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A Fast Enhanced Algorithm of PRI Transform

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***Abstract*: The problem of estimating pulse repetition interval (PRI) of an interleaved pulse train which consist of several independent radar signals, is the main issue of signal processing in electronic support systems. PRI Transform algorithm is one of the well known and effective methods of PRI detection which is capable of detecting several close jittered signals and surpassing subharmonics, but have some drawbacks especially because of small PRI dynamic range and heavy computations. In this paper a modified PRI transform is introduced which manage wide range of PRIs simultaneously, and speed up the algorithm by significantly reducing the computations. Moreover an efficient threshold is set for increasing detection rate. Simulation results proves that the modified algorithm shows much better performance in case of wide range PRI detection and processing speed while retaining the advantages the original algorithm.**

Keywords: Pulse Repetition Interval, PRI Transform, processing speed

I. Introduction

Deinterleaving of radar pulses which is an import part of electronic support systems, is employed to detect and identify different simultaneously active radar emitters. The main parameters used in deinterleaving process are three of monopulse features: Radio frequency, Direction of arrival and Time of arrival (TOA). The first two parameters are usually used for pre-processing and clustering received pulse stream to smaller groups, and the TOA is used for PRI estimation which is the most important parameter of a radar emitter.

Several algorithm based on TOA analysis have been developed to detect potential PRIs of interleaved radar pulse trains such as statistical histograms, autocorrelation based techniques and algorithms using time- frequency analysis such as wavelet transform. A review of these algorithms is given in [5]. Each of these algorithms is good on some aspects. For example two of most common histogram based algorithms are (sequential difference) SDIF and (cumulative difference) CDIF [3,4]. The former shows good performance on detecting fixed PRI but is unable to detect Jittered PRI. CDIF works well on low noise pulse sequence, but its performance highly degrades in present of missing pulses and detects subharmonics instead of correct PRI. Other methods such as wavelet analysis have not shown yet as good performance on jittered signals and unable to distinguish between radars having close PRIs. To overcome all these problem a new algorithm called PRI Transform is proposed in [1]. This algorithm is based on a complex autocorrelation of TOA of pulse stream which helps to surpass PRI subharmonics in TOA histogram. In addition, using overlapped bins and concept of shifting time origins make PRI Transform algorithm well suited for detecting jittered PRIs. Moreover the algorithm is robust to missing and spurious pulses and also able to discriminate pulse trains containing close PRIs.

Despite of all the mentioned advantages for PRI Transform algorithm yet it has some shortcomings. One is that the ratio of maximum PRI to minimum PRI which can be detected on a PRI transform spectrum is limited and is inversely proportional to bin width. In addition, as the computation order is of $O(N^2)$, increasing the time resolution is obtained by squaring of bin density.

In this paper we analyze the cause of these problems and by devising new ways for making bins, shifting time origins, computing local phases and setting proper thresholds, we propose the modified PRI Transform algorithm which is much faster and capable of detecting wide range of PRIs simultaneously. Some other enhancement of the algorithm and its applications could be find in [6], [7], and [9].

The organization of this paper is as follows. In section II, an overview of PRI Transform algorithm is given and its subharmonic suppression technique and overlapped bins are discussed. In section III, performance of the original algorithm is shown and its merits and weaknesses is presented. In Section IV, the cause of the problems are analyzed and the enhanced algorithm is proposed then, in section V the performance of the original and enhanced algorithm are compared, and eventually some conclusions are provided in section VI.

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II. PRI TRANSFORM

In this section we review the PRI Transform algorithm proposed in [1]. Although this algorithm itself is an improved version of older PRI Transform algorithm devised in [2], but throughout this paper we consider the former as PRI Transform algorithm.

Let pulse train consist of N pulses. We can model the pulse train as sum of unit impulse,

$$g(t) = \sum_{n=0}^{N-1} \delta(t - t_n) \qquad (1)$$

where t_n is time of arrival of the n'th pulse and $\delta(.)$ is the Dirac delta function. PRI Transform is defined as complex autocorrelation function of TOA difference by the following equation,

$$D(\tau) = \sum_{n=1}^{N-1} \sum_{m=0}^{n-1} \delta(\tau - t_n + t_m) \exp\left[\frac{2\pi i t_n}{t_n - t_m}\right] \qquad (2)$$

which differs from autocorrelation function, C(τ), only by the phase factor $exp(2\pi it/\tau)$ or $exp(2\pi itn/(t_n - t_m)]$. This factor plays an important role in suppressing the subharmonics which appears in autocorrelation function. The details of how adding phase factor damp the subharmonics could be find in [1].

