مدل دینامیکی سیستم پیل سوختی مرتبه پنجم غیرخطی با در نظر گرفتن نامعینی و اغتشاش به‌صورت زیر در نظر می‌گیریم:

$$\begin{aligned}
\dot{x}_1(t) &= \frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \frac{x_1(t)}{x_1(t)+x_2(t)})k_a u_{a_1}(t) + \frac{RTC_1(1-\gamma)}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)} - 1)I(t) + \Delta f_1(x) + d_1 \\
\dot{x}_2(t) &= \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})k_a u_{a2}(t) + \frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)} - 1)I(t) + \Delta f_2(x) + d_2 \\
\dot{x}_3(t) &= \frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_3}(t) + \frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)} - 1)I(t) + \Delta f_3(x) + d_3 \\
\dot{x}_4(t) &= \frac{RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_4}(t) + \Delta f_4(x) + d_4 \\
\dot{x}_5(t) &= \frac{RT\lambda_{air}}{V_C}(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}} - \frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_5}(t) + \frac{RTC_1(1-\gamma)}{V_C} \\
&\times(\frac{C_2}{C_1}(1 - \frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}) - 1 - \frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)})I(t) + \Delta f_5(x) + d_5
\end{aligned} \qquad (1)$$

خروجی سیستم به‌صورت زیر است که فشار هیدروژن و اکسیژن در دو سمت آند و کاتد باید کنترل شود.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} = \begin{bmatrix} P_{H_2}(t) \\ P_{H_2O_A}(t) \\ P_{O_2}(t) \\ P_{N_2}(t) \\ P_{H_2O_C}(t) \end{bmatrix} \qquad (2)$$

در سیستم مدل (۱)، R ثابت گاز، T دما، V_A, V_C مقدار ولتاژ آند و کاتد، C_1, C_2اعداد ثابت و معلوم، φ_a, φ_c ثابت رطوبت، P_{VS} فشار اشباع، $\lambda_{H_2}, \lambda_{air}$ ثابت‌های استوکیومتری، I(t) جریان پیل سوختی که به‌عنوان اغتشاش در نظر گرفته می‌شود چون به‌صورت $\frac{V_{fc}}{R_L}$ تغییر می‌کند که V_{fc} ولتاژ تولیدی پیل سوختی و R_L مجموع مقاومت‌های بارهای فعال هست. γ هم طبق مرجع [19] برابر صفر در نظر گرفته می‌شود.

نامعینی و اغتشاش که در نظر گرفتیم کران‌دار بوده و کران آن نامعلوم در نظر گرفته‌شده است.

$$\begin{cases} \Delta f_i < \alpha_i \\ d_i < \beta_i \end{cases} \qquad i = 1, 2, \ldots, 5 \qquad (3)$$

که α_i و β_i کران نامعینی و اغتشاش بوده و نامعلوم فرض شده‌اند.

.طراحی مود لغزشی ترمینال با در نظر گرفتن نامعینی و اغتشاش:

هدف طراحی مود لغزشی زمان محدود است به‌طوری‌که خروجی y(t) یک مقدار ثابت $y_{desired}$ را ردیابی کند.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} = \begin{bmatrix} P_{H_2}(t) \\ P_{H_2O_A}(t) \\ P_{O_2}(t) \\ P_{N_2}(t) \\ P_{H_2O_C}(t) \end{bmatrix} \qquad (4)$$

$e_i(t)$ خطای ردیابی را نشان می‌دهد که به‌صورت زیر تعریف می‌کنیم:

$$e_i(t) = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = y_i(t) - y_{desired} \qquad (5)$$

سطح لغزش را به‌صورت زیر تعریف می‌کنیم:

$$s_i = \xi e_i + \int \operatorname{sgn}(e_i)\left|e_i\right|^{\alpha} \qquad i = 1, 2, \ldots, 5 \qquad (6)$$

$$\dot{s} = \begin{bmatrix} \dot{s}_1(t) \\ \dot{s}_2(t) \\ \dot{s}_3(t) \\ \dot{s}_4(t) \\ \dot{s}_5(t) \end{bmatrix} = \zeta \dot{e}_i + \operatorname{sgn}(e_i)\left|e_i\right|^{\alpha} = \zeta \dot{y}_i + \operatorname{sgn}(e_i)\left|e_i\right|^{\alpha} = \begin{bmatrix} \zeta \dot{x}_1(t) + \operatorname{sgn}(e_1)\left|e_1\right|^{\alpha} \\ \zeta \dot{x}_2(t) + \operatorname{sgn}(e_2)\left|e_2\right|^{\alpha} \\ \zeta \dot{x}_3(t) + \operatorname{sgn}(e_3)\left|e_3\right|^{\alpha} \\ \zeta \dot{x}_4(t) + \operatorname{sgn}(e_4)\left|e_4\right|^{\alpha} \\ \zeta \dot{x}_5(t) + \operatorname{sgn}(e_5)\left|e_5\right|^{\alpha} \end{bmatrix} =$$

$$\begin{bmatrix} \zeta\left[\dfrac{RT\lambda_{H_2}}{V_A}\left(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)}\right)k_a u_{a_1}(t) + \dfrac{RTC_1(1-\gamma)}{V_A}\left(\dfrac{x_1(t)}{x_1(t)+x_2(t)} - 1\right)I(t) + \Delta f_1(x) + d_1\right] + \operatorname{sgn}(e_1)\left|e_1\right|^{\alpha} \\ \zeta\left[\dfrac{RT\lambda_{H_2}}{V_A}\left(\dfrac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \dfrac{x_2(t)}{x_1(t)+x_2(t)}\right)k_a u_{a2}(t) + \dfrac{RTC_1}{V_A}\left(\dfrac{x_2(t)}{x_1(t)+x_2(t)} - 1\right)I(t) + \Delta f_2(x) + d_2\right] \\ + \operatorname{sgn}(e_2)\left|e_2\right|^{\alpha} \\ \\ \end{bmatrix} = 0$$

$$\begin{bmatrix} \zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_3}(t)+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f_3(x)+d_3\right]+\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha} \\ \zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{N_2}-\frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_4}(t)+\Delta f_4(x)+d_4\right]+\mathrm{sgn}(e_4)\left|e_4\right|^{\alpha} \\ \zeta\left[\frac{RT\lambda_{air}}{V_C}(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_5}(t)+\frac{RTC_1(1-\gamma)}{V_C}\right. \\ \left.\times(\frac{C_2}{C_1}(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)})-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)})I(t)+\Delta f_5(x)+d_5\right]+\mathrm{sgn}(e_5)\left|e_5\right|^{\alpha} \end{bmatrix}=0$$

γ را طبق مرجع ۱۹ مقاله اصلی برابر صفر در نظر می‌گیریم:

$$\zeta\left[\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a u_a(t)+\frac{RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_1(x)+d_1\right]+\mathrm{sgn}(e_1)\left|e_1\right|^{\alpha}=0$$

$$\zeta\left[\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a u_a(t)+\frac{RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_1(x)+d_1\right]=-\mathrm{sgn}(e_1)\left|e_1\right|^{\alpha}$$

$$\left[\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a u_a(t)+\frac{RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_1(x)+d_1\right]=-\frac{1}{\zeta}\mathrm{sgn}(e_1)\left|e_1\right|^{\alpha}$$

$$\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a u_a(t)=-\frac{RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)-\frac{1}{\zeta}\mathrm{sgn}(e)\left|e\right|^{\alpha}-\Delta f_1(x)-d_1$$

$$u_{a_1}=\frac{\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)})I(t)-\frac{1}{\zeta}\mathrm{sgn}(e_1)\left|e_1\right|^{\alpha}-\Delta f_1(x)-d_1}{\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a}$$

در اینجا چون نمی‌توانیم از d و $\Delta f(x)$ در U (ورودی کنترل) استفاده کنیم، لذا از تقریب حد بالای کران آن‌ها استفاده کردیم.

$$u_{a_1}=\frac{\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)})I(t)}{\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a}-\frac{\frac{1}{\zeta}\mathrm{sgn}(e_1)\left|e_1\right|^{\alpha}}{\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a}-\frac{(\hat{\alpha}_1+\hat{\beta}_1)\mathrm{sgn}(s_1)}{\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a}$$

$$u_{a_1}=\frac{(\frac{RTC_1x_2(t)}{V_A(x_1(t)+x_2(t))})I(t)}{RT\lambda_{H_2}(\frac{\gamma_{H_2}(x_1(t)+x_2(t))-x_1(t)}{V_A(x_1(t)+x_2(t))})k_a}-\frac{\frac{1}{\zeta}\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}}{\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a}-\frac{(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)}{\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a}$$

$$u_{a_1}=\frac{C_1x_2(t)I(t)}{k_a\lambda_{H_2}(\gamma_{H_2}(x_1(t)+x_2(t))-x_1(t))}-\frac{\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}}{\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a\zeta}-\frac{(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)}{\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a} \qquad (٧)$$

$$\dot{s}_2(t)=\zeta\dot{e}_2+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}=\zeta\dot{y}_2+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}=\zeta\dot{x}_2(t)+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}=0$$

$$\zeta\left[\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_aP_{VS}}{x_1(t)+x_2(t)-\varphi_aP_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_au_{a2}(t)+\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_2(x)+d_2\right]$$
$$+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}=0$$

$$\zeta\left[\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_aP_{VS}}{x_1(t)+x_2(t)-\varphi_aP_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_au_{a2}(t)+\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_2(x)+d_2\right]$$
$$=-\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_aP_{VS}}{x_1(t)+x_2(t)-\varphi_aP_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_au_{a2}(t)+\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_2(x)+d_2$$
$$=-\frac{1}{\zeta}\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_aP_{VS}}{x_1(t)+x_2(t)-\varphi_aP_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_au_{a2}(t)=-\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)$$
$$-\Delta f_2(x)-d_2-\frac{1}{\zeta}\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$u_{a2}(t)=\frac{-\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)-\Delta f_2(x)-d_2-\frac{1}{\zeta}\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_aP_{VS}}{x_1(t)+x_2(t)-\varphi_aP_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$

$$u_{a2}(t)=\frac{-\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\frac{\frac{1}{\zeta}\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$
$$-\frac{\Delta f_2(x)+d_2}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$

$$u_{a2}(t)=\frac{-C_1(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)}{\lambda_{H_2}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\frac{\frac{1}{\zeta}\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$
$$-\frac{\Delta f_2(x)+d_2}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$

$$u_{a2}(t)=\frac{C_1(\frac{x_1(t)}{x_1(t)+x_2(t)})I(t)}{\lambda_{H_2}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\frac{\frac{1}{\zeta}\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$
$$-\frac{\Delta f_2(x)+d_2}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$

