a path from the origin to the current state is x , $\psi \in \mathbf{R}$ is a dummy vector for the integral, $f(x) \in \mathbf{R}^{\mathbf{r}}$ is a vector function of the state x , and $d(\psi)$ is an infinitesimal displacement vector.

Derived Lyapunov function is defined as follows:

$$\dot{V}(x) = \sum_{i=1}^{r} h_i \left\{\dot{x}^T \bar{P}_i x + x^T \bar{P}_i \dot{x}\right\} \tag{12}$$

Notice that $\dot{V}(x)$ does not depend on the time-derivative of the membership functions.

With :

$$\overline{P_i} = D_0 + D_i \tag{13}$$

$$D_0 = \begin{bmatrix} 0 & d_{12} & \cdots & d_{1r} \\ d_{12} & 0 & \cdots & d_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ d_{1r} & d_{2r} & \cdots & 0 \end{bmatrix}, \; D_i = diag\left(d_{jj}^{\alpha_{ij}}\right) \tag{14}$$

$i, j = 1..r$

This notation indicates that the off-diagonal elements of $\bar{P}_i$ are the same.The diagonal elements are altered according to the fuzzy sets in the IF$-$THEN rules. If a premise variable belongs to the same fuzzy set in different rules, then the diagonal elements related to that variable will be equal. We quote [12] and [14], which use this type of functions.

C. Theorem 4 [12]

The T-S fuzzy system is asymptotically stable if there exist $D_0, D_i \;\; and \; X \geq 0$ satisfying

$$\bar{P}_i = D_0 + D_i > 0 \tag{15}$$

$$A_i^T \bar{P}_i + \bar{P}_i A_i + (s-1)X < 0, \quad i = 1..r, 1 < s \leq r \tag{16}$$

$$A_i^T \bar{P}_j + \bar{P}_j A_i + A_j^T \bar{P}_i + \bar{P}_i A_j - 2X \leq 0, \quad i < j \tag{17}$$

D. Theorem 5 [14]

The T-S fuzzy system is asymptotically stable if there exist symmetric matrix $\bar{P}_i$, M_1 and M_2 satisfying

$$\bar{P}_i = \bar{P}_i^T > 0, \; i = 1..r \tag{18}$$

$$\begin{bmatrix} -M_1 A_i - A_i^T M_1^T & (*)^T \\ \bar{P}_i - M_2 A_i + M_1^T & M_2 + M_2^T \end{bmatrix} < 0, \quad i = 1..r \tag{19}$$

V. New Stability Condition for T-S system

In this paragraph, we establish a new stability condition for for T-S systems, which is derived from [13]and [14].The idea is to change the relationship which bound the time-derivative of the membership functions. In fact, we use $\dot{h}_i \leq \varphi_i h_i$ instead of $\dot{h}_i \leq \varphi_i$ [20]. So, we obtain new conditions less conservative than [13] and [14].

A. Theorem 6

Assume that $\dot{h}_i \leq \varphi_i h_i, \; \varphi_i \geq 0, \; i = 1, ..., r,$ T-S fuzzy system is asymptotically stable if there exist symmetric matrices $\bar{P}_i$ and X satisfying:

$$\begin{array}{l} P_i = {P_i}^T \geq 0 \\ P_i + X \geq 0, \;\; i = 1..r \\ \varphi_i (P_i + X) + \frac{1}{2}\left\{A_i^T P_j + P_j A_i + A_j^T P_i + P_i A_j\right\} < 0, \\ \hfill i \leq j \end{array} \tag{20}$$

Proof.

Consider the following Lyapunov function :

$$V(x(t)) = x^T(t)\left(\sum_{i=1}^{n} h_i P_i\right) x(t), \quad P_i \in \mathbf{R^{n \times n}}, \; P_i = P_i^T > 0 \tag{21}$$

So, $\dot{V}$ is written as follows:

$$\dot{V}(x) = \sum_{i=1}^{r} h_i \left\{\dot{x}^T P_i x + x^T P_i \dot{x}\right\} + \sum_{i=1}^{r} \dot{h}_i x^T P_i x \tag{22}$$

