Article - Control

TRANSACTIONS OF THE INSTITUTE OF MEASUREMENT & CONTROL

Transactions of the Institute of Measurement and Control
1–13

DOI: 10.1177/0142331220940205
journals.sagepub.com/home/tim
SAGE

A decentralized observer-based optimal control for interconnected systems using the block pulse functions

Soumaya Ghali[1], Abdelaziz Benallegue[2] and Salwa Elloumi[1]

Abstract

The paper proposes a method to integrate numerically an interconnected system, based on an idea of orthogonal approximation of functions. Here, block pulse functions (BPFs) are chosen as the orthogonal set. The main advantage of using this technique is its ability to transform the original optimal control problem to a mathematical programming problem relatively easier to solve. The primary focus of this paper is to exploit and rigorously develop the BPFs parametrization technique for the synthesis of a decentralized observer-based optimal control for large-scale interconnected systems. In addition, we develop a mathematical model of a double-parallel inverted pendulum coupled by a spring, taking into account all possible changes of the connecting position of the elastic spring. In so doing, we conducted advanced simulations applying the new optimal control method to the studied interconnected system. Our results demonstrate the validity and the effectiveness of the developed decentralized observer-based optimal control approach.

Keywords

Decentralized observer, optimal control, interconnected systems, block pulse functions (BPFs), double-parallel inverted pendulum

Introduction

In recent decades, an important part of the research activities in automatic has focused on the optimal control problem with numerous contributions to the theory. The fundamental optimal control problem formulation of a process can be summarized as follows: Since a process is given and defined by its model, we need to select among the admissible commands the one which allows, at the same time, verifying initial and final conditions given, satisfying various imposed constraints on the motion of the system and optimizing (minimize or maximize) a specified performance index. It has been applied to diverse fields such as automotive systems, energy systems, aerospace, robotics, power generation, distribution systems, etc. Because of the complexity of most applications, optimal control problems are most often solved numerically (Anderson and Moore, 2007; Anil, 2009; Fleming and Rishel, 2012; Kirk, 2012).

Numerical methods are divided into two major classes: indirect and direct methods (Von and Bulirsch, 1992). In an indirect method, the two popular solution techniques of an optimal control problem are Pontryagin's maximum principle and the Hamilton-Jacobi-Bellman equation (Bardi and Capuzzo-Dolcetta, 2008; Hartl et al., 1995). In a direct method, the optimal control problem is transcribed to a nonlinear programming problem by using discretization or parametrization techniques (Kraft, 1985; Sie-Long, 2013).

The second category of problem resolution has caught the attention of many researchers. In fact, many direct methods have been developed in the literature, such as Hermit-Simpson collocation method (Becerra, 2010), Chebyshev polynomials (Jaddu, 1998), quasi linearization (Nayyeri and Kamyad, 2014) and the block pulse functions method (Abdollahi and Babolian, 2016; Khajehnasiri, 2014; Mohan and Kar, 2012; Younespour and Ghaffarzadeh, 2015; Zarrini and Torkaman, 2014), which is the subject of our study.

Recently, the block pulse functions (BPFs) have been successfully applied to system analysis, optimal control and identification. The main characteristic of this approach is that it converts calculus (differential or integral) to algebra, which simplifies the problem solution. Dealing with the direct approach, the original optimal control problem will be converted to a mathematical programming problem relatively easier to solve by using the parameterization technique. This approach is based on the approximation of state and control variables. Compared with other approaches, the BPFs have several advantages, mainly: little sensitive to the choice of the

[1]Advanced Systems Laboratory (Laboratoire des Systèmes Avancés – LSA), Polytechnic School of Tunisia – EPT, University of Carthage, La Marsa, Tunisia
[2]Versailles Engineering Systems Laboratory (Laboratoire d'ingènierie des systémes de Versailles – LISV), University of Versailles Saint-Quentin–UVSQ, Versailles, France

Corresponding author:
Soumaya Ghali, Advanced Systems Laboratory (Laboratoire des Systèmes Avancés – LSA), Polytechnic School of Tunisia – EPT, University of Carthage, BP 743, 2078 La Marsa, Tunisia.
Email: soumayaghali1991@gmail.com

initial condition, efficiency, even complex control or state constraints can be handled easily.

On the other hand, one of the recent and most challenging problems in system theory and control is dealing with the ever-growing size and complexity of mathematical models of real-world processes. This kind of problem may become unwise, costly or even impossible in practice to solve by simply using faster computers and larger memories. What motivates this study is that this type of problem is recurrent in most of the vital aspects of our modern society.

Therefore, for stabilization analysis, control design, optimization and implementation of the strategies and algorithms, it is necessary to develop newer and more efficient methods and ideas.

The notion of 'large-scale' systems has been introduced for more than three decades as it became clear that there are practical control problems that cannot be tackled by classical on-shot methods. The general approach consists of dividing the problem into simpler and smaller subproblems which are only weakly dependent or can be treated completely independently and hence are easier to deal with. Since introducing decentralized control, it has been always considered an important control choice for interconnected systems. In this way, the decentralized optimal control partitions the measurement information and elaborates local and independent control law for each subsystem (Bakule, 2014; Feydi et al., 2019; Khorsand and Gattami, 2011; Mahmoud, 2011; Rtibi et al., 2017, 2018).

Besides, it is well known that observer design is an important problem in many practical situations since the state variables are often incompletely known at each subsystem for decentralized optimal control. In this context, one has to consider state decentralized feedback control based on designing a state observer able to reconstruct the state of individual subsystems (Abidi and Elloumi, 2020; Bakule, 2008; Elloumi et al., 2016; Gao et al., 2014; Lendek et al., 2008).

In this work, we introduced a novel numerical method, based on the principle of orthogonal approximation of functions, to solve the optimal control problem. By this means, we transform the original optimal control problem into a mathematical programming problem relatively easier to solve by the parameterization technique. We used the BPFs to parameterize the state and control variables, then synthesize a solution for decentralized observer-based optimal control for large-scale interconnected systems. Consequently, we reduced the performance index to a single algebraic equation in terms of the block pulse coefficients of the state and control variables. Besides, we developed in detail a mathematical model of two inverted pendulums interconnected by a spring, taking into account all possible changes of the connecting position of the elastic spring. Indeed, for the design of the decentralized control scheme, each pendulum should be seen as a subsystem.

The paper is organized according to the following outline: The second section is devoted to formulate the problem and to describe the studied system. In the third section, first we expose the basic properties of the BPFs, and then focus on the extension to decentralized optimal control with observers using BPFs. In the fourth section, in order to develop an efficient control of the double-parallel inverted pendulum, the system configuration and its mathematical model are established in detail in the first part, and simulations results are implemented in the second part in order to show the efficiency of the proposed investigated approach. Finally, conclusions and future scope of the study are drawn.

System description and problem formulation

The first definition of an 'interconnected system' is a system that can be decomposed into a set of elementary systems S_i, $i = 1, ..., n$ and an interconnection H.

In this paper, we study a class of large-scale interconnected systems; each subsystem is described by the following state equations:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + h_i(t,x) \\ y_i(t) = C_i x_i(t); \quad i = 1, 2, .., n \end{cases} \tag{1}$$

where

- $x_i(t) = x_i \in \Re^{n_i}$ is the state vector of the i^{th} subsystem;
- $u_i(t) = u_i \in \Re^{m_i}$ is the control vector of the i^{th} subsystem;
- $y_i(t) = y_i \in \Re^{p_i}$ is the output vector of the i^{th} subsystem;
- $h_i(t,x)$ reflects the interconnection term of the i^{th} subsystem with other subsystems and the uncertainty dynamics from the i^{th} subsystem (Siljak and Stipanovic, 2000)

$$h_i^T(t,x) h_i(t,x) \leq \alpha_i^2 x^T H_i^T H_i x \tag{2}$$

- $\alpha_i > 0$ are bounding parameters to be maximized and H_i are bounding matrices.
- $A_i \in \Re^{n_i \times n_i}$, $B_i \in \Re^{n_i \times m_i}$ and $C_i \in \Re^{p_i \times n_i}$ denote respectively the state matrix, the control matrix and the output matrix of the i^{th} subsystem.
- (A_i, B_i) is assumed to be controllable.
- (A_i, C_i) is assumed to be observable.

