

# Non-cooperative game theory based energy management systems for energy district in the retail market considering DER uncertainties

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**Abstract:** In this study, a general framework for implementing a retail energy market based on the Nikaido–Isoda relaxation algorithm is proposed as an electricity market structure with large distributed energy resources (DERs) penetration and demand side management of consumers. Moreover, the consumers are able to participate in the market as prosumers (i.e. producer and consumer at the same time). By considering the related uncertainties, the DERs can maximise their expected payoff or profit by undertaking strategies through the price bidding strategy, based on the proposed structure, considering Nash equilibrium. The results show the effectiveness and accuracy of the proposed framework in determining the optimal power set-points of players participating in the market to achieve the objectives.

## 1 Introduction

In many systems, the electricity market has evolved to a deregulated market where providers and consumers may be active participants in the electricity price pool. The evolution of power systems into more controllable systems leads to a significant increase in the number of market players. These players are presently able to freely enter the market and access new economic opportunities provided by the market. The participation of the market actors guides the market toward cost reduction and increasing market reliability, increasing the profit of its players [1, 2]. All market participants present an offer which is used for computing the real market clearing price (MCP). Market deregulation laws have been adjusted given possible technical problems such as system disconnections as well as economic decisions (e.g. unacceptable offers). The strong effect of technical grid constraints may lead to economic decisions impacting the behaviour of the consumers. The electricity market is defined by complex concepts such as unhealthy competition, strategy interference, unbalanced information and the possibility of multi-phase equilibrium [3–5]. Traditional economic modelling methods have faced problems in confronting these concepts. The smart competitive models based on agents, are an attractive candidate for modelling and simulating such problems. Each agent states an independent participant with independent pricing strategies that can respond to the outcomes of the market. Moreover, it has the ability to learn and draw conclusions from experience. In traditional power markets, the number of generators is significantly higher than the number of sellers, and most of the research done in the past years has been focused on the supply side. In [6–10], different scenarios of playing individually or cooperatively were simulated and the results showed that it is of great benefit to cooperate, but the free-rider problem may arise. In [11] an improved dynamic programming algorithm is developed that requires fewer operations and less memory than the original one. Given the large number of agents, this property is a major drawback because agents usually have limited time and would not be able to wait until the end of the execution of the algorithm.

In the global competitive market, electricity buyers are no longer price takers since they are able to influence the market by using

different bidding strategies, as well as cooperating with other buyers. Therefore, it is necessary to develop and investigate individual and cooperative strategies of electricity buyers. However, most of the research efforts concentrate on the generation of the test cases and how to solve the constraint problem; whereas research on demand side has not been sufficiently forthcoming.

In this market structure, the agents of both sides of supply and consumption continuously adapt their strategies according to their objective function.

An evolutionary model based on the agent can simulate a two direction auction market. Optimum pricing strategies for the generators and a countless number of consumers in a competitive market have been studied in [12] by using Monte–Carlo sampling for estimating the behaviour of rivals. Contreras *et al.* [13] has focused on minimising the locational marginal price of purchasers by using different evolutionary algorithms and adding decision modules based on game theory. The mechanisms of the solution of this method are related to fairness, efficiency and stability in distributing the payoffs among the agents. Furthermore, widespread efforts have been devoted to the area of coalition formation. The method of interest investigated in the researches has focused on dividing the agents inside the coalition such that the sum of payoffs in all of the coalition become maximum, which optimal coalition structure. The alliance strategy of buyers has been studied in [13] and it has been shown that the buyers could lower their costs by evolving their group sizes and memberships.

Economic operations of future residential distribution systems with high participation of distributed electricity prosumers (i.e. producer and consumer at the same time) with various non-convex objective function including generation, storage and responsive load offers is presented in previous literature [14–21]. However, previously published works have been mainly concentrated on the control and operation problem of individual energy district (ED) with particular attention to frequency stability and reliability improvement, and thus, coupled operation in systems based on producers and prosumers is still considered as an emerging area of research. Furthermore, while the existing studies have focused on simplifying market models by introducing only a small number of

participants, the proposed model can easily be implemented over a grid with a large number of purchasers and sellers. This is done by completely taking into account the technical and economic constraints and, further, can be easily developed for consideration of different market models. The residential customers here are not only electricity consumers but can through local exploitation of distributed generators, energy storage (ES) devices and their distributable loads become electricity supplies. The proposed framework provides significant economic incentives for installing and commissioning their distributed renewable generators [e.g. wind turbine (WT) in small scale and photovoltaic (PV) panel installed over the roof of houses].