We have $\sum\limits_{i=1}^{r} \dot{h}_i = 0$, So there exists a symmetric matrix X which verify

$$\sum_{i=1}^{r} \dot{h}_i X = 0 \tag{23}$$

We insert (23) in (22), we obtain:

$$\dot{V}(x) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j x^T \left\{A_j^T P_i + P_i A_j\right\} x + \sum_{i=1}^{r} \dot{h}_i x^T (P_i + X) x \tag{24}$$

It's also equivalent to :

$$\dot{V}(x) = \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T \left\{A_j^T P_i + P_i A_j + A_i^T P_j + P_j A_i\right\} x + \sum_{i=1}^{r} \dot{h}_i x^T (P_i + X) x \tag{25}$$

We propose to use the following expression to bound $\dot{h}_i$:

$$\dot{h}_i \leq \varphi_i h_i \tag{26}$$

We obtain:

$$\dot{V}(x) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T \left\{\frac{1}{2}\begin{pmatrix} A_j^T P_i + P_i A_j \\ +A_i^T P_j + P_j A_i \end{pmatrix} + \varphi_i (P_i + X)\right\} x \tag{27}$$

End proof.

B. *Theorem 7*

Assume that $\dot{h}_i \leq \varphi_i h_i, \varphi_i \geq 0, \rho = 1, ..., r,$ T-S fuzzy system is asymptotically stable if there exist symmetric matrices $\bar{P}_i$ and X satisfying

$$\begin{array}{c} P_i = P_i^T \geq 0 \\ P_i + X > 0, \ i = 1..r \\ \Xi_i < 0, \quad i = 1..r \end{array} \tag{28}$$

$$\Xi_i = \begin{bmatrix} \varphi_i (P_i + X) - M_1 A_i - A_i^T M_1^T & (*)^T \\ P_i - M_2 A_i + M_1^T & M_2 + M_2^T \end{bmatrix} \tag{29}$$

Proof.

Consider the following Lyapunov function :

$$V(x(t)) = x^T(t)\left(\sum_{i=1}^{n} h_i P_i\right) x(t), \quad P_i \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}, \ P_i = P_i^T > 0 \tag{30}$$

$\dot{V}(x)$ is defined by:

$$\dot{V}(x) = 2\sum_{i=1}^{r} h_i x^T P_i \dot{x} + \sum_{i=1}^{r} \dot{h}_i x^T P_i x \tag{31}$$

Consider the following null term that will serve stability analysis purposes:

$$2\left[x^T(t) M_1 + \dot{x}^T(t) M_2\right] \times \left[\dot{x}(t) - \sum_{i=1}^{r} h_i(z(t)) A_i x(t)\right] = 0 \tag{32}$$

We add this term and (23) to (31), we obtain the following expression:

$$\dot{V}(x) = 2\sum_{i=1}^{r} h_i x^T(t) P_i \dot{x}(t) + x^T \sum_{i=1}^{r} \dot{h}_i (P_i + X) x(t) + 2\left[x^T(t) M_1 + \dot{x}^T(t) M_2\right] \times \left[\dot{x}(t) - \sum_{i=1}^{r} h_i A_i x(t)\right] \tag{33}$$

We use inequality (26). So, the next expression is obtained:

$$\dot{V}(x) \leq 2\sum_{i=1}^{r} h_i x^T(t) P_i \dot{x}(t) + x^T \sum_{i=1}^{r} h_i \varphi_i (P_i + X) x(t) + 2\left[x^T(t) M_1 + \dot{x}^T(t) M_2\right] \times \left[\dot{x}(t) - \sum_{i=1}^{r} h_i A_i x(t)\right] \tag{34}$$

Then,

$$\dot{V}(x) \leq \sum_{i=1}^{r} h_i \left\{\begin{array}{c} 2x^T P_i \dot{x} + x^T \varphi_i (P_i + X) x + 2x^T M_1 \dot{x} \\ -2x^T M_1 A_i x + 2\dot{x}^T M_2 \dot{x} - 2\dot{x}^T M_2 A_i x \end{array}\right\} \tag{35}$$

Let

$$\xi \triangleq \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix}^T \tag{36}$$

Inequality (35) is equivalent to :

$$\dot{V}(x) \leq \sum_{i=1}^{r} h_i \xi^T(t) \Xi_i \xi \tag{37}$$

We get then the conditions given in theorem 7. Thus concludes the proof.