The overall interconnected system formed by n subsystems can then be represented by the following compact form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + h(t,x) \\ y(t) = Cx(t) \end{cases} \tag{3}$$

where

- $x^T = [x_1{}^T, x_2{}^T, ..., x_n{}^T]$ is the state vector of the global system;
- $u^T = [u_1{}^T, u_2{}^T, ..., u_n{}^T]$ is the control vector of the global system;
- $y^T = [y_1{}^T, y_2{}^T, ..., y_n{}^T]$ is the output vector of the global system;
- $h^T(t,x) = \left[h_1^T, ..., h_n^T\right]^T$ is the nonlinear interconnection functions, where $h(t,x)$ is bounded as follows:

$$h^T(t,x)h(t,x) \leqslant x^T\left(\sum_{i=1}^{N}\alpha_i^2 H_i^T H_i\right)x \tag{4}$$

- $A = diag[A_i]$, $B = diag[B_i]$, $C = diag[C_i]$
- the pair (A, B) is controllable.
- the pair (A, C) is observable.

To respect the decentralized information structure constraint, the i^{th} subsystem is controlled by the local control law:

$$u_i(x_i) = -K_i x_i, \qquad i = 1, ..., n \tag{5}$$

The control law of the overall system, which is a collection of the individual local laws, represents the block-diagonal form

$$u(x) = -Kx \tag{6}$$

where $K = diag(K_1, K_2, \ldots, K_n)$ is the block diagonal control gain matrix.

Applying this feedback optimal control law, one obtains the closed-loop equation for the dynamical interconnected system

$$\dot{x} = (A - BK)x + h(t,x) \tag{7}$$

where

$$(A - BK) = \begin{bmatrix} A_1 - B_1K_1 & A_{12} & \ldots & A_{1n} \\ A_{21} & A_2 - B_2K_2 & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & \ldots & \ldots & A_n - B_nK_n \end{bmatrix} \tag{8}$$

We now come back to some very important concepts in system theory. The methods which use state feedback naturally assume that the states of each subsystem are ready to use. Therefore, if we are not able to directly measure the states of the system, a decentralized observer should be designed. To construct state estimators, we will form the dynamic equation of the subsystem observers as follows:

$$\begin{cases} \dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_iu_i(t) + L_i[y_i(t) - \hat{y}_i(t)] \\ \hat{y}_i(t) = C_i\hat{x}_i(t); \quad i = 1, \ldots, n \end{cases} \tag{9}$$

where

- $L_i \in \Re^{n_i \times p_i}$ is the observation gain matrix of the i^{th} subsystem;
- $\hat{x}_i$ is the state observation of the i^{th} subsystem.

The control law of each subsystem using the estimated state is given by

$$u_i(x_i) = -K_i\hat{x}_i \tag{10}$$

forming the estimation error equation between the i^{th} actual state and the i^{th} observer output

$$e_i = x_i - \hat{x}_i \tag{11}$$

So, the dynamic error becomes

$$\dot{e}_i = [A_i - L_iC_i]e_i + h_i(t,x) \tag{12}$$

Consider again the overall system, hence, the decentralized observer of the global system formed by local observers can be rewritten as

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - \hat{y}(t)] \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{13}$$

where

- $\hat{x}^T(t) = \hat{x}^T = \left[\hat{x}_1{}^T, \hat{x}_2{}^T, \ldots, \hat{x}_n{}^T\right]$ is the state observer of the global system;
- $L = diag[L_i]$ is the block diagonal observation gain matrix of the overall system.

Thus, we consider the global system observation error described by the following dynamical equation:

$$\dot{e} = [A - LC]e + h(t,x) \tag{14}$$

and then, the control law of the global system will be designed using the observed state vector as follows:

$$u(x) = -K\hat{x} \tag{15}$$

Finally, the development of the overall interconnected system, using the control law, leads to obtain the following augmented state model:

$$\begin{cases} \dot{x} = (A - BK)x + BKe + h(t,x) \\ \dot{e} = [A - LC]e + h(t,x) \\ y = Cx \end{cases} \tag{16}$$

Then, considering the global system (16) and the overall system observation error (14), the augmented system can be expressed as

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{H} \tag{17}$$

with $\tilde{A} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}$, $\tilde{x} = \begin{bmatrix} x \\ e \end{bmatrix}$ and $\tilde{H} = \begin{bmatrix} h(t,x) \\ h(t,x) \end{bmatrix}$

The last stage in this section is to optimize a quadratic cost performance defined by

$$J_i = \frac{1}{2}\int_0^\infty \left(y_i{}^TQ_iy_i + e_i{}^TQ_{i0}e_i + u_i{}^TR_iu_i\right)dt \tag{18}$$

where

- $Q_i(n_i \times n_i)$ and $Q_{i0}(n_i \times n_i)$ are semi positive definite matrices, that represent respectively the output and the error weighting matrices;
- $R_i(m_i \times m_i)$ are positive definite matrices, representing the input weighting matrices.

The overall criterion is expressed as

$$J = \sum_{i=1}^{N} J_i = \frac{1}{2}\int_0^\infty \left(y^T Qy + e^T Q_0 e^T + u^T Ru\right) dt \qquad (19)$$

where $Q = diag[Q_i]$; $Q_0 = diag[Q_{0i}]$; $R = diag[R_i]$

By using the control law, the last equation can be written as

$$J = \frac{1}{2}\int_0^\infty \tilde{x}^T \tilde{Q}\tilde{x}dt \qquad (20)$$

with $\tilde{Q} = \begin{bmatrix} Q_1 + K^T RK & -K^T RK \\ -K^T RK & Q_2 + K^T RK \end{bmatrix}$

$$Q_1 = C^T QC\,;\, Q_2 = Q_0$$

Proposed approach for decentralized observer-based optimal control resolution

First, we give a brief overview to specify our strict framework, that is to say, the BPFs parameterization technique. Then, the challenging tasks investigated in this section will concern the extension of the direct approach (BPFs) to decentralized optimal control with observers of interconnected systems (Bichiou et al., 2018; Dadkhah and Farahi, 2015; Ghali et al., 2017a, b; Hosseinpour and Nazemi, 2016; Marzban, 2016; Tang et al., 2017; Warrad et al., 2018; Xie and Huang, 2016; Ziari et al., 2019).

Properties of block pulse functions

Recently, the BPFs technique has been successfully applied to system analysis, control and identification. The main characteristic of this approach is that it converts calculus (differential or integral) to algebra, thus simplifying the problem solution.

In this part, the BPFs and its important properties are introduced in detail. We discuss the integration and multiplication properties formulas associated with BPFs that are important for this work.

Definition of BPFs. An m-set of BPFs defined on a unit interval $[0, 1)$ as follows:

For each integer i, $1 \leqslant i \leqslant m$, the function $\varphi_i(t)$ is given by

$$\varphi_i(t) = \begin{cases} 1 & \text{if } t \in [t_0 + (i-1)h\,,\, t_0 + ih[\\ 0 & \text{otherwise;} \end{cases} \qquad (21)$$

where $i = 1, \ldots, m$ with positive integer values, φ_i is called the *ith* BPF.

$$h = \frac{t_f - t_0}{m} \qquad (22)$$

is the block-pulse width along the time scale, and

$$\varphi_m(t_f) = 1 \qquad (23)$$

These BPFs verify the under mentioned properties. The most important of these properties are disjointness, orthogonality and completeness that can be expressed as follows:

Disjointness: The BPFs are disjoint with each other

$$\varphi_i(t)\varphi_j(t) = \begin{cases} \varphi_i(t) & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \qquad (24)$$

with $t \in [0, h)\,;\, i, j = 1, \ldots, m$

Orthogonality: The BPFs are orthogonal with each other, it is clear that

$$\int_{t_0}^{t_f} \varphi_i(t)\varphi_j(t)dt = \begin{cases} h & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \qquad (25)$$

with $t \in [0, h)\,;\, i, j = 1, \ldots, m$

Completeness: For every $f \in L^2$, $\{\varphi\}$ is complete if $\int \varphi f = 0$ then $f = 0$ almost everywhere. Because of completeness of $\{\varphi\}$, Parseval's identity holds:

$$\int_0^T f^2(t)dt = \sum_{i=1}^{\infty} \|\varphi_i(t)\|^2 \qquad (26)$$

Function approximation. This set of functions can be concisely described by an m-vector $\varphi(t)$ with $\varphi_i(t)$ as its i^{th} component. It is well known that a function which is integrated into $(0, 1)$ can be approximated as

$$f(t) \approx \sum_{i=1}^{m} f_i \varphi_i(t) = F^T \Phi(t) \qquad (27)$$

where F is a m-vector given by

$$F = [f_1(t), f_2(t), \ldots, f_m(t)]^T \qquad (28)$$

Also, the set of BPFs is written as

$$\Phi(t) = [\varphi_1(t), \varphi_2(t), \ldots, \varphi_m(t)]^T \qquad (29)$$

where the coefficients f_i are determined such that the following integral square error is minimized:

$$\varepsilon = \int_0^1 \left(f(t) - \sum_{i=1}^{m} f_i \varphi_i(t)\right)^2 dt \qquad (30)$$

In fact, f_i is defined by

$$f_i = \frac{1}{h}\int_{t_0 + (i-1)h}^{t_0 + ih} f(t)dt \qquad (31)$$

where f_i is the average value of $f(t)$ over $t_0 + (i-1)h \leqslant t \leqslant t_0 + ih$

Note that the approximation in (27) gives the best m-segment piecewise constant approximation of $f(t)$ and is unique whatever may be the set of basis functions.