To numerically compute the PRI transform defined by (2), the τ-axis is divided to some countable discrete points which called bins. Let $[\tau_{min}, \tau_{max}]$ be the range of PRI to be analyzed. Then we equally divide this range into K small intervals with bin centers given by,

$$\tau_k = (k - 1/2)b + \tau_{min}, \quad k = 1,2, \dots, K \qquad (3)$$

where $b = (\tau_{max} - \tau_{min})/K$, is distance between adjustant bins and also the minimum bin width in non-overlapped type of the algorithm. The discrete version of PRI transform is defined as follows:

$$D_k = \int_{\tau_k - b_k/2}^{\tau_k + b_k/2} D(\tau) d\tau$$
$$= \sum_{\{(m,n);\tau_k - b_k/2 < t_n - t_m < \tau_k + b_k/2\}} \exp\left[\frac{2\pi i t_n}{t_n - t_m}\right] \qquad (4)$$

where b_k is bin width of $k'th$ bin. For detecting the jittered PRIs and avoiding PRI value being distributed into many bins, overlapped bins have been used in which bin widths are much bigger than adjacent bins distance and wide enough two include all value of $|D_k|$. Therefore if ϵ be the upper limit of PRI jitter, bin width may be set as,

$$b_k = 2\epsilon\tau_k \qquad (5)$$

Finally for avoiding enlargement of phase errors phase factor of each bin is measured regards to its unique phase origin and may be updated only at integer multiple of the bin center value, τ_k. Rate of updating time origin is fixed for all bins and defined by constant mobility factor, ζ_0 . we further give more details of the effect of this factor in shape of PRI Transform spectrum in section IV.

Now we can compute absolute value of PRI Transform, $|D_k|$, for all the N pulses in the input pulse train, which needs $N(N-1)/2$ calculation of bins. Each bin itself includes n_b iteration for phase calculation and updating time origins and $|D_k|$ values. So making PRI Trasnform histogram is of order of $\sum_{k=1}^{K} N^2 n_{b_k}$ in which $n_b$is given by:

$$n_{b_k} = \frac{bin\ width}{bin\ distance} = \frac{2\epsilon\tau_k}{(\tau_{max} - \tau_{min})/K} \qquad (6)$$

It can be seen that n_{b_k} is proportional to $2\epsilon k$ where k is bin index. Therefore generally bins with higher indices need much more computations that is not desirable. Future in section IV we propose a method for setting optimum bin width to reduce n_{b_k}.

After creating bins, and filling the PRI histogram with regards to given pulse train, the final step is to detect peaks of the spectrum by setting a proper threshold. The threshold used to discriminate PRIs from other bins is set by three criteria: a criterion by observation time, a criterion for eliminating subharmonics, and a criterion for eliminating noise caused by the mutual interference between different single pulse trains inside the given pulse train. It is proved in [1] that these criteria leads to three conditions $|D_k| \geq \alpha \frac{T}{t_k}$, $|D_k| \geq \beta C_k$, and $|D_k| \geq \gamma\sqrt{T\rho^2 b_k}$ respectively. Where T is total observation time, C_k is autocorellation function at bin index k, ρ is the pulse density, and parameters α, β, and γ are tunable parameters. Combining above three criteria the threshold value at each bin established as

$$A_k = max\left\{\alpha\frac{T}{t_k}, \beta C_k, \gamma\sqrt{T\rho^2 b_k}\right\} \qquad (7)$$

The threshold parameters are tuned under various simulation scenario to increase probability of detection and to reduce false alarms. It should be mentioned that optimum threshold function has room for further improvement [1].

III. PERFROMANCE ANALYSIS OF PRI TRANSFORM ALGORITHM

To assess the performance and capabilities of PRI Transform, we use the algorithm for detected PRIs in three different category of pulse train which are

i) pulse train includes single PRI

ii) pulse train include 2-5 PRIs in wide range

iii) pulse train includes close PRIs

The algorithm parameters for all the simulations are set according to Table I. Summary of the simulations are provided in Table II.