Remark.

Due to the Assumption $\dot{h}_i \leq \varphi_i h_i, \ \varphi_i \geq 0, \ i = 1, ..., r,$ we can obtain more relaxed matrices. In fact, the term (1,1) in matrix Ξ_i contains $\varphi_\rho (P_\rho + X)$ instead of $P_\phi = \sum_{\rho=1}^{r} \varphi_\rho (P_\rho + X)$. In the next section , we will present numerical simulations to show the improvement provided by the proposed method.

VI. STABILITY ANALYSIS

To illustrate the effectiveness of the new non-quadratic stability conditions, that we propose in this paper, we consider the following T-S system [14]

$$
\begin{array}{l}
R_1 : if\, x_1\,(t)\, is\, F_1^1\, and\, x_2\,(t)\;\; is\;\; F_2^1\;\; then\;\; \dot{x}\,(t)\; = A_1\, x\,(t) \\
R_2 : if\, x_2\,(t)\, is\, F_1^1\, and\, x_2\,(t)\;\; is\;\; F_2^2\;\; then\;\; \dot{x}\,(t)\; = A_2\, x\,(t) \\
R_3 : if\, x_1\,(t)\, is\, F_1^2\, and\, x_2\,(t)\;\; is\;\; F_2^1\;\; then\;\; \dot{x}\,(t)\; = A_3\, x\,(t) \\
R_4 : if\, x_2\,(t)\, is\, F_1^2\, and\, x_2\,(t)\;\; is\;\; F_2^2\;\; then\;\; \dot{x}\,(t)\; = A_4\, x\,(t)
\end{array}
$$

with

$$
A_1 = \begin{bmatrix} -5 & -4 \\ -1 & a \end{bmatrix},\; A_2 = \begin{bmatrix} -4 & -4 \\ \frac{1}{5}\,(3b-2) & \frac{1}{5}\,(3a-4) \end{bmatrix}
$$

$$
A_3 = \begin{bmatrix} -3 & -4 \\ \frac{1}{5}\,(2b-3) & \frac{1}{5}\,(2a-6) \end{bmatrix},\; A_4 = \begin{bmatrix} -2 & -4 \\ b & -2 \end{bmatrix}
$$

Fig. 1. Stable region provided by quadratic approaches [2],[4] and [5].

Figure (1) shows that feasibility domain is very restrictive and the various quadratic approaches give the same domain even with relaxation. So, those methods are very conservative for stability analysis of T-S system.

Fig. 2. Stable region provided by non quadratic Approaches and integral Lyapunov function theorem 4 (blue circle) and theorem 5 (red circle)

In figure (2) we present the stability domain obtained by non quadratic approaches using integral Lyapunov function. As shown figure (2) that theorem 5 gives a domain of feasibility larger than theorem 4. In fact, the LMIs of theorem 7 contain slack matrices which decrease the conservatism. Consequently, the stability domain is extended.

Fig. 3. Stable region obtained with theorem 2 (blue $+$), theorem 3 (black circle) and new Approaches: theorem 6 (blue tile) theorem 7 (red circle)

In comparison with figures 1 and 2 we note that stability domain is extended ($b \in [0, 1200]$ instead of $[0, 200]$). As shown figure (3), feasibility domain obtained by Theorem 6 is larger than theorems 2 and 3. Moreover, the stable region indicated by theorem 7 includes all approaches. In fact, the new relationship which bound the time-derivative of the membership functions and slack matrices introduced in theorem 7 involve less conservatism. Also, we note that the stability region presented by theorems 6 and 7 include the region obtained by theorem 4 and 5. In fact, the line-integral Lyapunov function assumes that the off-diagonal elements of the Lyapunov matrices $\bar{P}_i$ should be same, and this strong restriction could also introduce some conservatism. So, we conclude that theorem 7 gives the largest stability domain as shown figure (3) .

VII. CONCLUSION

In this paper, we have derived a new LMI stability conditions based on fuzzy Lyapunov function for T-S fuzzy systems. To do this, we propose another upper bounds on the first time-derivative of the membership functions. Numerical examples have been demonstrated the advantage and effectiveness of the proposed methods compared with the previous approaches. For this reason, we will extend them to design control in future work.

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