Matrix of integration. In the beginning, to get an interrelationship between the matrices, we use the following identity:

$$\int_0^t \dot{x}(t)dt = x(t) - x(0) \tag{32}$$

Now we would like to expand the integral of $\Phi(t)$ concerning t and we express the result into its BPF series. Arranging the coefficients of expansion in a matrix form, we have

$$\int_{t_0}^t \phi(\tau)d\tau \approx M\Phi(t) \tag{33}$$

These integrals are

$$\int_{t_0}^{t} \varphi_i(t)dt = \frac{h}{2}\varphi_i(t) + h\sum_{j=i+1}^{m}\varphi_j(t) \tag{34}$$

$$\int_{t_f}^{t} \varphi_i(t)dt = -\frac{h}{2}\varphi_i(t) - h\sum_{j=1}^{i-1}\varphi_j(t) \tag{35}$$

$i = 1, 2, ..., m$ where M is an $m \times m$ matrix, which is upper triangular given by

$$M = h\begin{pmatrix} \frac{1}{2} & 1 & 1 & \dots & 1 \\ 0 & \frac{1}{2} & 1 & \dots & 1 \\ 0 & 0 & \frac{1}{2} & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \frac{1}{2} \end{pmatrix} \tag{36}$$

The significance of the matrix M is that the integration of $\Phi(t)$ can be approximately (in the least-square error sense) achieved by pre-multiplying $\Phi(t)$ by the constant matrix M. For this reason, M is called the operational matrix for BPFs.

Extension to decentralized optimal control with observers using BPFs

Having considered larger and more complex processes, the optimal control problem of these systems arises with new acuity. Direct application of optimal control to large systems poses problems – that is to say, calculation problems directly related to the size of these systems, implementation problems of the structure control, etc.

Decentralized control responds directly to its obstacles. The general idea is to decompose a system into several subsystems that are easier to handle (the interconnections are handled by a coordinator managing the entire structure).

In addition to that, control systems designed for the decoupled problems provide a completely decentralized control structure for the original problem, as illustrated in Figure 1.

First of all, the BPFs have exceptional properties that act directly on the problem of optimal control, whatever its nature. Using this direct approach, the optimal control problem can be converted to a mathematical programming problem solved numerically by using the discretization or the parametrization technique. In this part, we propose a study related to the analysis of parameterization methods. Three

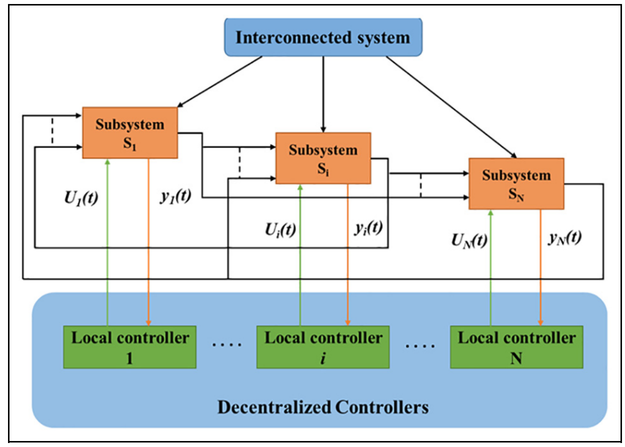

Figure 1. Decentralized control structure.

cases can be considered: the control parameterization, the state parameterization and the state-control parameterization.

Thanks to the integration properties, we first implement the BPFs as follows:

$$\begin{cases} x(t) - x(0) = (A - BK)\int_0^t x(t)dt + BK\int_0^t e(t)dt + h(t,x) \\ e(t) - e(0) = (A - LC)\int_0^t e(t)dt + h(t,x) \end{cases} \tag{37}$$

We have opted for the state-control parameterization technique. The idea behind this technique is to approximate, at the same time, the state and control variables

$$x(t) \cong \sum_{i=1}^{m} x_i\varphi_i(t) \cong x_N\Phi_N(t) \tag{38}$$

$$u(t) \cong \sum_{i=1}^{m} u_i\varphi_i(t) \cong u_N\Phi_N(t) \tag{39}$$

where $x_N \in \Re^{n\times m}$, $u_N \in \Re^{r\times m}$ and $\Phi_N(t) \in \Re^{m\times 1}$.

Similarly, we expand the observer error $e(t)$ into BPFs as follows:

$$e(t) = e_N\Phi_N \tag{40}$$

Let consider the integration operational matrix represented by (37), which becomes:

$$\begin{cases} (x_N - x_{0N})\Phi_N = (A - BK)x_N M\Phi(t) + BKe_N M\Phi(t) + h(t,x) \\ (e_N - e_{0N})\Phi_N = (A - LC)e_N M\Phi(t) + h(t,x) \end{cases} \tag{41}$$

It leads us easily to get the following algebraic equation:

$$\begin{cases} x_N - x_{0N} = (A - BK)x_N M + BKe_N M + h(t,x) \\ e_N - e_{0N} = (A - LC)e_N M + h(t,x) \end{cases} \tag{42}$$

Combining the application of the integration and multiplication properties, the state equation and the observation error can be transformed into a sequence of algebraic equations which have the following form:

$$\begin{cases} x_1 - x(0) = \frac{1}{2}h[(A_1 - B_1K_1)x_1 + B_1K_1e_1 + h_1(t,x)] \\ \vdots \\ x_k - x(0) = \frac{1}{2}h[(A_k - B_kK_k)x_k + B_kK_ke_k + h_k(t,x)] \\ \qquad + h\sum_{i=1}^{k-1}[(A_i - B_iK_i)x_i + B_iK_ie_i + h_i(t,x)] \\ \vdots \\ x_m - x(0) = \frac{1}{2}h[(A_m - B_mK_m)x_m + B_mK_me_m + h_m(t,x)] \\ \qquad + h\sum_{i=1}^{m-1}[(A_i - B_iK_i)x_i + B_iK_ie_i + h_i(t,x) \end{cases} \tag{43}$$

$k = 1, \ldots, m$

$$\begin{cases} e_1 - e(0) = \frac{1}{2}h[(A_1 - L_1C_1)e_1 + h_1(t,x)] \\ \vdots \\ e_k - e(0) = \frac{1}{2}h[(A_k - L_kC_k)e_k + h_k(t,x)] + h\sum_{i=1}^{k-1}[(A_i - L_iC_i)e_i + h_i(t,x)] \\ \vdots \\ e_m - e(0) = \frac{1}{2}h[(A_m - L_mC_m)e_m + h_m(t,x)] + h\sum_{i=1}^{m-1}[(A_i - L_iC_i)e_i + h_i(t,x)] \end{cases} \tag{44}$$

where

$$x_k = \begin{pmatrix} {X_1}^{x_k} \\ {X_2}^{x_k} \\ \vdots \\ {X_m}^{x_k} \end{pmatrix} \text{ and } u_k = \begin{pmatrix} {U_1}^{u_k} \\ {U_2}^{u_k} \\ \vdots \\ {U_m}^{u_k} \end{pmatrix}$$

We notice that the differential equations given by (16) are transformed, through BPFs, in a set of algebraic equations given by (43) and (44).