TABLE I. PARAMETERS OF SIMULATIONS

Parameter	***Description***	***Value***
PRI	Values of PRI	1, $\sqrt{2}$, $\sqrt{5}$, 4.5, 13, 100
$\tau_{max} - \tau_{min}$	Range of PRI	[0.5 - 150]
N	Number of Pulses	<8000
K	Number of bins	5000
ζ_0	Mobility of time origins	0.03
ϵ_0	Bin width parameter	Relative to max jitter
α	Threshold parameter	0.3
β	Threshold parameter	0.15
γ	Threshold parameter	3

For testing single PRI pulse stream we simulate one emitter source with PRI value of 1 consist of 2883 pulses. The PRI transform spectrum is seen in Fig.1.

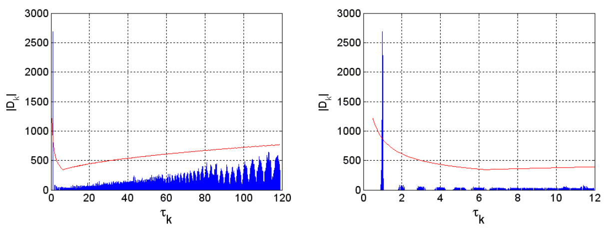

Fig.1. PRI spectrum by PRI Transform. Input data is a single emitter with mean PRI 1, and 10% peak to peak jitter.

As it can be seen in Fig.1, the peak is exactly appeared in 1 with value of 2694, but noise increases in higher order subharmonics of the peak, especially when bin index exceeds ${}^{1}/_{2\epsilon}$multiple of main PRI that makes it difficult to detect another emitter with PRI much smaller than the dominant PRI in the pulse sequence. To show this behavior we perform following tests.

In the second test two emitters with PRI 1, and 100 were generated, and in the third test four emitters with PRI 1,$\sqrt{2}$, $\sqrt{5}$, and 100 were generated with 2911 and 6156 pulses respectively . Fig.2 and 3 show the PRI spectrum of the two tests. In both tests, emitters with small PRI, high density emitters, were perfectly detected which demonstrates power of PRI transform in estimating PRI of multiple emitters with relatively near frequency. But emitter with PRI 100, which is about two decades far of other dominant PRIs is not detected in third test, and imprecisely detected in second test.

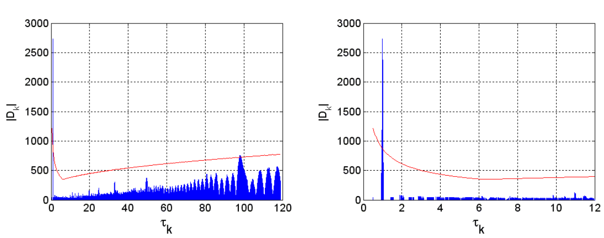

Fig.2. PRI spectrum by PRI Transform. Input data contains two emitters with mean PRI 1, and 100..

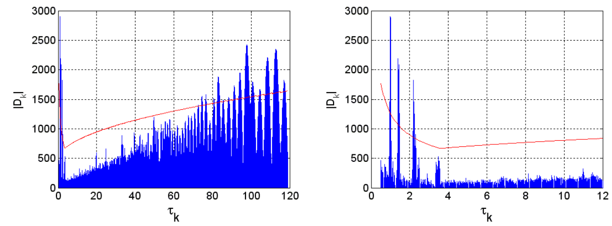

Fig.3. PRI spectrum by PRI Transform. Input data contains four emitters with mean PRI 1,$\sqrt{2}$, $\sqrt{5}$,and 100.

this is mostly because of noise level is much higher than the signal value at bin value 100. Therefore either emitter with PRI 100, is lost in noise produced by high density emitters, or its peak value at the spectrum is randomly moved around the PRI bin 100. It should be mentioned that detection is done by excluding autocorrelation criterion, βC_k, from threshold function, A_k, because this factor makes threshold extremely large at bins decades far away from average interarrival times, and prevents any probable detection at such bins.

IV. ENAHNCED ALGORITHM OF PRI TRANSFOM

In this section we propose an enhanced algorithm of PRI transform by introducing new ways for making bins, shifting time origins, computing local phases and setting proper thresholds, increase detection of wide range PRIs as well as to increase the speed of the algorithm.