از تقریب کران بالای d_2 و $\Delta f_2(x)$ استفاده می‌کنیم، داریم:

$$u_{a2}(t)=\frac{C_1(\frac{x_1(t)}{x_1(t)+x_2(t)})I(t)}{\lambda_{H_2}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\frac{\frac{1}{\zeta}\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$
$$-\frac{(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2)}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}$$

$$\zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_3}(t)+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f_3(x)+d_3\right]+\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}=0$$

$$\zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_3}(t)+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f_3(x)+d_3\right]=-\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}$$

$$\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_3}(t)+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f_3(x)+d_3=-\frac{1}{\zeta}\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}$$

$$\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_3}(t)=-\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)-\frac{1}{\zeta}\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}-\Delta f_3(x)-d_3$$

$$u_{c_3}=\frac{-\frac{1}{\zeta}\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c}+\frac{\frac{RTC_1(x_4(t)+x_5(t))I(t)}{2V_C(x_3(t)+x_4(t)+x_5(t))}}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c}$$

$$-\frac{\Delta f_3(x)+d_3}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k}$$

$$u_{c_3}=\frac{-\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c}+\frac{C_1(x_4(t)+x_5(t))I(t)}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))}$$

$$-\frac{\Delta f_3(x)+d_3}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k}$$

$$u_{c_3}=\frac{-\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}}{k_c\zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\right]}+\frac{C_1(x_4(t)+x_5(t))I(t)}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))}$$

$$-\frac{\Delta f_3(x)+d_3}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k}$$

$$u_{c_3}=\frac{-\operatorname{sgn}(e_3)\left|e_3\right|^{\alpha}}{k_c\zeta\left[\frac{RT\lambda_{air}}{V_C}\left(\frac{\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t))-x_3(t)}{x_3(t)+x_4(t)+x_5(t)}\right)\right]}+\frac{C_1(x_4(t)+x_5(t))I(t)}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))}$$

$$-\frac{\Delta f_3(x)+d_3}{\frac{RT\lambda_{air}}{V_C}\left(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k}$$

در این مرحله هم طبق مرحله قبل از تقریب کران بالای d و $\Delta f(x)$ استفاده می‌کنیم.

$$u_{c_3}=\frac{-V_C(x_3(t)+x_4(t)+x_5(t))\operatorname{sgn}(e)\left|e\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t))-x_3(t))\right]}+\frac{C_1(x_4(t)+x_5(t))I(t)}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))}$$

$$-\frac{(\hat{\alpha}_3+\hat{\beta}_3)\operatorname{sgn}(s_3)}{\frac{RT\lambda_{air}}{V_C}\left(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k} \qquad (۸)$$

$$\zeta\left[\frac{RT\lambda_{air}}{V_C}\left(\gamma_{N_2}-\frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c u_{c_4}(t)+\Delta f_4(x)+d_4\right]+\operatorname{sgn}(e_4)\left|e_4\right|^{\alpha}=0$$

$$\zeta\left[\frac{RT\lambda_{air}}{V_C}\left(\gamma_{N_2}-\frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c u_{c_4}(t)+\Delta f_4(x)+d_4\right]=-\operatorname{sgn}(e_4)\left|e_4\right|^{\alpha}$$

$$\frac{RT\lambda_{air}}{V_C}\left(\gamma_{N_2}-\frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c u_{c_4}(t)+\Delta f_4(x)+d_4=-\frac{1}{\zeta}\operatorname{sgn}(e_4)\left|e_4\right|^{\alpha}$$

$$u_{c_4}=\frac{-\frac{1}{\zeta}\operatorname{sgn}(e_4)\left|e_4\right|^{\alpha}}{\frac{RT\lambda_{air}}{V_C}\left(\gamma_{N_2}-\frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}-\frac{\Delta f_4(x)+d_4}{\frac{RT\lambda_{air}}{V_C}\left(\gamma_{N_2}-\frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

در این مرحله هم طبق مرحله قبل از تقریب کران بالای d و $\Delta f_4(x)$ استفاده می‌کنیم.

$$u_{c_4}=-\frac{\operatorname{sgn}(e_4)\left|e_4\right|^{\alpha}}{\frac{\zeta RT\lambda_{air}}{V_C}\left(\gamma_{N_2}-\frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}-\frac{(\hat{\alpha}_4+\hat{\beta}_4)\operatorname{sgn}(s_4)}{\frac{RT\lambda_{air}}{V_C}\left(\gamma_{N_2}-\frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c} \qquad (۹)$$

$$\zeta\left[\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c u_{c_5}(t)+\frac{RTC_1(1-\gamma)}{V_C}\right.$$
$$\left.\times\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)+\Delta f_5(x)+d_5\right]+\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}=0$$

$$\zeta\left[\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c u_{c_5}(t)+\frac{RTC_1}{V_C}\right.$$
$$\left.\times\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)+\Delta f_5(x)+d_5\right]=-\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}$$

$$\left[\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c u_{c_5}(t)+\frac{RTC_1}{V_C}\right.$$
$$\left.\times\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)+\Delta f_5(x)+d_5\right]=-\frac{1}{\zeta}\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}$$

$$\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c u_{c_5}(t)=-\frac{1}{\zeta}\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}$$
$$-\frac{RTC_1}{V_C}\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)-\Delta f_5(x)-d_5$$

$$u_{c_5}=\frac{-\frac{1}{\zeta}\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}}{\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$+\frac{-\frac{RTC_1}{V_C}\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)-\Delta f_5(x)-d_5}{\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

در این مرحله هم طبق مرحله قبل از تقریب کران بالای d و $\Delta f_4(x)$ استفاده می‌کنیم.

$$u_{c_5}=\frac{-\frac{1}{\zeta}\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}}{\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$-\frac{\frac{RTC_1}{V_C}\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)}{\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$-\frac{\Delta f_5(x)+d_5}{\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$u_{c_5}=-\frac{\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}}{\frac{\zeta RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$-\frac{C_1\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)}{\lambda_{air}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$-\frac{\Delta f_5(x)+d_5}{\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

در این مرحله هم طبق مرحله قبل از تقریب کران بالای d و $\Delta f_5(x)$ استفاده می‌کنیم.

$$u_{c_5}=-\frac{\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}}{\frac{\zeta RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$-\frac{C_1\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)}{\lambda_{air}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$-\frac{(\hat{\alpha}_5+\hat{\beta}_5)\operatorname{sgn}(s_5)}{\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

باید روابط زیر را هم در نظر بگیریم:

$$\gamma_{H_2} - \frac{x_1(t)}{x_1(t)+x_2(t)} > 0$$

(۱۰)

$$\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)} > 0$$

تغییرات جریان را هم محدود در نظر می‌گیریم:

$$0 < I_{min} < I(t) < I_{max}$$

$I(t) \quad I_{min} > 0$ and $I_{max} \quad I(t) > 0.$

$$s_i s_i < 0, \;\; i = 1,2,3,4,5 \qquad (۱۱)$$

درنهایت ورودی کنترلی را بدون در نظر گرفتن ورودی اشباع به‌صورت زیر طراحی می‌کنیم:

$$u_{a_1} = \begin{cases} \dfrac{C_1 x_2(t) I_{\min}}{k_a \lambda_{H_2}(\gamma_{H_2}(x_1(t)+x_2(t)) - x_1(t))} - \dfrac{\operatorname{sgn}(e)|e|^{\alpha}}{\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)})k_a \zeta} \\ - \dfrac{(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_1)}{\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)})k_a} - \delta_1 sat(s_1) \\ \dfrac{C_1 x_2(t) I_{\max}}{k_a \lambda_{H_2}(\gamma_{H_2}(x_1(t)+x_2(t)) - x_1(t))} - \dfrac{\operatorname{sgn}(e)|e|^{\alpha}}{\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)})k_a \zeta} \\ - \dfrac{(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_1)}{\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)})k_a} - \delta_2 sat(s_1) \end{cases} \qquad (۱۲)$$