In the same way, the performance index is reduced to a single algebraic equation having the following form:

$$J = \frac{1}{2}h\sum_{k=1}^{m}\left({x_k}^T {c_k}^T Q_k c_k x_k + {e_k}^T Q_{0k} {e_k}^T + {u_k}^T R_k u_k\right) \tag{45}$$

We get the expression of the optimal state-feedback of the global system using the observed state vector as follows:

$$u_k(x) = -K\hat{x}_k \tag{46}$$

For this reason, the initial optimal control problem, which is described by (16) and (19), is converted to a numerical resolution problem of a set of algebraic equations expressed by (43), (44) and (45).

The resolution of this mathematical programming problem leads to the determination of the control gain K and the observed gain L.

Application of the proposed optimal control approach to a double-parallel inverted pendulum

The problem of balancing an inverted pendulum has been a reference problem demonstrating various control design techniques. The main reasons for its popularity are its nonlinear, unstable and non-minimum-phase characteristics. This system has been widely applied in different areas such as rocket control, robot system, balancing mechanism and wheeled motion.

Mathematical modelling of the studied system

To prove the efficiency of the proposed approach, we establish a mathematical model of a double-parallel inverted pendulum coupled by a spring, taking into account all possible changes of the connecting position of the elastic spring.

The studied system is graphically depicted in Figure 2. It consists of two identical inverted pendulums directly mounted on the motor shafts in parallel where $\tau_i (i = 1, 2)$ are the input torques of each motor. These pendulums are connected by an elastic spring of constant k which is mounted at the height a.

θ_1 and θ_2 are the angular displacement of each inverted pendulum from the vertical, respectively.

To derive its equations of motion, one of the possible ways is to use Lagrange equations, which are given as follows:

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = \tau_1 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = \tau_2 \end{cases} \tag{47}$$

The Lagrangian, denoted by L, is expressed as follows:

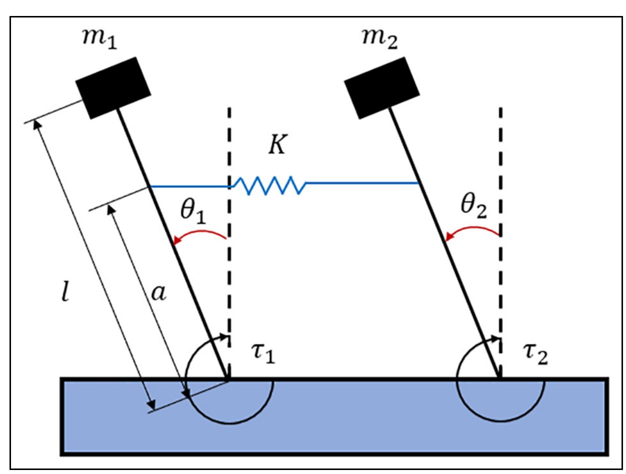

Figure 2. System configuration: Double-parallel inverted pendulum.

$$L = E_c - E_p \tag{48}$$

where E_c is the kinetic energy and E_p is the potential energy. The kinetic energy for each pendulum is described by the following form:

$$E_{ci} = \frac{1}{2} J_i \dot{\theta}_i^{\,2} \tag{49}$$

where J_i is the moment of inertia and $\dot{\theta}_i$ is the angular velocity of the i^{th} pendulum. The total kinetic energy of the interconnected system is gained from the sum of two equations:

$$E_c = \frac{1}{2} J_1 \dot{\theta}_1^{\,2} + \frac{1}{2} J_2 \dot{\theta}_2^{\,2} = \frac{1}{2} m_1 l^2 \dot{\theta}_1^{\,2} + \frac{1}{2} m_2 l^2 \dot{\theta}_2^{\,2} \tag{50}$$

Now we calculate the potential energy of the system that consists of three terms: one for each mass and one for the spring.

Indeed, the gravitational potential energy for each mass is expressed as follows:

$$E_{pi} = m_i g l cos \theta_i \tag{51}$$

The potential energy of the spring is calculated using Hooke's law:

$$U_{spring} = \frac{1}{2} k x^2 \tag{52}$$

where

$$x = \sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin(\theta_1) - \sin(\theta_2)) - 2a^2 \cos(\theta_1 - \theta_2)} - l_0 \tag{53}$$

$$\begin{aligned} x^2 &= 2a^2 + 2l_0^{\,2} + 2al_0(\sin(\theta_1) - \sin(\theta_2)) - 2a^2 \cos(\theta_1 - \theta_2) \\ &\quad - 2l_0 \sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin(\theta_1) - \sin(\theta_2)) - 2a^2 \cos(\theta_1 - \theta_2)} \end{aligned} \tag{54}$$

Then

$$\begin{aligned} U_{spring} &= k\big(a^2 + l_0^{\,2} + al_0(\sin(\theta_1) - \sin(\theta_2)) - a^2 \cos(\theta_1 - \theta_2) \\ &\quad - l_0 \sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin(\theta_1) - \sin(\theta_2)) - 2a^2 \cos(\theta_1 - \theta_2)}\Big) \end{aligned} \tag{55}$$

where x is the spring length change and l_0 represents the spring slack length.

Thus, the total potential energy of the system is represented as follows:

$$\begin{aligned} E_p &= m_1 g l \cos\theta_1 + m_2 g l \cos\theta_2 \\ &\quad + k\big(a^2 + l_0^{\,2} + al_0(\sin(\theta_1) - \sin(\theta_2)) \\ &\quad - a^2 \cos(\theta_1 - \theta_2) \\ &\quad - l_0 \sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin(\theta_1) - \sin(\theta_2)) - 2a^2 \cos(\theta_1 - \theta_2)}\Big) \end{aligned} \tag{56}$$

By substituting equations (50) and (56) in equation (48), the Lagrangian of the interconnected studied system is rewritten as follows:

$$\begin{aligned} L &= \frac{1}{2} m_1 l^2 \dot{\theta}_1^{\,2} + \frac{1}{2} m_2 l^2 \dot{\theta}_2^{\,2} - m_1 g l \cos\theta_1 - m_2 g l \cos\theta_2 \\ &\quad - k\big(a^2 + l_0^{\,2} + al_0(\sin(\theta_1) - \sin(\theta_2)) - a^2 \cos(\theta_1 - \theta_2) \\ &\quad - l_0 \sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin(\theta_1) - \sin(\theta_2)) - 2a^2 \cos(\theta_1 - \theta_2)} \end{aligned} \tag{57}$$

We apply the equations (47) separately on the equation (57). Then, we obtain the nonlinear equations of motion for the system.

For the first inverted pendulum:

$$\begin{aligned} &\frac{\partial L}{\partial \dot{\theta}_1} = m_1 l^2 \dot{\theta}_1 \\ &\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}_1}\right] = m_1 l^2 \ddot{\theta}_1 \\ &\frac{\partial L}{\partial \theta_1} = m_1 g l \sin\theta_1 \\ &- k\left(al_0 \cos(\theta_1) - l_0 \left(\frac{2al_0 \cos(\theta_1)}{2\sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin\theta_1 - \sin\theta_2) - 2a^2 \cos(\theta_1 - \theta_2)}}\right)\right) \end{aligned} \tag{58}$$

So, for the first pendulum the equation of motion is

$$\begin{aligned} &m_1 l^2 \ddot{\theta}_1 - m_1 g l \sin\theta_1 \\ &+ k\left(al_0 \cos(\theta_1) - \frac{al_0^{\,2} \cos(\theta_1)}{\sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin\theta_1 - \sin\theta_2) - 2a^2 \cos(\theta_1 - \theta_2)}}\right) = \tau_1 \end{aligned} \tag{59}$$

Now, for the second inverted pendulum:

$$\begin{aligned} &\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l^2 \dot{\theta}_2 \\ &\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}_2}\right] = m_2 l^2 \ddot{\theta}_2 \\ &\frac{\partial L}{\partial \theta_2} = m_2 g l \sin\theta_2 \\ &- k(-al_0 \cos(\theta_2) - l_0 \\ &\left(\frac{-2al_0 \cos(\theta_2)}{2\sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin\theta_1 - \sin\theta_2) - 2a^2 \cos(\theta_1 - \theta_2)}}\right)\Bigg) \end{aligned} \tag{60}$$

The motion equation of the second pendulum is then

$$\begin{aligned} &m_2 l^2 \ddot{\theta}_2 - m_2 g l \sin\theta_2 \\ &+ k\left(-al_0 \cos(\theta_2) + \frac{al_0^{\,2} \cos(\theta_2)}{\sqrt{2a^2 + l_0^{\,2} + 2al_0(\sin\theta_1 - \sin\theta_2) - 2a^2 \cos(\theta_1 - \theta_2)}}\right) = \tau_2 \end{aligned} \tag{61}$$