A. Shifting Time Origins

Shifting time origins is to prevent phase error enlargement. When a PRI bin is much greater than average interarrival times of pulses, shifting time origin too fast leads to imperfect harmonic suppression at that bin and increase of noise. So we should set an optimum point for shifting time origins. It could be proved that optimum changing of time origins is proportional to ratio of average interarrival time and bin value:

$$\zeta_k = \zeta_0 \frac{\bar{x}}{\tau_k} \qquad (8)$$

where ζ_k is mobility factor for bin index k, ζ_0 is initial mobility factor which is considered for single PRI pulse stream, and $\bar{x}$ is average interarrival time that is $\bar{x} = \sum_{i=2}^{N}(t_i - t_{i-1})/(N-1)$, and t_i is TOA of the ith pulse.

B. Symmetric Bin Updating

Another problem which is emerge on PRI bins much greater than $\bar{x}$, is the noise of asymmetric filling the bins. In this case, $d \gg \bar{x}$, as pulse trains under test is time limited, each bin is updated more by lefthand adjacent bins than the righthand. This noise is proportional to square of $\rho\epsilon\tau_k$ regardless of length of the pulse train. This noise is negligible if number of pulses increases, but the limitation is that in general when frame length is become large, the interarrival time process is no more obey the Homogenous Poisson process relation, so the assumption for making threshold function will not be valid. Therefore we consider this noise and try to reduce its effect by adaptive windowing for each PRI bin so that each bin specially in high density pulse sequence, be updated symmetrically. So we update PRI spectrum , D_k, only for the TOAs lie in integer multiple of τ_k.

C. Constant Bin Resolution

The overlapped bins are used to resolve the problem of jittered PRI radars. But regardless of we use simple bins or overlapped ones, what determines the accuracy of PRI detection is bins resolution which is given by:

$$r_k = \frac{\tau_k - \tau_{k-1}}{\tau_k} \cong \frac{\tau_{k+1} - \tau_k}{\tau_k} \tag{9}$$

where r_k is PRI resolution at bin index k. in the original version of PRI transform bins are equally distanced because the whole interval, $\tau_{max} - \tau_{min}$is uniformly divided into N bins. So bins with smaller center value have less resolution which is not desirable. As an example if we divide PRI range 0 to 100 into 5000 bins, then PRI resolution at bin index 1 would be 2% and at bin index 100 would be 0.02%. Which means we lose the appropriate PRI detection resolution at lower bins because of low density of bins at small PRIs, and obtain over fitted bins at larger PRIs. But it is desirable to detect PRI at any range with constant accuracy. For this purpose we define the bin grow rate as ratio of two consecutive bins which is constant for any bin index which means

$$r = \frac{\tau_k}{\tau_{k-1}} \tag{10}$$

If we use above concept to make spectrum bins of range [0..100] by 5000 bins, we reach resolution better than 0.1% for any PRI in the specified range which is much more efficient than the uniform method. On the other word we say creating bins based on a geometry progression is much more beneficial than arithmetic progression approach. For fixed number of bins, and given PRI range, the bin grow rate is obtain by the following equation,

$$r = \sqrt[N]{1 + (\tau_{max} - \tau_{min})/\tau_0} \tag{11}$$

To use specific common ratio between bins, number of bins is obtained by:

$$r^N - (r-1)(\tau_{max} - \tau_{min})\tau_0 - 1 = 0 \tag{12}$$

Where τ_0 is start bin value and r is typically in range of 1.005 to 1.001 for reaching desirable detection accuracy between 0.5% to 0.1%. This is because for detecting PRI, specially emitters with fixed PRI with high accuracy, and to avoid extra computations and making the algorithm too time consuming.

D. Making the Spectrum

At this step we can define the enhanced PRI algorithm by inserting adaptive shifting time origins, rounded boundaries as follows :

1) Make Bins and their corresponding PRI value, $cT(k)$. This is function that map bin index k, to its PRI.

2) Init spectrum, D.

3) For n=start:N do loop step 4

4) For m=n-1:1 repeat steps 5 to 10.

5) Let $d = t_n - t_m$, if $d \in [lBand, uBand]$goto step 6, else back to step 4.

6) Calculate the range of PRI bins:

$$k1 = \mathrm{cT}[\mathrm{d}\ (1 - \epsilon)]$$
$$k2 = \mathrm{cT}[\mathrm{d}\ (1 + \epsilon)]$$

7) Let $\zeta_{ref} = min\left(\zeta_{max}, \zeta_0 \frac{\bar{x}}{\tau_k}\right)$

8) For k=k1:k2 repeat steps 9 to 12

9) Init the time origin if the kth PRI bins is used for the first time, Let $O[k] = 0$.