$$u_{a_2}=\begin{cases}\dfrac{C_1(\dfrac{x_1(t)}{x_1(t)+x_2(t)})I_{\min}}{\lambda_{H_2}(\dfrac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\dfrac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\dfrac{\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}}{\dfrac{\zeta RT\lambda_{H_2}}{V_A}(\dfrac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\dfrac{x_2(t)}{x_1(t)+x_2(t)})k_a}\\ -\dfrac{(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2)}{\dfrac{RT\lambda_{H_2}}{V_A}(\dfrac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\dfrac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\delta_2 sat(s_2)\\ \dfrac{C_1(\dfrac{x_1(t)}{x_1(t)+x_2(t)})\mathrm{I}_{\max}}{\lambda_{H_2}(\dfrac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\dfrac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\dfrac{\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}}{\dfrac{\zeta RT\lambda_{H_2}}{V_A}(\dfrac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\dfrac{x_2(t)}{x_1(t)+x_2(t)})k_a}\\ -\dfrac{(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2)}{\dfrac{RT\lambda_{H_2}}{V_A}(\dfrac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\dfrac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\delta_2 sat(s_2)\end{cases} \tag{۱۳}$$

$$u_{c_3}=\begin{cases}\dfrac{C_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))}-\dfrac{V_C(x_3(t)+x_4(t)+x_5(t))\operatorname{sgn}(e)\left|e\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t))-x_3(t))\right]}\\ -\dfrac{(\hat{\alpha}_3+\hat{\beta}_3)\operatorname{sgn}(s_3)}{\dfrac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\dfrac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k}-\delta_3 sat(s_3)\\ \dfrac{C_1(x_4(t)+x_5(t))I_{\max}}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))}-\dfrac{V_C(x_3(t)+x_4(t)+x_5(t))\operatorname{sgn}(e)\left|e\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t))-x_3(t))\right]}\\ -\dfrac{(\hat{\alpha}_3+\hat{\beta}_3)\operatorname{sgn}(s_2)}{\dfrac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\dfrac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k}-\delta_3 sat(s_3)\end{cases} \tag{۱۴}$$

$$u_{c_4}=-\dfrac{\operatorname{sgn}(e_4)\left|e_4\right|^{\alpha}}{\dfrac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2}-\dfrac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c}-\dfrac{(\hat{\alpha}_4+\hat{\beta}_4)\operatorname{sgn}(s_4)}{\dfrac{RT\lambda_{air}}{V_C}(\gamma_{N_2}-\dfrac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c}-\delta_4 sat(s_4) \tag{۱۵}$$

$$u_{c_5}=\begin{cases} -\dfrac{\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}}{\dfrac{\zeta RT\lambda_{air}}{V_C}\left(\dfrac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c} \\ -\dfrac{C_1\left(\dfrac{C_2}{C_1}\left(1-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I_{\min}}{\lambda_{air}\left(\dfrac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c} \\ -\dfrac{(\hat{\alpha}_5+\hat{\beta}_5)\operatorname{sgn}(s_5)}{\dfrac{RT\lambda_{air}}{V_C}\left(\dfrac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}-\delta_5 sat(s_5) \\ \\ -\dfrac{\operatorname{sgn}(e_5)\left|e_5\right|^{\alpha}}{\dfrac{\zeta RT\lambda_{air}}{V_C}\left(\dfrac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c} \\ -\dfrac{C_1\left(\dfrac{C_2}{C_1}\left(1-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I_{\max}}{\lambda_{air}\left(\dfrac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c} \\ -\dfrac{(\hat{\alpha}_5+\hat{\beta}_5)\operatorname{sgn}(s_5)}{\dfrac{RT\lambda_{air}}{V_C}\left(\dfrac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\dfrac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}-\delta_5 sat(s_5) \end{cases} \tag{١٦}$$

$$\dot{s}_1(t)=\zeta\dot{e}_1+\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}=\zeta\dot{x}_1+\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}$$

$$=\zeta\left[\frac{RT\lambda_{H_2}}{V_A}\left(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)}\right)k_a u_{a_1}(t)+\frac{RTC_1}{V_A}\left(\frac{x_1(t)}{x_1(t)+x_2(t)}-1\right)I(t)+\Delta f(x)+d\right]+\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}$$

$$=\zeta\left(\frac{RT\lambda_{H_2}}{V_A}\left(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)}\right)k_a\right)\left[\frac{C_1x_2(t)I_{\min}}{k_a\lambda_{H_2}(\gamma_{H_2}(x_1(t)+x_2(t))-x_1(t))}-\frac{\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}}{\dfrac{RT\lambda_{H_2}}{V_A}\left(\gamma_{H_2}-\dfrac{x_1(t)}{x_1(t)+x_2(t)}\right)k_a\zeta}\right.$$

$$\left.-\frac{(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)}{\dfrac{RT\lambda_{H_2}}{V_A}\left(\gamma_{H_2}-\dfrac{x_1(t)}{x_1(t)+x_2(t)}\right)k_a}-\delta_1 sat(s_1)\right]+\zeta\left[\frac{RTC_1}{V_A}\left(\frac{x_1(t)}{x_1(t)+x_2(t)}-1\right)I(t)+\Delta f_1(x)+d_1\right]+\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}$$

$$=\zeta\frac{RT}{V_A}(\frac{\gamma_{H_2}(x_1(t)+x_2(t))-x_1(t)}{x_1(t)+x_2(t)})(\frac{C_1x_2(t)I_{\min}}{(\gamma_{H_2}(x_1(t)+x_2(t))-x_1(t))})-\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}$$
$$-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)-\zeta(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)$$
$$\zeta\left[\frac{RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_1(x)+d_1\right]+\operatorname{sgn}(e_1)\left|e_1\right|^{\alpha}$$

$$=\zeta\frac{RT}{V_A}(\frac{C_1x_2(t)I_{\min}}{x_1(t)+x_2(t)})+\zeta\left[\frac{RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_1(x)+d_1\right]-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)$$
$$-\zeta(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)$$

$$=\zeta\frac{RT}{V_A}(\frac{C_1x_2(t)I_{\min}}{x_1(t)+x_2(t)})+\zeta\left[\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)})I(t)\right]-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)$$
$$-\zeta(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta\Delta f_1(x)+\zeta d_1$$

$$=\frac{\zeta RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)$$
$$-\zeta(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta\Delta f_1(x)+\zeta d_1$$

$$=\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)$$
$$-\zeta(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta\Delta f_1(x)+\zeta d_1 \tag{١٧}$$

$$\dot{s}_1=\begin{cases}\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t))-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\\ -\zeta(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta\Delta f_1(x)+\zeta d_1<0 \qquad if \quad s_1>0\\ \frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\max}-I(t))-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_2 sat(s_1)\\ -\zeta(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta\Delta f_1(x)+\zeta d_1>0 \qquad if \quad s_1<0\end{cases} \tag{١٨}$$

تابع لیاپانف را به‌صورت زیر در نظر می‌گیریم:

$$V_1=\frac{1}{2}S_1{}^2+\frac{1}{2}(\hat{\alpha}_1-\alpha_1)+\frac{1}{2}(\hat{\beta}_1-\beta_1)$$

$$\dot{V}_1=s_1\dot{s}_1+(\hat{\alpha}_1-\alpha_1)\dot{\hat{\alpha}}_1+(\hat{\beta}_1-\beta_1)\dot{\hat{\beta}}_1$$

از رابطه (۱۷) جایگذاری می‌کنیم:

$$\dot{V}_1=s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right.$$
$$\left.-\zeta(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta\Delta f_1(x)+\zeta d_1\right]+(\hat{\alpha}_1-\alpha_1)\dot{\hat{\alpha}}_1+(\hat{\beta}_1-\beta_1)\dot{\hat{\beta}}_1$$

$$\dot{V}_1=s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta s(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta s\Delta f_1(x)+\zeta sd_1+(\hat{\alpha}_1-\alpha_1)\dot{\hat{\alpha}}_1+(\hat{\beta}_1-\beta_1)\dot{\hat{\beta}}_1$$

قوانین تطبیقی را به‌صورت زیر تعریف می‌کنیم:

$$\dot{\hat{\alpha}}_i=\zeta|s_i| \qquad i=1,2,...,5$$
$$\dot{\hat{\beta}}_i=\zeta|s_i| \qquad i=1,2,...,5 \tag{۱۹}$$

با جایگذاری قوانین تطبیقی داریم:

$$\dot{V}_1=s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta s_1(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta s_1\Delta f_1(x)+\zeta s_1 d_1+\zeta|s_1|(\hat{\alpha}_1-\alpha_1)+\zeta|s_1|(\hat{\beta}_1-\beta_1)$$

$$\dot{V}_1\le s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta s_1(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta|s_1|\alpha_1+\zeta|s_1|\beta_1+\zeta|s_1|\hat{\alpha}_1-\zeta|s_1|\alpha_1+\zeta|s_1|\hat{\beta}_1-\zeta|s_1|\beta_1$$

$$\dot{V}_1 \le s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta s_1(\hat{\alpha}_1+\hat{\beta}_1)\operatorname{sgn}(s_1)+\zeta\left|s_1\right|\hat{\alpha}_1+\zeta\left|s_1\right|\hat{\beta}_1$$

با توجه به رابطه روبه‌رو می‌توان نوشت:

$$\operatorname{sgn}(s)=\frac{\left|s_i\right|}{s_i} \qquad i=1,2,...,5$$

$$\dot{V}_1 \le s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta\left|s_1\right|(\hat{\alpha}_1+\hat{\beta}_1)+\zeta\left|s_1\right|\hat{\alpha}_1+\zeta\left|s_1\right|\hat{\beta}_1$$

$$\dot{V}_1 \le s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta\left|s_1\right|\hat{\alpha}_1-\zeta\left|s_1\right|\hat{\beta}_1+\zeta\left|s_1\right|\hat{\alpha}_1+\zeta\left|s_1\right|\hat{\beta}_1$$

درنهایت می‌توان نوشت:

$$\dot{V}_1 \le s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$

که با توجه به رابطه (۱۵)، می‌توان نوشت:

$$\dot{s}_1=\begin{cases}\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t))-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)<0 & if \quad s_1>0\\ \frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\max}-I(t))-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_2 sat(s_1)>0 & if \quad s_1<0\end{cases}$$

$$s_1\dot{s}_1=\begin{cases} s_1\left[\dfrac{\zeta RTC_1}{V_A}(1-\dfrac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t))-\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\dfrac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]<0 & if \quad s_1>0 \\ s_1\left[\dfrac{\zeta RTC_1}{V_A}(1-\dfrac{x_1(t)}{x_1(t)+x_2(t)})(I_{\max}-I(t))-\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\dfrac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_2 sat(s_1)\right]>0 & if \quad s_1<0 \end{cases}$$

که در هر حالت $s_1\dot{s}_1$ یک عدد منفی بوده و $\dot{V}\le s_1\dot{s}_1$ می‌باشد.