Using Lagrange equations (47), we showed that the nonlinear equations of motion of the double inverted pendulum system are

$$\begin{cases} m_1 l^2 \ddot{\theta}_1 - m_1 g l sin\theta_1 + k\left(a l_0 cos(\theta_1) - \dfrac{a l_0^2 cos(\theta_1)}{\sqrt{2a^2 + l_0^2 + 2a l_0(sin\theta_1 - sin\theta_2) - 2a^2 cos(\theta_1 - \theta_2)}}\right) = \tau_1 \\ m_2 l^2 \ddot{\theta}_2 - m_2 g l sin\theta_2 + k\left(-a l_0 cos(\theta_2) + \dfrac{a l_0^2 cos(\theta_2)}{\sqrt{2a^2 + l_0^2 + 2a l_0(sin\theta_1 - sin\theta_2) - 2a^2 cos(\theta_1 - \theta_2)}}\right) = \tau_2 \end{cases} \quad (62)$$

Assuming a small angular displacement, we linearize this set of equations around the equilibrium point $\theta_1 = \theta_2 = 0$. In addition, for simplicity reasons, rigid body motion, lumped mass and linear spring are assumed; the dynamics of the motor, the mechanical friction in the motor and the spring guider are neglected. So, the nonlinear equations of motion (62) can be replaced by the following linear model:

$$\begin{cases} m_1 l^2 \ddot{\theta}_1 = m_1 g l \theta_1 - k a^2(\theta_1 - \theta_2) - \tau_1 \\ m_2 l^2 \ddot{\theta}_2 = m_2 g l \theta_1 - k a^2(\theta_1 - \theta_2) - \tau_2 \end{cases} \quad (63)$$

First, each inverted pendulum is seen as a subsystem having a state vector

$$x_i(t) = \begin{bmatrix} \theta_i(t) \\ \dot{\theta}_i(t) \end{bmatrix} \quad (64)$$

Then, the system composed of two parallel inverted pendulums is described by the following state equations:

$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_1 u_1 + H_1 x_2 \\ \dot{x}_2 = A_2 x_2 + B_2 u_2 + H_2 x_1 \end{cases} \quad (65)$$

with

- x_1, x_2 the state vectors of the subsystems;
- u_1, u_2 the control vectors of the subsystem such as the input torque of each motor.

Thus, the disconnected subsystem can be expressed by the following system matrices:

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{l} - \dfrac{ka^2}{m_1 l^2} & 0 \end{bmatrix};\ A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{l} - \dfrac{ka^2}{m_2 l^2} & 0 \end{bmatrix};$$

$$B_1 = \begin{bmatrix} 0 \\ \dfrac{-1}{m_1 l^2} \end{bmatrix};\ B_2 = \begin{bmatrix} 0 \\ \dfrac{-1}{m_2 l^2} \end{bmatrix};$$

$$H_1 = \begin{bmatrix} 0 & 0 \\ \dfrac{ka^2}{m_1 l^2} & 0 \end{bmatrix};\ H_2 = \begin{bmatrix} 0 & 0 \\ \dfrac{ka^2}{m_2 l^2} & 0 \end{bmatrix}$$

Accordingly, the global system can be expressed by the following state vector:

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \theta_1(t) \\ \dot{\theta}_1(t) \\ \theta_2(t) \\ \dot{\theta}_2(t) \end{bmatrix} \quad (66)$$

Thus, the global system formed by two identical inverted pendulum coupled by a spring can be defined by the following global state representation:

$$\dot{x} = Ax + Bu + Hx \quad (67)$$

with

- $x^T = [x_1^T, x_2^T]$ is the state vector;
- $u^T = [u_1^T, u_2^T]$ is the control vector;
- $A = diag(A_1, A_2)$ is the characteristic matrix of the global system:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{g}{l} - \dfrac{ka^2}{m_1 l^2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{g}{l} - \dfrac{ka^2}{m_2 l^2} & 0 \end{bmatrix}$$

- $B = diag(B_1, B_2)$ is the control matrix:

$$B = \begin{bmatrix} 0 & 0 \\ \dfrac{-1}{m_1 l^2} & 0 \\ 0 & 0 \\ 0 & \dfrac{-1}{m_2 l^2} \end{bmatrix}$$

- H is the matrix formed by the terms of interconnection:

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{ka^2}{m_1 l^2} & 0 \\ 0 & 0 & 0 & 0 \\ \dfrac{ka^2}{m_2 l^2} & 0 & 0 & 0 \end{bmatrix}$$

Simulation study

In order to check the validity and the effectiveness of the proposed decentralized optimal control strategy developed in the

Table 1. Detailed specifications.

Notations	Physical Meaning	Value	Unit
m_1	The mass of the first pendulum	0.4489	kg
m_2	The mass of the second pendulum	0.4496	kg
l	The rod length	0.325	m
a	The connecting position of the spring	0.21	m
k	The stiffness of spring	340.22	N/m
g	The acceleration of gravity	9.81	Ms^{-2}

previous section, a numerical simulation test has been performed on a double interconnected inverted pendulum.

Detailed specifications used in the studied system are listed in Table 1.

According to the model (3), the studied double inverted pendulum system can be described by the following state equations:

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 286.2486 & 0 & 316.4332 & 0 \\ 0 & 0 & 0 & 1 \\ 315.9406 & 0 & -285.7560 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ -21.0903 & 0 \\ 0 & 0 \\ 0 & -21.0575 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x \end{cases} \tag{68}$$

It is interesting, first of all, to note that the multiplication of the weightings factors Q, Q_0 and R by the same scalar leaves the gains K and L unchanged. Indeed, the weights are generally chosen diagonally.

By this way, the weighting factors are chosen such that

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, Q_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$and\, R = \begin{bmatrix} 0.0043 & 0 \\ 0 & 0.0043 \end{bmatrix}$$

After applying the BPFs parameterization and the use of the gain matrices, the problem described by (16) will be transformed into a nonlinear programming problem solved numerically using Fmincon function of Matlab toolbox.

We obtain the control gain matrices:

$$K_1 = [-5.0148 \quad -0.4657]$$

$$K_2 = [-5.0134 \quad -0.4418]$$

and the observation gain matrices:

$$L_1 = \begin{bmatrix} 2.2533 \\ -0.1423 \end{bmatrix}, L_2 = \begin{bmatrix} 2.2633 \\ -0.1432 \end{bmatrix}$$

It is well known that this problem can be reduced to a linear optimization problem. Under equations (43) and (44), the problem is stated as: it is a question of determining the coefficients $X_i^{x_1}$, $X_i^{x_2}$, $X_i^{x_3}$, $X_i^{x_4}$, $U_i^{u_1}$ and $U_i^{u_2}$ that minimize the following performance criterion:

$$\begin{aligned} J = \frac{1}{2}h\sum_{i=1}^{m}\left((X_i^{x_1})^2 + (X_i^{x_2})^2 + (X_i^{x_3})^2 + (X_i^{x_4})^2\right) \\ + \frac{1}{2}h\sum_{i=1}^{m}\left(0.0043(U_i^{u_1})^2 + 0.0043(U_i^{u_2})^2\right) \end{aligned} \tag{69}$$

The performances of the proposed decentralized observer-based optimal control approach using the BPFs are demonstrated in Figures 3 to 8 and implemented on a double-parallel inverted pendulum coupled by a spring, in which are simulated the evolution of the real state variables, their observers and the corresponding decentralized optimal control signals of the studied system respectively to $m = 20$, $m = 40$, $m = 60$, $m = 80$, $m = 100$ and $m = 120$.

From the simulation results shown in these curves, it can be clearly seen that the decentralized control approach using the BPFs produces a piecewise constant solution for each value of $k = 1, ..., m$. Also, it is shown that the direct method is able to enhance the stability of the studied system in approximately 4 seconds. Consequently, we can demonstrate numerically that the decentralized optimal state observers for the interconnected system allow the reconstruction of the state variables with a good performance. This proves the applicability, the validity and the effectiveness of the BPFs technique.

To show the efficiency of the BPFs designed, the following scenario is considered:

1. Note that these curves are quantified; nevertheless, this difficulty has become indiscernible if the parameter m is large.
2. The calculation time takes account of one parameter: the number m.
3. By taking for m a big number, that is to say, for a good approximation we risk wasting precious time in the execution of the algorithm, then we should make a compromise between these parameters.