10) Calculate the primarily phase and decompose it:

$$\eta_0 = (t_n - O_k)/\tau_k$$
$$\nu = [\eta_0 + 0.49999 \ldots]$$
$$\zeta = 1 - \eta_0/\nu$$

11) Shift the time origin if either of the following conditions are satisfied, then let $O_k = t_n$.

$$\nu = 1 \ and \ O_k = t_m$$
$$\nu \geq 2 \ and \ \zeta \leq \zeta_{ref}$$

12) Calculate the phase: $\eta = (t_n - O_k)/\tau_k$

13) Update the PRI transform if t_n meet the symmetric condition, then Let, $D_k = D_k + \mathrm{e}^{2\pi i \eta}$.

E. Threshold for Detection of PRIs

The last step our algorithm is detection of potential PRIs from enhanced PRI spectrum. The threshold function used for this purpose is like that of the original algorithm in case of density criterion, but it is different for noise rejection criterion because of different noise reduction method used in this paper. The threshold function is given:

$$A_k = max\left\{\alpha_k \frac{T}{t_k}, \gamma\sqrt{T\rho^2 b_k} + \beta 2\epsilon\rho T\right\} \tag{13}$$

where α, β, and γ are tunable parameters. We tuned these values according to simulations to reach best detection rate, and reduce false alarms. All the simulations given in next section are performed by the following values: $\alpha_0 = 0.3$, $\beta = 0.75$, $\gamma = 1$.

V. SIMULATIONS AND PERFORMANCE ANALYSIS

In this section we analyze the performance of the proposed algorithm based on the test scenarios given in section III, and compare the performance of the original algorithm and the enhanced one, for correct detection rate and computation complexity.

Four test scenarios are designed according to Table II, covering PRI range of two logarithmic decades. For each test performance of original PRI transform algorithm is compared with proposed algorithm with the same number of bins as original algorithm, and with the same average PRI bin resolution. Each emitter is either fixed PRI or has up to 12% peak to peak jitter. For all the tests a fixed $\epsilon = 0.05$ is used. Fig. 4-6 shows the test result of proposed algorithm for equal bin resolution,0.3% , according to test number 1,2, and 4 of Table II.

Comparing the results, it is observed that the proposed algorithm has better detection performance of emitter with PRI 100, also produces less bin noise.

To analyze speed of the algorithms, we focus on phase computation loop which is the main part of the both algorithms. For the same bin resolution, 0.3%, the original algorithm needs about seven times iterations, and more than five times processing time. It worth noting that, at the same bin resolution, the proposed method needs only about half number of bins to the original algorithm. If we use the same number of bins for our algorithm, according to (12), we reach 0.1% bin resolution which is three times more than the original algorithm, still the proposed algorithm is more than two times faster, and needs about one third number of phase computations.

The main reason for different processing speed of the two algorithms is that in the original version, bin width are variant and increase linearly by increasing PRI value, but in our method the bin widths are always fixed, and include specific number of bins, $n_{b_k} = n_b = 2\epsilon/(r-1)$ in comparison to (6).

Considering n_{b_k}, a rough estimate for number of main loop iterations for both algorithm is obtained by (15).

TABLE II. SIMULATIONS SUMMARY

Test	Emitters	N pulses	Algorithm	Number of Bins	Ave PRI bin resolution	Detection-False alarm	Number of Main Loop Iterations	Elapsed Time (ms)
1	PRI = 1	2883	Original	5000	% 0.3	1/1 - 0	92295012	9701
			Proposed	5000	% 0.1	1/1 - 0	30245472	3737
			Proposed	2539	% 0.3	1/1 - 0	13699674	1738
2	PRI = 1, 100	2911	Original	5000	% 0.3	2/2 - 0 (but Imprecise at 100)	94118067	9835
			Proposed	5000	% 0.1	2/2 - 0	30435617	3845
			Proposed	2539	% 0.3	2/2 - 0	13966291	1779
3	PRI = $\sqrt{2}$, 4.5, 13, 100	2925	Original	5000	% 0.3	4/4 - 0 (but Imprecise at 100)	95454524	10008
			Proposed	5000	% 0.1	4/4 - 0	30722707	3824
			Proposed	2539	% 0.3	4/4 - 0	14052725	1820
4	PRI = 1, $\sqrt{2}$, $\sqrt{5}$, 100	6156	Original	5000	% 0.3	3/4 - Many	422707987	45876
			Proposed	5000	% 0.1	4/4 - 1	136265304	16981
			Proposed	2539	% 0.3	4/4 - 0	62443426	7928