برای حالت دوم نیز داریم:

$$\dot{s}_2(t)=\zeta\dot{e}_2+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}=\zeta\dot{x}_2+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$=\zeta\left[\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a u_{a2}(t)+\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_2(x)+d_2\right]+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$=\zeta(\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a)\left[\frac{C_1(\frac{x_1(t)}{x_1(t)+x_2(t)})I(t)}{\lambda_{H_2}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}\right.$$

$$\left.-\frac{\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}}{\frac{\zeta RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\frac{(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2)}{\frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a}-\delta_2 sat(s_2)\right]$$

$$+\zeta\left[\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_2(x)+d_2\right]+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$=\zeta\frac{RT}{V_A}(C_1(\frac{x_1(t)}{x_1(t)+x_2(t)})I(t))-\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}$$
$$-\zeta(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2)-\delta_2 sat(s_2)(\frac{\zeta RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a)$$
$$+\zeta\left[\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_2(x)+d_2\right]+\operatorname{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$=\zeta\frac{RT}{V_A}(C_1(\frac{x_1(t)}{x_1(t)+x_2(t)})I(t))-(\frac{\zeta RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}}-\frac{x_2(t)}{x_1(t)+x_2(t)})k_a)\delta_2 sat(s_2)$$
$$-\zeta(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2)+\zeta\left[\frac{RTC_1}{V_A}(\frac{x_2(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f_2(x)+d_2\right]$$

$$
\begin{aligned}
&= \zeta \frac{RT}{V_A}(C_1(\frac{x_1(t)}{x_1(t)+x_2(t)})I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2) \\
&-\zeta(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta \Delta f_2(x) + \zeta d_2 - \zeta\left[\frac{RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})I(t)\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t)) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2) \\
&-\zeta(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta \Delta f_2(x) + \zeta d_2
\end{aligned} \qquad (۲۰)
$$

$$
\dot{s}_2 = \begin{cases}
\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t)) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2) \\
-\zeta(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta \Delta f_2(x) + \zeta d_2 < 0 \qquad if \quad s_2 > 0 \\
\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t)) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2) \\
-\zeta(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta \Delta f_2(x) + \zeta d_2 > 0 \qquad if \quad s_2 < 0
\end{cases} \qquad (۲۱)
$$

تابع لیاپانف را به‌صورت زیر در نظر می‌گیریم:

$$
V_2 = \frac{1}{2}S_2{}^2 + \frac{1}{2}(\hat{\alpha}_2 - \alpha_2) + \frac{1}{2}(\hat{\beta}_2 - \beta_2)
$$

$$
\dot{V}_2 = s_2\dot{s}_2 + (\hat{\alpha}_2 - \alpha\ )\dot{\hat{\alpha}}_2 + (\hat{\beta}_2 - \beta_2)\dot{\hat{\beta}}_2
$$

از رابطه (۲۰) جایگذاری می‌کنیم:

$$
\begin{aligned}
\dot{V}_2 = s_2 \Bigg[&\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2) \\
&-\zeta(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta \Delta f_2(x) + \zeta d_2 \Bigg] + (\hat{\alpha}_2 - \alpha_2)\dot{\hat{\alpha}}_2 + (\hat{\beta}_2 - \beta_2)\dot{\hat{\beta}}_2
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_2 &= s_2 \left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2) \right] \\
&-\zeta s_2(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta s \Delta f_2(x) + \zeta s d_2 + (\hat{\alpha}_2 - \alpha_2)\dot{\hat{\alpha}}_2 + (\hat{\beta}_2 - \beta_2)\dot{\hat{\beta}}_2
\end{aligned}
$$

قوانین تطبیقی را به‌صورت زیر تعریف می‌کنیم:

$$\dot{\hat{\alpha}}_i = \zeta \left| s_i \right| \qquad i = 1,2,...,5$$
$$\dot{\hat{\beta}}_i = \zeta \left| s_i \right| \qquad i = 1,2,...,5 \tag{۲۱}$$

با جایگذاری قوانین تطبیقی داریم:

$$\dot{V}_2 = s_2\left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2)\right]$$
$$-\zeta s_2(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta s_2 \Delta f_2(x) + \zeta s_2 d_2 + \zeta\left|s_2\right|(\hat{\alpha}_2-\alpha_2) + \zeta\left|s_2\right|(\hat{\beta}_2-\beta_2)$$

$$\dot{V}_2 \le s_2\left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2)\right]$$
$$-\zeta s_2(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta\left|s_2\right|\alpha_2 + \zeta\left|s_2\right|\beta_2 + \zeta\left|s_2\right|\hat{\alpha}_2 - \zeta\left|s_2\right|\alpha_2 + \zeta\left|s_2\right|\hat{\beta}_2 - \zeta\left|s_2\right|\beta_2$$

$$\dot{V}_2 \le s_2\left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2)\right]$$
$$-\zeta s_2(\hat{\alpha}_2+\hat{\beta}_2)\operatorname{sgn}(s_2) + \zeta\left|s_2\right|\hat{\alpha}_2 + \zeta\left|s_2\right|\hat{\beta}_2$$

با توجه به رابطه روبه‌رو می‌توان نوشت:

$$\operatorname{sgn}(s) = \frac{\left|s_i\right|}{s_i} \qquad i = 1,2,...,5$$

$$\dot{V}_2 \le s_2\left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2)\right]$$
$$-\zeta\left|s_2\right|(\hat{\alpha}_2+\hat{\beta}_2) + \zeta\left|s_2\right|\hat{\alpha}_2 + \zeta\left|s_2\right|\hat{\beta}_2$$

$$\dot{V}_2 \le s_2\left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2)\right]$$
$$-\zeta\left|s_2\right|\hat{\alpha}_2 - \zeta\left|s_2\right|\hat{\beta}_2 + \zeta\left|s_2\right|\hat{\alpha}_2 + \zeta\left|s_2\right|\hat{\beta}_2$$

درنهایت می‌توان نوشت:

$$\dot{V}_2 \le s_2\left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2)\right]$$

که با توجه به رابطه (۱۵)، می‌توان نوشت:

$$\dot{s}_2 = \begin{cases} \frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min} - I(t)) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2) < 0 & if \quad s_2 > 0 \\ \frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\max} - I(t)) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2) > 0 & if \quad s_2 < 0 \end{cases}$$

$$s_2\dot{s}_2 = \begin{cases} s_2\left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min} - I(t)) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2)\right] < 0 & if \ s_2 > 0 \\ s_2\left[\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\max} - I(t)) - \frac{RT\lambda_{H_2}}{V_A}(\frac{\varphi_a P_{VS}}{x_1(t)+x_2(t)-\varphi_a P_{VS}} - \frac{x_2(t)}{x_1(t)+x_2(t)})\zeta k_a \delta_2 sat(s_2)\right] > 0 & if \ s_2 < 0 \end{cases}$$

که در هر حالت $s_2\dot{s}_2$ یک عدد منفی بوده و $\dot{V} \le s_2\dot{s}_2$ می‌باشد.

برای حالت سوم نیز داریم:

ورودی کنترلی را طبق رابطه (۱۳) در نظر می‌گیریم.