Conclusion

The content of this paper concerns the synthesis of decentralized observer-based optimal control for large-scale interconnected systems using the BPFs parametrization technique. Based on this approach, the optimal control problem is transformed to a nonlinear programming problem solved mathematically.

The BPFs direct approach was implemented on a double-parallel inverted pendulum coupled by a spring. Simulation results have shown the aptitude of the direct approach, using BPFs, to be implemented easily and to give a satisfactory result, which succeeds to ensure quickly the stability and improve the performance of the studied system.

Many interesting directions for future research remain. One of the possible perspectives is to develop a receding horizon control through the BPFs for interconnected systems.

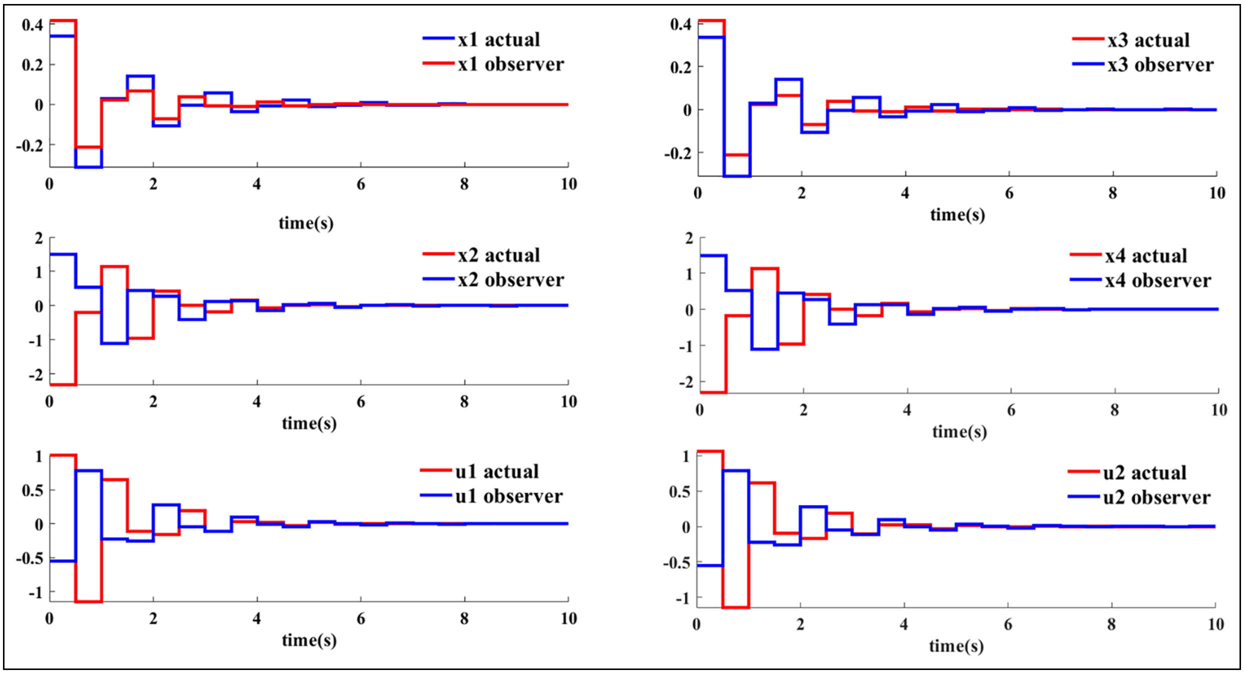

Figure 3. Evolution of the real state variables, their observers and the corresponding decentralized optimal control signals of the global system for $m = 20$.

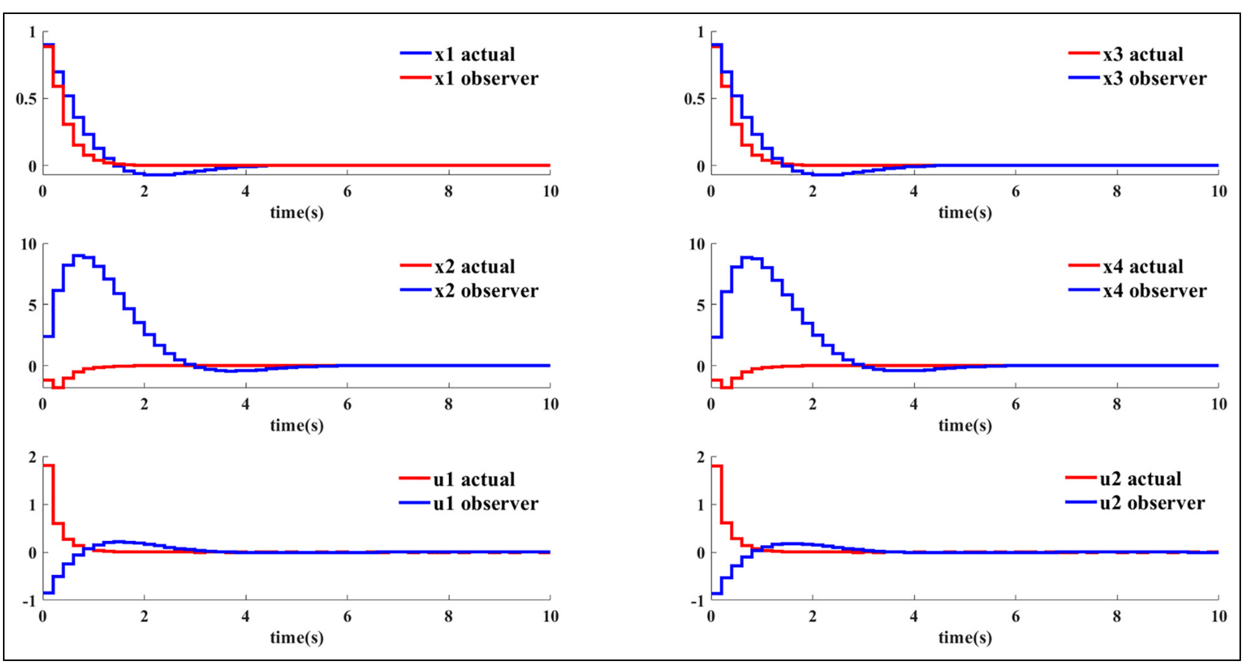

Figure 4. Evolution of the real state variables, their observers and the corresponding decentralized optimal control signals of the global system for $m = 40$.

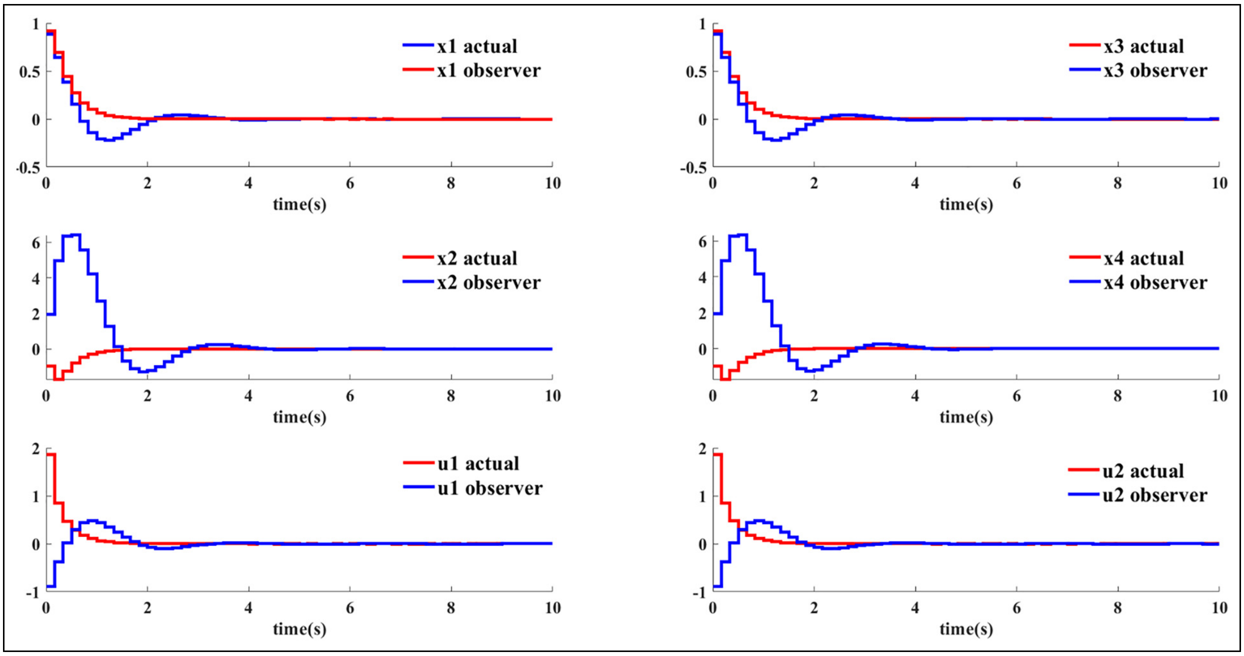

Figure 5. Evolution of the real state variables, their observers and the corresponding decentralized optimal control signals of the global system for $m = 60$.