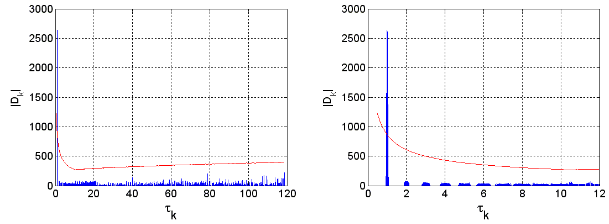

Fig.4. PRI spectrum by the proposed algorithm. Input data is a single emitter with mean PRI 1, and 10% peak to peak jitter.

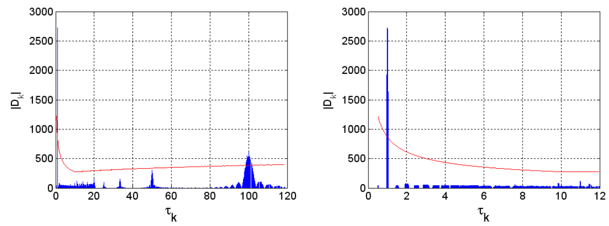

Fig.5. PRI spectrum by the proposed algorithm. Input data contains two emitters with mean PRI 1, and 100..

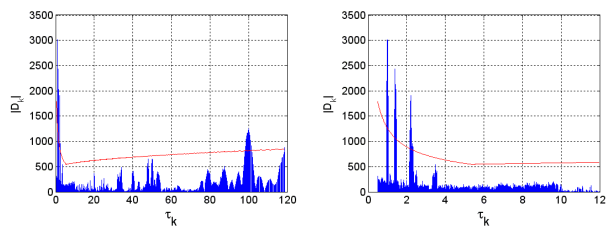

Fig.6. PRI spectrum by PRI Transform. Input data contains four emitters with mean PRI 1,$\sqrt{2}$, $\sqrt{5}$,and 100.

$$N_k = N^2.Prob(\tau_k - b_k/2 < t_n - t_m < \tau_k + b_k/2)$$

$$N_k = N^2\left(\frac{T-\tau_k}{T^2}\right)b_k \qquad (14)$$

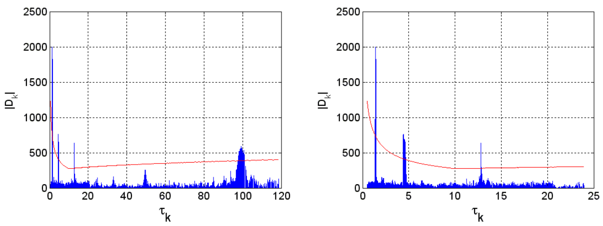

Fig.7. PRI spectrum by PRI Transform. Input data contains four emitters with mean PRI $\sqrt{2}$, 4.5 , 13, and 100.

where for calculating probability that $t_n - t_m$ is included in the kth PRI bin, we have modeled pulse sequence as Poisson process [1,8], and N_k is expecting value for number of phase computation loop at bin index k. Considering each loop needs n_{b_k} iterations, we can write,.

$$N_{it} = \sum_{k=1}^{N_{bins}} N^2 \left(\frac{T-\tau_k}{T^2}\right) b_k.n_{b_k}$$

$$\cong N\rho \sum_{k=1}^{N_{bins}} b_k.n_{b_k} \qquad (15)$$

where N_{it} is total number of phase calculations and bin updating for making PRI spectrum, and can be used for both the original and the proposed algorithm.

The final point worth to mention is that both of the algorithms are suitable for parallel computing, and implementation on digital signal processing units such as FPGA or GPU.

VI. CONCLUSIONS

In this paper an improved version of the PRI transform algorithm for detection of pulse repetition interval of active emitters in an interleaved pulse train is proposed. It is tried to enhance the original algorithm to manage wide range PRI detection as well as to increase the speed of the algorithm. For this purpose, some enhancements are performed in shifting time origins, setting thresholds, and making PRI bins. Simulation results show that the proposed algorithm has better performance in analyze of pulse train containing wide range PRIs. Moreover, It is much faster than the original algorithm specially when the range of PRI getting large.

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