برای اثبات پایداری تابع لیاپانف را به‌صورت زیر در نظر می‌گیریم:

$$V_3 = \frac{1}{2}S_2{}^2 + \frac{1}{2}(\hat{\alpha} - \alpha) + \frac{1}{2}(\hat{\beta} - \beta)$$

که α و β حد بالای نامعینی و اغتشاش بوده و در تابع لیاپانف حتماً باید در نظر بگیریم.

$$\dot{V}_3 = s_2\dot{s}_2 + (\hat{\alpha} - \alpha)\dot{\hat{\alpha}} + (\hat{\beta} - \beta)\dot{\hat{\beta}}$$

$$\dot{s}_3(t) = \zeta \dot{e}_3 + \operatorname{sgn}(e_3)\left|e_3\right|^{\alpha} = \zeta \dot{x}_3 + \operatorname{sgn}(e_3)\left|e_3\right|^{\alpha}$$

$$= \zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_c(t) + \frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)} - 1)I(t) + \Delta f_3(x) + d_3\right] + \operatorname{sgn}(e_3)\left|e_3\right|^{\alpha}$$

$$= \zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\left[\frac{C_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2} - x_3(t))}\right.$$

$$\left. - \frac{V_C(x_3(t)+x_4(t)+x_5(t))\operatorname{sgn}(e_3)\left|e_3\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t)) - x_3(t))\right]} - \frac{(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_3)}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c} - \delta_3 sat(s_3)\right]$$

$$+ \zeta\left[\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)} - 1)I(t) + \Delta f_3(x) + d_3\right] + \operatorname{sgn}(e_3)\left|e_3\right|^{\alpha}$$

$$= \left[\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\right] \times \left[\frac{C_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2} - x_3(t))}\right]$$

$$- \left[\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\right] \times \left[\frac{V_C(x_3(t)+x_4(t)+x_5(t))\operatorname{sgn}(e_3)\left|e_3\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t)) - x_3(t))\right]}\right]$$

$$- \left[\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\right] \times \left[\frac{(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_3)}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c}\right]$$

$$- \left[\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\right] \times \delta_3 sat(s_3)$$

$$+ \zeta\left[\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)} - 1)I(t) + \Delta f_3(x) + d_3\right] + \operatorname{sgn}(e_3)\left|e_3\right|^{\alpha}$$

$$=\zeta\frac{RTC_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2V_C}-\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}-\zeta(\hat{\alpha}+\hat{\beta})\,\mathrm{sgn}(s_3)$$
$$-(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\zeta\delta_3 sat(s_3)$$
$$+\zeta\left[\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)+d\right]+\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}$$

$$=\zeta\frac{RTC_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2V_C(x_3(t)+x_4(t)+x_5(t))}-\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}-\zeta(\hat{\alpha}+\hat{\beta})\,\mathrm{sgn}(s_3)$$
$$-(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\zeta\delta_3 sat(s_3)$$
$$-\zeta\frac{RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I(t)+\zeta\Delta f_3(x)+\zeta d_3+\mathrm{sgn}(e_3)\left|e_3\right|^{\alpha}$$

$$=\zeta\frac{RTC_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2V_C(x_3(t)+x_4(t)+x_5(t))}-\zeta\frac{RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I(t)$$

$$+\zeta\Delta f_3(x)+\zeta d_3-\zeta(\hat{\alpha}+\hat{\beta})\,\mathrm{sgn}(s_3)-(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\zeta\delta_3 sat(s_3)$$

$$=\zeta\frac{RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\zeta\frac{RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I(t)$$

$$+\zeta\Delta f_3(x)+\zeta d_3-\zeta(\hat{\alpha}+\hat{\beta})\,\mathrm{sgn}(s_3)-(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\zeta\delta_3 sat(s_3)$$

درنتیجه به رابطه‌ی زیر می‌رسیم:

$$s_3\dot{s}_3=\begin{cases}\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})(I_{\min}-I(t))-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_3)\\ +\zeta\Delta f_3(x)+\zeta d_3-\zeta(\hat{\alpha}+\hat{\beta})\,\mathrm{sgn}(s_3)<0 \qquad if\ \ s_3>0\\ \\ \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})(I_{\max}-I(t))-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_4 sat(s_3)\\ +\zeta\Delta f_3(x)+\zeta d_3-\zeta(\hat{\alpha}+\hat{\beta})\,\mathrm{sgn}(s_3)>0 \qquad if\ \ s_3<0\end{cases} \qquad (۲۲)$$

از طرفی هم داریم:

$$\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)=-\frac{\zeta RTC_1}{V_A}\frac{x_2(t)}{x_1(t)+x_2(t)}I<0$$

$$\frac{\zeta RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I=-\frac{\zeta RTC_1}{2V_C}\frac{x_4(t)+x_5(t)}{x_3(t)+x_4(t)+x_5(t)}I<0$$

در ادامه برای اثبات پایداری باید $\dot{V}$ را حساب کنیم.

$$\dot{V}_3=s_3\dot{s}_3+(\hat{\alpha}-\alpha)\dot{\hat{\alpha}}+(\hat{\beta}-\beta)\dot{\hat{\beta}}$$

$s_3\dot{s}_3$ را از رابطه ۱۷ جایگذاری می‌کنیم.

$$\dot{V}_3=\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_3)$$
$$+\zeta\Delta f_3(x)+\zeta d_3-\zeta(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_3)+(\hat{\alpha}-\alpha)\dot{\hat{\alpha}}+(\hat{\beta}-\beta)\dot{\hat{\beta}}$$

قوانین تطبیقی را از رابطه ۱۶ جایگذاری می‌کنیم:

$$\dot{V}_3=\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_3)$$
$$+\zeta\Delta f_3(x)+\zeta d_3-\zeta(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_3)+\zeta|s_3|(\hat{\alpha}-\alpha)+\zeta|s_3|(\hat{\beta}-\beta)$$

$$\dot{V}_3=\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_3)$$
$$+\zeta\Delta f_3(x)+\zeta d_3-\zeta(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_3)+\zeta|s_3|\hat{\alpha}-\zeta|s_3|\alpha+\zeta|s_3|\hat{\beta}-\zeta|s_3|\beta$$

حال حد بالای نامعینی و اغتشاش را با توجه به رابطه ۳ جایگذاری می‌کنیم:

$$\dot{V}_3\le\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_3)$$
$$+\zeta|s_3|\alpha+\zeta|s_3|\beta-\zeta(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_3)+\zeta|s_3|\hat{\alpha}-\zeta|s_3|\alpha+\zeta|s_3|\hat{\beta}-\zeta|s_3|\beta$$

$$\dot{V}_3 \le \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min} - \frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c \delta_3 sat(s_3)$$
$$-\zeta(\hat{\alpha}+\hat{\beta})\,\mathrm{sgn}(s_3)+\zeta|s_3|\hat{\alpha}+\zeta|s_3|\hat{\beta}$$

با توجه به رابطه $\mathrm{sgn}(s)=\frac{|s|}{s}$ داریم:

$$\dot{V}_3 \le \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min} - \frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c \delta_3 sat(s_3)$$
$$-\zeta|s_3|\hat{\alpha}-\zeta|s_3|\hat{\beta}+\zeta|s_3|\hat{\alpha}+\zeta|s_3|\hat{\beta}$$

که درنهایت به رابطه زیر می‌رسیم:

$$\dot{V}_3 \le \begin{cases} \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})(I_{\min}-I(t)) - \frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c \delta_3 sat(s_3) < 0 \quad if \ \ s_3 > 0 \\ \\ \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})(I_{\max}-I(t)) - \frac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c \delta_4 sat(s_3) > 0 \quad if \ \ s_3 < 0 \end{cases}$$

که در هر حالت $\dot{V}_3$ کران‌دار و منفی به دست می‌آید و سیستم حلقه بسته پایدار است.

برای حالت چهارم نیز داریم:

ورودی کنترلی را طبق رابطه (۱۳) در نظر می‌گیریم.

برای اثبات پایداری تابع لیاپانف را به‌صورت زیر در نظر می‌گیریم:

$$V_4 = \frac{1}{2}S_4{}^2 + \frac{1}{2}(\hat{\alpha}_4 - \alpha_4) + \frac{1}{2}(\hat{\beta}_4 - \beta_4)$$

که α_4 و β_4 حد بالای نامعینی و اغتشاش بوده و در تابع لیاپانف حتماً باید در نظر بگیریم.

$$\dot{V}_4 = s_4\dot{s}_4 + (\hat{\alpha}_4-\alpha_4)\dot{\hat{\alpha}}_4 + (\hat{\beta}_4-\beta_4)\dot{\hat{\beta}}_4$$

$$\dot{s}_4(t) = \zeta \dot{e}_4 + \mathrm{sgn}(e_4)\left|e_4\right|^{\alpha} = \zeta \dot{x}_4 + \mathrm{sgn}(e_4)\left|e_4\right|^{\alpha}$$
$$= \zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_{c_4}(t) + \Delta f_4(x) + d_4\right] + \mathrm{sgn}(e_4)\left|e_4\right|^{\alpha}$$

$$= \zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c \left[-\frac{\mathrm{sgn}(e_4)\left|e_4\right|^{\alpha}}{\frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c}\right.\right.$$
$$\left.\left. -\frac{(\hat{\alpha}_4 + \hat{\beta}_4)\mathrm{sgn}(s_4)}{\frac{RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c} - \delta_4 sat(s_4)\right] + \Delta f_4(x) + d_4\right] + \mathrm{sgn}(e_4)\left|e_4\right|^{\alpha}$$

$$= \zeta\left[-(\hat{\alpha}_4 + \hat{\beta}_4)\mathrm{sgn}(s_4) - \frac{RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c \delta_4 sat(s_4)\right]$$
$$+\zeta\Delta f_4(x) + \zeta d_4 + \mathrm{sgn}(e_4)\left|e_4\right|^{\alpha} - \mathrm{sgn}(e_4)\left|e_4\right|^{\alpha}$$

$$= -\zeta(\hat{\alpha}_4 + \hat{\beta}_4)\mathrm{sgn}(s_4) - \frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c \delta_4 sat(s_4)$$
$$+\zeta\Delta f_4(x) + \zeta d_4 + \mathrm{sgn}(e_4)\left|e_4\right|^{\alpha} - \mathrm{sgn}(e_4)\left|e_4\right|^{\alpha}$$

$$s_4\dot{s}_4 = -\zeta(\hat{\alpha}_4 + \hat{\beta}_4)\mathrm{sgn}(s_4) - \frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c \delta_4 sat(s_4) + \zeta\Delta f_4(x) + \zeta d_4$$
$$= s_4\left[-\frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c \delta_4 sat(s_4) - \zeta(\hat{\alpha}_4 + \hat{\beta}_4)\mathrm{sgn}(s_4) + \zeta\Delta f_4(x) + \zeta d_4\right] < 0$$