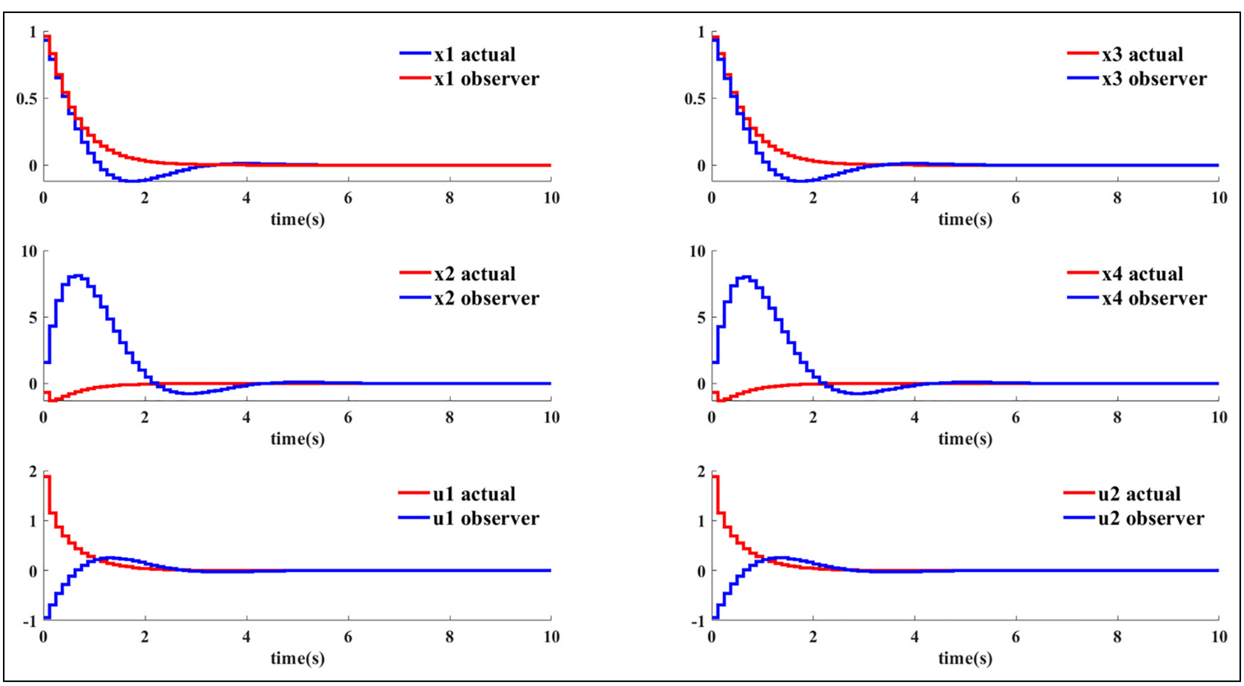

Figure 6. Evolution of the real state variables, their observers and the corresponding decentralized optimal control signals of the global system for $m = 80$.

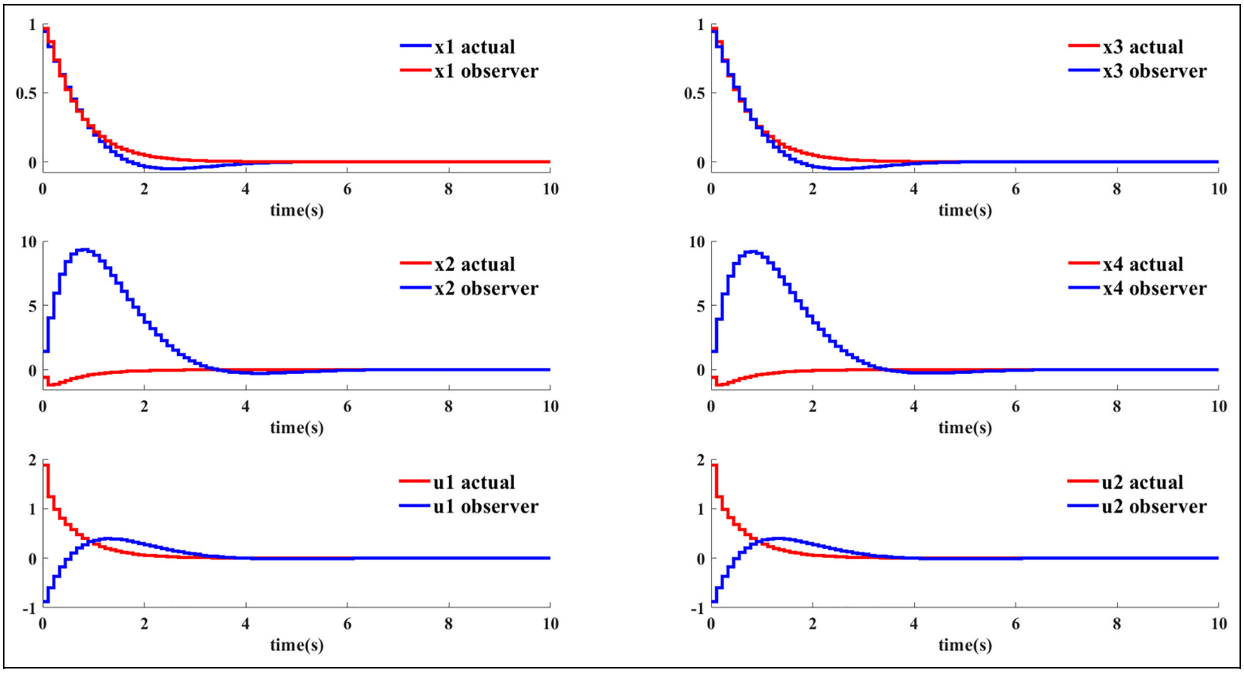

Figure 7. Evolution of the real state variables, their observers and the corresponding decentralized optimal control signals of the global system for $m = 100$.

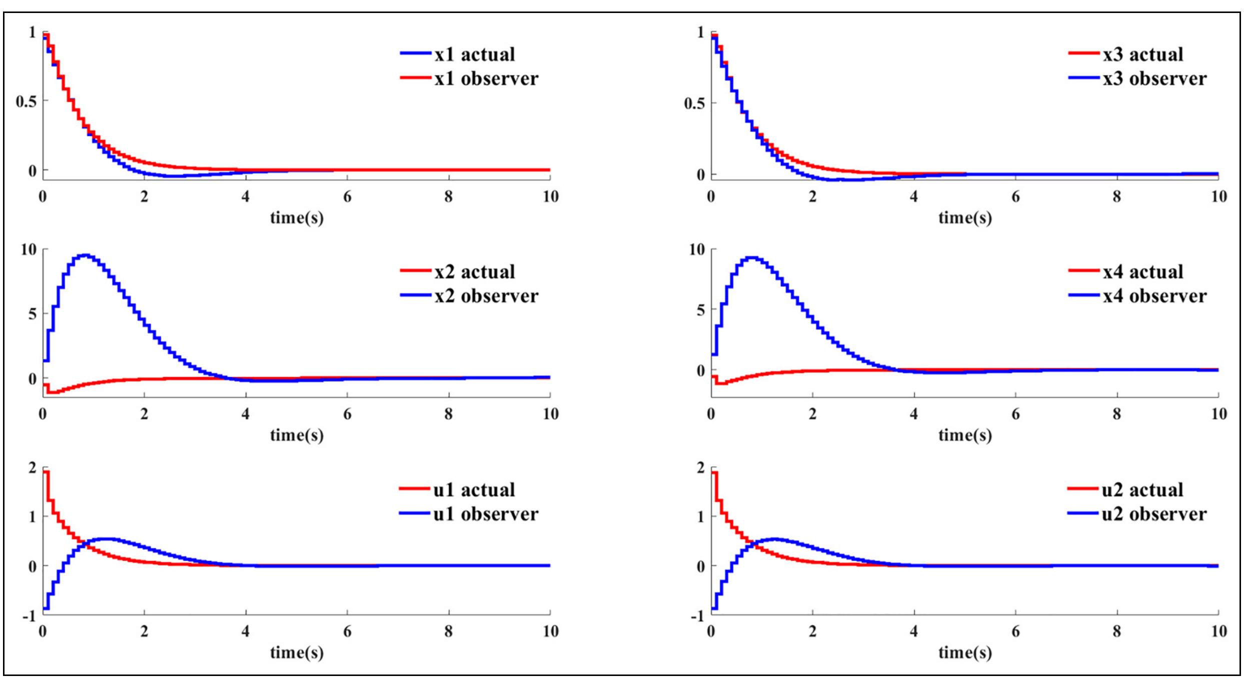

Figure 8. Evolution of the real state variables, their observers and the corresponding decentralized optimal control signals of the global system for $m = 120$.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