(۱۷)

$$s_4\dot{s}_4 < 0$$

در ادامه برای اثبات پایداری باید $\dot{V}_4$ را حساب کنیم.

$$\dot{V}_4 = s_4\dot{s}_4 + (\hat{\alpha}_4 - \alpha_4)\dot{\hat{\alpha}}_4 + (\hat{\beta}_4 - \beta_4)\dot{\hat{\beta}}_4$$

$s_4\dot{s}_4$ را از رابطه ۱۸ جایگذاری می‌کنیم.

$$\dot{V}_4 = s_4\dot{s}_4 + (\hat{\alpha}_4 - \alpha_4)\dot{\hat{\alpha}}_4 + (\hat{\beta}_4 - \beta_4)\dot{\hat{\beta}}_4$$

$$\dot{V}_4 = s_4\left[-\frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\delta_4 sat(s_4) - \zeta(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4)\right]$$
$$+\zeta s_4\Delta f_4(x) + \zeta s_4 d_4 - \zeta s_4(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4) + (\hat{\alpha}_4 - \alpha_4)\dot{\hat{\alpha}}_4 + (\hat{\beta}_4 - \beta_4)\dot{\hat{\beta}}_4$$

قوانین تطبیقی را جایگذاری می‌کنیم:

$$\dot{V}_4 = s_4\left[-\frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\delta_4 sat(s_4) - \zeta(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4)\right]$$
$$+\zeta s_4\Delta f_4(x) + \zeta s_4 d_4 - \zeta s_4(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4) + \zeta|s_4|(\hat{\alpha}_4 - \alpha_4) + \zeta|s_4|(\hat{\beta}_4 - \beta_4)$$

$$\dot{V}_4 = s_4\left[-\frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\delta_4 sat(s_4) - \zeta(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4)\right]$$
$$+\zeta s_4\Delta f_4(x) + \zeta s_4 d_4 - \zeta s_4(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4) + \zeta|s_4|\hat{\alpha}_4 - \zeta|s_4|\alpha_4 + \zeta|s_4|\hat{\beta}_4 - \zeta|s_4|\beta_4$$

$$\dot{V}_4 \le s_4\left[-\frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\delta_4 sat(s_4) - \zeta(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4)\right]$$
$$+\zeta s_4\alpha_4 + \zeta s_4\beta_4 - \zeta s_4(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4) + \zeta|s_4|\hat{\alpha}_4 - \zeta|s_4|\alpha_4 + \zeta|s_4|\hat{\beta}_4 - \zeta|s_4|\beta_4$$

با توجه به رابطه $\operatorname{sgn}(s_i) = \frac{|s_i|}{s_i}$ داریم:

$$\dot{V}_4 \le s_4\left[-\frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\delta_4 sat(s_4) - \zeta(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4)\right]$$
$$-\zeta|s_4|\hat{\alpha}_4 - \zeta|s_4|\hat{\beta}_4 + \zeta|s_4|\hat{\alpha}_4 + \zeta|s_4|\hat{\beta}_4$$

$$\dot{V}_4 \le s_4\left[-\frac{\zeta RT\lambda_{air}}{V_C}(\gamma_{N_2} - \frac{x_4(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\delta_4 sat(s_4) - \zeta(\hat{\alpha}_4 + \hat{\beta}_4)\operatorname{sgn}(s_4)\right] \le 0$$

برای حالت پنجم نیز داریم:

ورودی کنترلی را طبق رابطه (۱۴) در نظر می‌گیریم.

برای اثبات پایداری تابع لیاپانف را به‌صورت زیر در نظر می‌گیریم:

$$V_5=\frac{1}{2}{S_5}^2+\frac{1}{2}(\hat{\alpha}_5-\alpha_5)+\frac{1}{2}(\hat{\beta}_5-\beta_5)$$

که α_5 و β_5 حد بالای نامعینی و اغتشاش بوده و در تابع لیاپانف حتماً باید در نظر بگیریم.

$$\dot{V}_5=s_5\dot{s}_5+(\hat{\alpha}_5-\alpha_5)\dot{\hat{\alpha}}_5+(\hat{\beta}_5-\beta_5)\dot{\hat{\beta}}_5$$

$$\dot{s}_5(t)=\zeta\dot{e}_5+\mathrm{sgn}(e_5)\left|e_5\right|^{\alpha}=\zeta\dot{x}_5+\mathrm{sgn}(e_5)\left|e_5\right|^{\alpha}$$

$$=\zeta\left[\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c u_{c_5}(t)+\frac{RTC_1}{V_C}\right.$$
$$\left.\times\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)+\Delta f_5(x)+d_5\right]+\mathrm{sgn}(e_5)\left|e_5\right|^{\alpha}$$
$$=\zeta\left(\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c\right)$$
$$\times\left[-\frac{\mathrm{sgn}(e_5)\left|e_5\right|^{\alpha}}{\frac{\zeta RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}\right.$$

$$-\frac{C_1\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)}{\lambda_{air}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}$$

$$\left.-\frac{(\hat{\alpha}_5+\hat{\beta}_5)\,\mathrm{sgn}(s_5)}{\frac{RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)k_c}-\delta_5 sat(s_5)\right]$$

$$+\zeta\left[\frac{RTC_1}{V_C}\times\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)+\Delta f_5(x)+d_5\right]+\mathrm{sgn}(e_5)\left|e_5\right|^{\alpha}$$

$$\dot{s}_5 = -\frac{\zeta RTC_1}{V_C}\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)-\zeta(\hat{\alpha}_5+\hat{\beta}_5)\,\mathrm{sgn}(s_5)-\mathrm{sgn}(e_5)\left|e_5\right|^{\alpha}$$

$$-\left(\frac{\zeta k_c RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)\right)\delta_5 sat(s_5)$$

$$+\zeta\left[\frac{RTC_1}{V_C}\times\left(\frac{C_2}{C_1}\left(1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)-1-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)I(t)+\Delta f_5(x)+d_5\right]+\mathrm{sgn}(e_5)\left|e_5\right|^{\alpha}$$

$$\dot{s}_5 = -\zeta(\hat{\alpha}_5+\hat{\beta}_5)\,\mathrm{sgn}(s_5)+\zeta\Delta f_5(x)+\zeta d_5$$

$$-\left(\frac{\zeta k_c RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)\right)\delta_5 sat(s_5)$$

$$\dot{V}_5 = s_5\dot{s}_5+(\hat{\alpha}_5-\alpha_5)\dot{\hat{\alpha}}_5+(\hat{\beta}_5-\beta_5)\dot{\hat{\beta}}_5$$

$$\dot{V}_5 = s_5\left[-\left(\frac{\zeta k_c RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)\right)\delta_5 sat(s_5)\right.$$
$$\left.-\zeta(\hat{\alpha}_5+\hat{\beta}_5)\,\mathrm{sgn}(s_5)+\zeta\Delta f_5(x)+\zeta d_5\right]+(\hat{\alpha}_5-\alpha_5)\dot{\hat{\alpha}}_5+(\hat{\beta}_5-\beta_5)\dot{\hat{\beta}}_5$$

قوانین تطبیقی را جایگذاری میکنیم:

$$\dot{V}_5 = s_5\left[-\left(\frac{\zeta k_c RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)\right)\delta_5 sat(s_5)\right]$$
$$-\zeta s_5(\hat{\alpha}_5+\hat{\beta}_5)\,\mathrm{sgn}(s_5)+\zeta s_5\Delta f_5(x)+\zeta s_5 d_5+(\hat{\alpha}_5-\alpha_5)\zeta\left|s_5\right|+(\hat{\beta}_5-\beta_5)\zeta\left|s_5\right|$$

$$\dot{V}_5 = s_5\left[-\left(\frac{\zeta k_c RT\lambda_{air}}{V_C}\left(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}\right)\right)\delta_5 sat(s_5)\right]$$
$$-\zeta s_5(\hat{\alpha}_5+\hat{\beta}_5)\,\mathrm{sgn}(s_5)+\zeta s_5\Delta f_5(x)+\zeta s_5 d_5+\hat{\alpha}_5\zeta\left|s_5\right|-\alpha_5\zeta\left|s_5\right|+\hat{\beta}_5\zeta\left|s_5\right|-\beta_5\zeta\left|s_5\right|$$

از کران بالای نامعینی و اغتشاش استفاده می‌کنیم.

$$\dot{V}_5 = s_5\left[-(\frac{\zeta k_c RT\lambda_{air}}{V_C}(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}))\delta_5 sat(s_5)\right]$$
$$-\zeta s_5(\hat{\alpha}_5+\hat{\beta}_5)\operatorname{sgn}(s_5)+\zeta s_5\alpha_5(x)+\zeta s_5\beta_5+\hat{\alpha}_5\zeta\left|s_5\right|-\alpha_5\zeta\left|s_5\right|+\hat{\beta}_5\zeta\left|s_5\right|-\beta_5\zeta\left|s_5\right|$$

با توجه به رابطه $\operatorname{sgn}(s_i)=\frac{\left|s_i\right|}{s_i}$ داریم:

$$\dot{V}_5 = s_5\left[-(\frac{\zeta k_c RT\lambda_{air}}{V_C}(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}))\delta_5 sat(s_5)\right]$$
$$-\zeta\left|s_5\right|\hat{\alpha}_5-\zeta\left|s_5\right|\hat{\beta}_5+\hat{\alpha}_5\zeta\left|s_5\right|+\hat{\beta}_5\zeta\left|s_5\right|$$

$$\dot{V}_5 = s_5\left[-(\frac{\zeta k_c RT\lambda_{air}}{V_C}(\frac{\varphi_a P_{VS}}{x_3(t)+x_4(t)+x_5(t)-\varphi_a P_{VS}}-\frac{x_5(t)}{x_3(t)+x_4(t)+x_5(t)}))\delta_5 sat(s_5)\right]\le 0$$

. طراحی ورودی کنترل با در نظر گرفتن ورودی اشباع:

سیستم پیل سوختی را با در نظر گرفتن ورودی اشباع، نامعینی و اغتشاش به‌صورت زیر می‌نویسیم:

$$\dot{x}_1(t)=\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a sat(u_a)(t)+\frac{RTC_1(1-\gamma)}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f(x)+d$$

$$\dot{x}_3(t)=\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c sat(u_c)(t)+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)+d$$

مدل اشباع را به‌صورت زیر در نظر می‌گیریم:

$$u(v(t))=sat(v(t))=\begin{cases} sign(v(t))u_M, & \left|v(t)\right|\ge u_M \\ v(t), & \left|v(t)\right|<u_M \end{cases} \qquad (۲۳)$$

$v(t) \in R$ بوده و ورودی کنترلی می‌باشد، $u(t)$ ورودی سیستم می‌باشد که اشباع در نظر می‌گیریم و با رابطه ۲۳ نشان می‌دهیم. u_M حد بالای $u(t)$ می‌باشد.

برای از بین بردن اثر ورودی اشباع، $u(v(t))$ را توسط تابع هموار زیر تقریب می‌زنیم.

$$h(v) = u_M \tanh(\frac{v}{u_M}) = u_M \frac{e^{\frac{v}{u_M}} - e^{\frac{-v}{u_M}}}{e^{\frac{v}{u_M}} + e^{\frac{-v}{u_M}}} \qquad (۲۴)$$

آنچه بدیهی است وجود اختلاف بین $sat(v(t))$ و $h(v)$ است که آن را با $\Delta(v)$ به‌صورت زیر بیان می‌کنیم.

$$\Delta(v) = sat(v(t)) - h(v) \qquad (۲۵)$$

با توجه به این‌که هر دو تابع $\tanh$ و sat کران‌دار بوده، پس می‌توانیم $\Delta(v)$ را هم به‌صورت زیر کران‌دار در نظر بگیریم.

$$\left|\Delta(v)\right| = \left|sat(v(t)) - h(v)\right| \le u_M (1 - \tanh(1)) = b$$

به‌طوری‌که b یک عدد ثابت و مثبت بوده و $0 \le v(t) \le u_M$ می‌باشد. این در حالی است که حد $\Delta(v)$ از 0 تا b افزایش یافته و $\left|v(t)\right|$ از 0 تا u_M کاهش می‌یابد و خارج از این محدوده $(0 \le \left|v(t)\right| \le u_M)$، $\Delta(v)$ از 0 تا b کاهش می‌یابد.
برای سهولت در طراحی می‌توان $h(v(t))$ را طبق روابط زیر ساده سازی کرد.

$$\left(v(t)\right) = \quad (v^0) + \frac{\partial h(w)}{\partial w}|\mathrm{v}=v^{\mu}(\mathrm{v}\text{-}v^0)$$

با توجه به روابط زیر:

$$v^{\mu} = \mu v + (1-\mu) v^0 \qquad , \qquad 0 < \mu < 1$$

با در نظر گرفتن $v^0 = 0$ به رابطه زیر می‌رسیم:

$$h(v(t)) = h(0) + \frac{\partial h}{\partial v}_{v=v^{\mu}} v \qquad , \qquad h(0) = 0$$

$$h(v(t)) = \frac{\partial h}{\partial v}_{v=v^{\mu}} v$$

با در نظر گرفتن رابطه ۲۴ و ۲۵ و جایگذاری آن در معادله دینامیکی سیستم به روابط زیر می‌رسیم:

$$\dot{x}_1(t)=\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a h(v(t))+\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a\Delta(v)$$
$$+\frac{RTC_1(1-\gamma)}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f(x)+d$$

$$\dot{x}_3(t)=\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c h(v(t))+\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k\Delta(v)$$
$$+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)+d$$

از روابط بالا داریم:

$$\dot{x}_1(t)=\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a v(t)+\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a\Delta(v)+d$$
$$+\frac{RTC_1(1-\gamma)}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f(x)$$

$$\dot{x}_3(t)=\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c v(t)+\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k\Delta(v)+d$$
$$+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)$$

$$D_1=\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a\Delta(v)+d$$

$$D_2=\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k\Delta(v)+d$$

درنتیجه مدل دینامیکی سیستم به فرم زیر درمی‌آید:

$$\dot{x}_1(t)=\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})k_a v(t)+\frac{RTC_1(1-\gamma)}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)+\Delta f(x)+D$$

$$\dot{x}_3(t)=\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c v(t)+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)+D$$

حال باید با قوانین تطبیقی D را حذف کنیم. D را به‌عنوان اغتشاش در نظر می‌گیریم که کران‌دار است ولی کران آن نامعلوم فرض می‌شود.

$D < \Psi$

Ψ را کران D در نظر گرفته و ورودی کنترلی را به‌صورت زیر تعریف می‌کنیم:

$$v_a = \begin{cases} \dfrac{C_1 x_2(t) I_{\min}}{k_a \lambda_{H_2}(\gamma_{H_2}(x_1(t)+x_2(t))-x_1(t))} - \dfrac{\operatorname{sgn}(e)\left|e\right|^{\alpha}}{\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)})k_a \zeta} \\ -\dfrac{(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)}{\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)})k_a} - \delta_1 sat(s_1) \\ \dfrac{C_1 x_2(t) I_{\max}}{k_a \lambda_{H_2}(\gamma_{H_2}(x_1(t)+x_2(t))-x_1(t))} - \dfrac{\operatorname{sgn}(e)\left|e\right|^{\alpha}}{\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)})k_a \zeta} \\ -\dfrac{(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)}{\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2} - \dfrac{x_1(t)}{x_1(t)+x_2(t)})k_a} - \delta_2 sat(s_1) \end{cases}$$

$$v_c = \begin{cases} \dfrac{C_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))} - \dfrac{V_C(x_3(t)+x_4(t)+x_5(t))\operatorname{sgn}(e)\left|e\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t))-x_3(t))\right]} \\ -\dfrac{(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_2)}{\dfrac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \dfrac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k} - \delta_3 sat(s_2) \\ \dfrac{C_1(x_4(t)+x_5(t))I_{\max}}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))} - \dfrac{V_C(x_3(t)+x_4(t)+x_5(t))\operatorname{sgn}(e)\left|e\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t))-x_3(t))\right]} \\ -\dfrac{(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_2)}{\dfrac{RT\lambda_{air}}{V_C}(\gamma_{O_2} - \dfrac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k} - \delta_3 sat(s_2) \end{cases}$$

تابع لیاپانف را به‌صورت زیر در نظر می‌گیریم:

$$V = \frac{1}{2}S_2{}^2 + \frac{1}{2}(\hat{\alpha}-\alpha) + \frac{1}{2}(\hat{\Psi}-\Psi)$$

$$\dot{V} = s_2\dot{s}_2 + (\hat{\alpha}-\alpha)\dot{\hat{\alpha}} + (\hat{\Psi}-\Psi)\dot{\hat{\Psi}}$$

$$\dot{s}_1=\begin{cases}\dfrac{\zeta RTC_1}{V_A}(1-\dfrac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t))-\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\dfrac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\\ -\zeta(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)+\zeta\Delta f(x)+\zeta D<0 \qquad\qquad if \quad s_1>0\\ \dfrac{\zeta RTC_1}{V_A}(1-\dfrac{x_1(t)}{x_1(t)+x_2(t)})(I_{\max}-I(t))-\dfrac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\dfrac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_2 sat(s_1)\\ -\zeta(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)+\zeta\Delta f(x)+\zeta D>0 \qquad\qquad if \quad s_1<0\end{cases}$$

$$\dot{V}=s\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right.$$
$$\left.-\zeta(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)+\zeta\Delta f(x)+\zeta D\right]+(\hat{\alpha}-\alpha)\dot{\hat{\alpha}}+(\hat{\Psi}-\Psi)\dot{\hat{\Psi}}$$

$$\dot{V}=s\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta s(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)+\zeta s\Delta f(x)+\zeta sD+(\hat{\alpha}-\alpha)\dot{\hat{\alpha}}+(\hat{\Psi}-\Psi)\dot{\hat{\Psi}}$$

قوانین تطبیقی را به‌صورت زیر تعریف می‌کنیم:

$$\begin{aligned}\dot{\hat{\alpha}}&=\zeta\left|s\right|\\ \dot{\hat{\Psi}}&=\zeta\left|s\right|\end{aligned}\qquad\qquad (۲۶)$$

با جایگذاری قوانین تطبیقی داریم:

$$\dot{V}=s\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta s(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)+\zeta s\Delta f(x)+\zeta sD+\zeta\left|s\right|(\hat{\alpha}-\alpha)+\zeta\left|s\right|(\hat{\Psi}-\Psi)$$

$$\dot{V}\le s\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta s(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)+\zeta\left|s\right|\alpha+\zeta\left|s\right|\Psi+\zeta\left|s\right|\hat{\alpha}-\zeta\left|s\right|\alpha+\zeta\left|s\right|\hat{\Psi}-\zeta\left|s\right|\Psi$$

$$\dot{V} \le s\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta s(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_1)+\zeta|s|\hat{\alpha}+\zeta|s|\hat{\Psi}$$