Soumaya Ghali https://orcid.org/0000-0002-6113-2248

References

Abdollahi A and Babolian E (2016) Theory of block-pulse functions in numerical solution of Fredholm integral equations of the second kind. *International Journal of Industrial Mathematics* 8(2): 157–163.

Abidi B and Elloumi S (2020) Decentralized observer-based optimal control using two-point boundary value successive approximation approach application for a multimachine power system. *Transactions of the Institute of Measurement and Control*. DOI:10.1177/0142331219893864.

Anderson B and Moore J (2007) *Optimal control: Linear quadratic methods*. 91.12 edition. Dover Publications, p. 464.

Anil VR (2009) A survey of numerical methods for optimal control. *Advances in the Astronautical Sciences* 135(1): 497–528.

Bakule L (2008) Decentralized control: An overview. *Annual Reviews in Control* 32(1): 87–98.

Bakule L (2014) Decentralized control: Status and outlook. *Annual Reviews in Control* 38(1): 71–80.

Bardi M and Capuzzo-Dolcetta I (2008) *Optimal Control and Viscosity Solutions of Hamilton-Jacobi-Bellman Equations*. Springer Science Business Media.

Becerra V (2010) Solving complex optimal control problems at no cost with PSOPT. In: *2010 IEEE International Symposium on Computer-Aided Control System Design*, Yokohama, Japan, pp. 1391–1396.

Bichiou S, Bouafoura M and Benhadj Braiek N (2018) Time optimal control laws for bilinear systems. *Journal of Mathematical Problems in Engineering* (2): 1–10.

Dadkhah M and Farahi M (2015) Optimal control of time delay systems via hybrid of block-pulse functions and orthonormal taylor series. *International Journal of Applied Computational Mathematics* (2): 137–152.

Elloumi S, Belhouane MM and Benhadj Braiek N (2016) Observer based controller for nonlinear analytical systems. *International Journal of Systems Science* 47(8): 1783–1790.

Feydi A, Elloumi S, Jammazi C and Benhadj Braiek N (2019) Decentralized finite-horizon suboptimal control for nonlinear interconnected dynamic systems using SDRE approach. *Transactions of the Institute of Measurement and Control* 41(11): 3264–3275.

Fleming W and Rishel R (2012) *Deterministic and Stochastic Optimal Control*. Springer Science Business Media.

Gao N, Darouach M, Alma M and Voos H (2014) Decentralized dynamic-observer-based control design for large-scale nonlinear interconnected systems. *Transaction on IoT and Cloud Computing* 2(2): 47–67.

Ghali S, Aousgi I and Elloumi S (2017a) Decentralized optimal control of interconnected systems using a direct approach. In: *International Conference on Advanced Systems and Electrical Technologies* (IC-ASET), Tunisia, pp. 79–84.

Ghali S, Aousgi I and Elloumi S (2017b) Synthesis of decentralized optimal control of nonlinear interconnected systems with block pulse functions application to a double-parallel inverted pendulum. In: *18th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA)*, Tunisia, pp. 24–29.

Hartl R, Sethi S and Vickson R (1995) A survey of the maximum principles for optimal control problems with state constraints. *SIAM Review* 37(2): 181–218.

Hosseinpour S and Nazemi A (2016) A collocation method via blockpulse functions for solving delay fractional optimal control problems. *IMA Journal of Mathematical Control and Information* 34(4): 1215–1237.

Jaddu HM (1998) Numerical methods for solving optimal control problems using Chebyshev polynomials. Thesis, School of Information Science, Japan Advanced Institute of Science and Technology, p.122.

Khajehnasiri AA (2014) Block-pulse functions and their application to solving systems of higher-order nonlinear Volterra integro-differential equations. *Electronic Journal of Differential Equations* 54: 1–9.

Khorsand O and Gattami A (2011) A decentralized stabilization scheme for a class of large-scale interconnected systems. *IFAC Proceedings Volumes* 44(1): 1404–1409.

Kirk DE (2012) *Optimal Control Theory: An Introduction*. Courier Corporation.

Kraft D (1985) On converting optimal control problems into nonlinear programming problems. In: *Computational Mathematical Programming*. Berlin: Springer, pp. 261–280.

Lendek Z, Babuska R and Schutter B (2008) Stability analysis and observer design for decentralized TS fuzzy systems. In: *2008 IEEE International Conference on Fuzzy Systems (IEEE World Congress on Computational Intelligence)*, Hong Kong, China, pp. 631–636.

Mahmoud MS (2011) *Decentralized Systems with Design Constraints*. London: Springer.

Marzban H (2016) Parameter identification of linear multi-delay systems via a hybrid of block-pulse functions and Taylor's polynomials. *International Journal of Control*. 90(3): 504–518.

Mohan BM and Kar SK (2012) *Continuous time dynamical systems: state estimation and optimal control with orthogonal functions*. 1 edition. CRC Press, p. 247.

Nayyeri MD and Kamyad AV (2014) A consecutive quasilinearization method for the optimal boundary control of semilinear parabolic equations. *Applied Mathematics* 5(4): 391–406.

Rtibi H, Elloumi S and Benhadj Braiek N (2017) Robust decentralized polynomial control for multimachine power systems. In: *Proceeding of the 1st International Conference on Advanced Systems and Electrical Technologies (IC-ASET)*, Tunisia, pp. 473–480.

Rtibi H, Elloumi S and Benhadj Braiek N (2018) Development of a decentralized nonlinear controller for a class of uncertain polynomial interconnected systems: Application for a large-scale power system. *Transactions of the Institute of Measurement and Control* 41(8): 2236–2249.

Sie-Long K (2013) Nonlinear programming approach for optimal control problems. In: *International Conference on Global Optimization and its Applications*, Malaysia, pp. 22–27.

Siljak DD and Stipanovic DM (2000) Robust stabilization of nonlinear systems: The LMI approach. *Mathematical Problems in Engineering* 6(5): 461–493.

Tang Y, Li N, Liu M, et al. (2017) Identification of fractional-order systems with time delays using block pulse functions. *Mechanical Systems and Signal Processing* 91: 382–394.

Von S and Bulirsch R (1992) Direct and indirect methods for trajectory optimization. *Annals of Operations Research* 37(1): 357–373.

Warrad B, Bouafoura MK and Benhadj Braiek N (2018) Static output tracking control for non-linear polynomial time-delay systems via block-pulse functions. *Journal of the Chinese Institute of Engineers* 41(3): 194–205.

Xie J and Huang Q (2016) Numerical solution of nonlinear Volterra-Fredholm-Hammerstein integral equations in two-dimensional spaces based on block pulse functions. *Journal of Computational and Applied Mathematics* 317: 565–572.

Younespour A and Ghaffarzadeh H (2015) Structural active vibration control using active mass damper by block pulse functions. *Journal of Vibration and Control* 21(14): 565–575.

Zarrini M and Torkaman S (2014) The application of hybrid orthonomal Bernstein and block-pulse functions in finding numerical solution of Fredholm fuzzy integral equations. *International Journal of Mathematical and Computational Sciences* 8(7): 1074–1078.

Ziari S, Perfilieva I and Abbasbandy S (2019) *Block-Pulse Functions in the Method of Successive Approximations for Nonlinear Fuzzy Fredholm Integral Equations*. Springer: Differential Equations and Dynamical Systems.