با توجه به رابطه روبه‌رو می‌توان نوشت:

$$\operatorname{sgn}(s)=\frac{|s|}{s}$$

$$\dot{V} \le s\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta|s|(\hat{\alpha}+\hat{\Psi})+\zeta|s|\hat{\alpha}+\zeta|s|\hat{\Psi}$$

$$\dot{V} \le s\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$
$$-\zeta|s|\hat{\alpha}-\zeta|s|\hat{\Psi}+\zeta|s|\hat{\alpha}+\zeta|s|\hat{\Psi}$$

درنهایت می‌توان نوشت:

$$\dot{V} \le s\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min})-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]$$

که با توجه به رابطه (۱۵)، می‌توان نوشت:

$$\dot{s}_1=\begin{cases}\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t))-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)<0 & if \quad s_1>0\\ \frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\max}-I(t))-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_2 sat(s_1)>0 & if \quad s_1<0\end{cases}$$

$$s_1\dot{s}_1=\begin{cases}s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\min}-I(t))-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_1 sat(s_1)\right]<0 & if \quad s_1>0\\ s_1\left[\frac{\zeta RTC_1}{V_A}(1-\frac{x_1(t)}{x_1(t)+x_2(t)})(I_{\max}-I(t))-\frac{RT\lambda_{H_2}}{V_A}(\gamma_{H_2}-\frac{x_1(t)}{x_1(t)+x_2(t)})\zeta k_a\delta_2 sat(s_1)\right]>0 & if \quad s_1<0\end{cases}$$

که در هر حالت $s_1\dot{s}_1$ یک عدد منفی بوده و $\dot{V}\le s_1\dot{s}_1$ می‌باشد.

$$\dot{s}_2(t)=\zeta\dot{e}_2+\mathrm{sgn}(e_2)\left|e_2\right|^{\alpha}=\zeta\dot{x}_3+\mathrm{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$=\zeta\left[\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c u_c(t)+\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)+D\right]+\mathrm{sgn}(e_1)\left|e_1\right|^{\alpha}$$

$$=\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\left[\frac{C_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))}\right.$$

$$\left.-\frac{V_C(x_3(t)+x_4(t)+x_5(t))\mathrm{sgn}(e)\left|e\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t))-x_3(t))\right]}-\frac{(\hat{\alpha}+\hat{\Psi})\mathrm{sgn}(s_1)}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c}-\delta_3 sat(s_2)\right]$$

$$+\zeta\left[\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)+D\right]+\mathrm{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$=\left[\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\right]\times\left[\frac{C_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2\lambda_{air}k_c(x_3(t)+x_4(t)+x_5(t)\gamma_{O_2}-x_3(t))}\right]$$

$$-\left[\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\right]\times\left[\frac{V_C(x_3(t)+x_4(t)+x_5(t))\mathrm{sgn}(e_2)\left|e_2\right|^{\alpha}}{k_c\zeta\left[RT\lambda_{air}(\gamma_{O_2}(x_3(t)+x_4(t)+x_5(t))-x_3(t))\right]}\right]$$

$$-\left[\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\right]\times\left[\frac{(\hat{\alpha}+\hat{\Psi})\mathrm{sgn}(s_2)}{\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c}\right]$$

$$-\left[\zeta(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c)\right]\times\delta_3 sat(s_2)$$

$$+\zeta\left[\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)+D\right]+\mathrm{sgn}(e_2)\left|e_2\right|^{\alpha}$$

$$=\zeta\frac{RTC_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2V_C}-\mathrm{sgn}(e_2)|e_2|^{\alpha}-\zeta(\hat{\alpha}+\hat{\Psi})\,\mathrm{sgn}(s_2)$$
$$-(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\zeta\delta_3 sat(s_2)$$
$$+\zeta\left[\frac{RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I(t)+\Delta f(x)+D\right]+\mathrm{sgn}(e_2)|e_2|^{\alpha}$$

$$=\zeta\frac{RTC_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2V_C(x_3(t)+x_4(t)+x_5(t))}-\mathrm{sgn}(e_2)|e_2|^{\alpha}-\zeta(\hat{\alpha}+\hat{\Psi})\,\mathrm{sgn}(s_2)$$
$$-(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\zeta\delta_3 sat(s_2)$$
$$-\zeta\frac{RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I(t)+\zeta\Delta f(x)+\zeta D+\mathrm{sgn}(e_2)|e_2|^{\alpha}$$

$$=\zeta\frac{RTC_1(x_4(t)+x_5(t))\mathrm{I}_{\min}}{2V_C(x_3(t)+x_4(t)+x_5(t))}-\zeta\frac{RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I(t)$$

$$+\zeta\Delta f(x)+\zeta D-\zeta(\hat{\alpha}+\hat{\Psi})\,\mathrm{sgn}(s_2)-(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\zeta\delta_3 sat(s_2)$$

$$=\zeta\frac{RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\zeta\frac{RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I(t)$$

$$+\zeta\Delta f(x)+\zeta D-\zeta(\hat{\alpha}+\hat{\Psi})\,\mathrm{sgn}(s_2)-(\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})k_c\zeta\delta_3 sat(s_2)$$

درنتیجه به رابطه‌ی زیر می‌رسیم:

$$s_2\dot{s}_2=\begin{cases}\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})(I_{\min}-I(t))-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_2)\\ +\zeta\Delta f(x)+\zeta D-\zeta(\hat{\alpha}+\hat{\Psi})\,\mathrm{sgn}(s_2)<0\quad if\;\; s_2>0\\ \\ \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})(I_{\max}-I(t))-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_4 sat(s_2)\\ +\zeta\Delta f(x)+\zeta D-\zeta(\hat{\alpha}+\hat{\Psi})\,\mathrm{sgn}(s_2)>0\quad if\;\; s_2<0\end{cases}$$

از طرفی هم دائم:

$$\frac{\zeta RTC_1}{V_A}(\frac{x_1(t)}{x_1(t)+x_2(t)}-1)I(t)=-\frac{\zeta RTC_1}{V_A}\frac{x_2(t)}{x_1(t)+x_2(t)}I<0$$

$$\frac{\zeta RTC_1}{2V_C}(\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)}-1)I=-\frac{\zeta RTC_1}{2V_C}\frac{x_4(t)+x_5(t)}{x_3(t)+x_4(t)+x_5(t)}I<0$$

در ادامه برای اثبات پایداری باید $\dot{V}$ را حساب کنیم.

$$\dot{V}=s_2\dot{s}_2+(\hat{\alpha}-\alpha)\dot{\hat{\alpha}}+(\hat{\Psi}-\Psi)\dot{\hat{\Psi}}$$

$s_2\dot{s}_2$ را از رابطه ۱۷ جایگذاری می‌کنیم.

$$\dot{V}=\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_2)$$
$$+\zeta\Delta f(x)+\zeta D-\zeta(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_2)+(\hat{\alpha}-\alpha)\dot{\hat{\alpha}}+(\hat{\Psi}-\Psi)\dot{\hat{\Psi}}$$

قوانین تطبیقی را از رابطه ۱۶ جایگذاری می‌کنیم:

$$\dot{V}=\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_2)$$
$$+\zeta\Delta f(x)+\zeta D-\zeta(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_2)+\zeta|s|(\hat{\alpha}-\alpha)+\zeta|s|(\hat{\Psi}-\Psi)$$

$$\dot{V}=\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_2)$$
$$+\zeta\Delta f(x)+\zeta D-\zeta(\hat{\alpha}+\hat{\beta})\operatorname{sgn}(s_2)+\zeta|s|\hat{\alpha}-\zeta|s|\alpha+\zeta|s|\hat{\Psi}-\zeta|s|\Psi$$

حال حد بالای نامعینی و اغتشاش را با توجه به رابطه ۳ جایگذاری می‌کنیم:

$$\dot{V}\le\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_2)$$
$$+\zeta|s|\alpha+\zeta|s|\Psi-\zeta(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_2)+\zeta|s|\hat{\alpha}-\zeta|s|\alpha+\zeta|s|\hat{\Psi}-\zeta|s|\Psi$$

$$\dot{V}\le \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_2)$$
$$-\zeta(\hat{\alpha}+\hat{\Psi})\operatorname{sgn}(s_2)+\zeta|s|\hat{\alpha}+\zeta|s|\hat{\Psi}$$

با توجه به رابطه $\operatorname{sgn}(s)=\frac{|s|}{s}$ داریم:

$$\dot{V}\le \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})I_{\min}-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_2)$$
$$-\zeta|s|\hat{\alpha}-\zeta|s|\hat{\Psi}+\zeta|s|\hat{\alpha}+\zeta|s|\hat{\Psi}$$

که درنهایت به رابطه زیر می‌رسیم:

$$\dot{V}\le\begin{cases}\frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})(I_{\min}-I(t))-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_3 sat(s_2)<0 \quad if\ \ s_2>0\\ \\ \frac{\zeta RTC_1}{2V_C}(1-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})(I_{\max}-I(t))-\frac{RT\lambda_{air}}{V_C}(\gamma_{O_2}-\frac{x_3(t)}{x_3(t)+x_4(t)+x_5(t)})\zeta k_c\delta_4 sat(s_2)>0 \quad if\ \ s_2<0\end{cases}$$

که در هر حالت $\dot{V}$ کران‌دار و منفی به دست می‌آید و سیستم حلقه بسته پایدار است.