It is expected that in the future, EDs became a common entity within the distribution grid, which will have the capability of import and export energy depending on the situation. Thus, it may have the chance on participate in the electricity market if several of them apply together (cooperate). The paper aims to present a methodology for ensuring profits for micro-prosumers (producers and consumers at the same time as autonomous houses can be [15]). The concept relies on the concept of aggregator [15], where different autonomous houses, forming an ED, may be big enough to participate within the energy market. Since, different ED are interacting together (cooperating) they are sharing load needs for minimising the power imported from the main grid. Moreover, such concept allows the EDs selling the power excedence in the market due to their bigger size. The earnings from such market participation must be shared; this paper presents a methodology based on game-theory for EDs participation on the retail market and a earning sharing procedure for maximising the earnings of each ED participating.

Furthermore, it is desirable to develop non-cooperative behaviour of players for maximising the benefits. The cooperation and pricing strategies of power sellers in a deregulated market have been studied using algorithms based on non-cooperative game theory in a framework based on agents. Complex systems have not allowed analysis of larger number of participants; but in this paper the proposed model can easily be implemented over a grid with a large number of buyers taking into account of technical and economic constraints and can be easily extended to other market models. The innovations of this paper can be listed as follows:

(i) Identification and formulation of the functionalities of generation units and distributed electricity prosumers in the future residential distribution systems considering uncertainties.

(ii) Development of package-based software, using a novel non-cooperative game-theoretic methodology and uncertainty algorithms to further analyse the behaviours of the key market participants in different scenarios, and to clear the retail electricity market price.

(iii) Investigation of the impact of distributed electricity prosumers on the economic operations of the future residential distribution systems.

(iv) Conduct of numerical case study to evaluate and validate the proposed framework.

## 2 Proposed stochastic market clearing procedure

The proposed structure for electricity markets presents a solution for enabling high share participation of distributed generation (DG). This market aims to reduce electricity price, maximising generation and consumption players profit through cooperation. The considered framework provides this possibility for the exploiters of distribution system and household customers to select in a proper method for their energy supply resource by noting different choices, such as selecting a wide range of renewable energy resources based on final electricity cost. In the proposed framework, the household consumers play a relevant role in the exploitation and management of distributed energy resources (DER) (DG resources, ES) and also responsive load demand (RLD). The DERs can also cooperate for obtaining more profit. On the other hand, the consumers while participating with the producers for managing their demand attempt to reduce final electricity price. The proposed structure is shown in Fig. 1. The proposed structure has four stages as follows:

*Stage 1:* The amount of energy produced by WT, PV and non-responsive load (NRL) demand are forecasted.

*Stage 2:* The scenarios of energy consumed by NRL demand, and the energy generated by both WT and PV are generated using the information obtained in the previous stage. Moreover, generation uncertainty is carried out through stochastic programming for considering generation variability.

*Stage 3:* The expected values of generation and consumption powers are randomly selected for participation in the game theory based on different scenarios and their occurrence probability. Then, the Nash equilibrium point in the market electricity price is obtained considering local and global constraints.

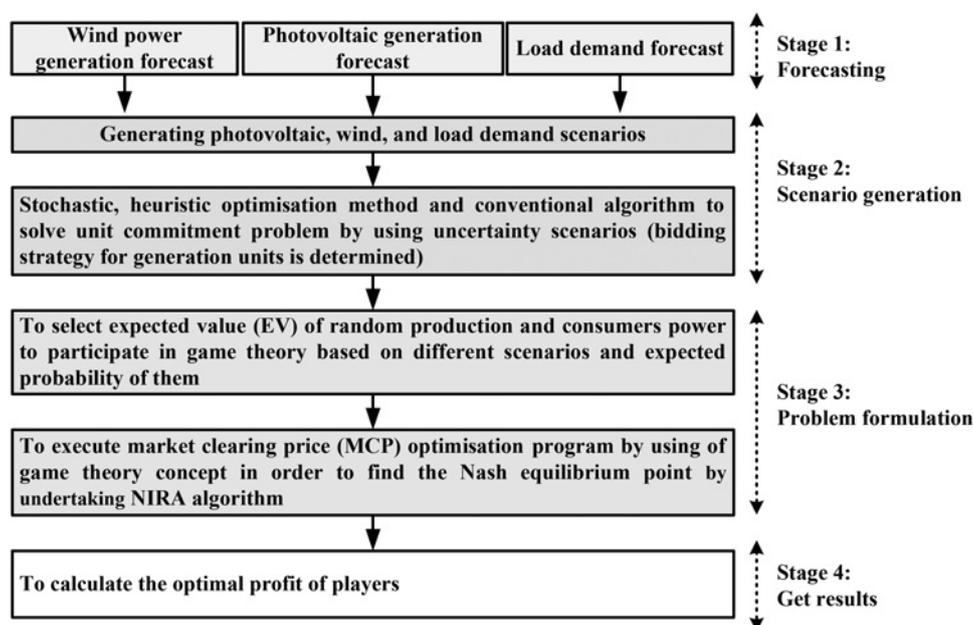


Fig. 1 Proposed market structure implementation process

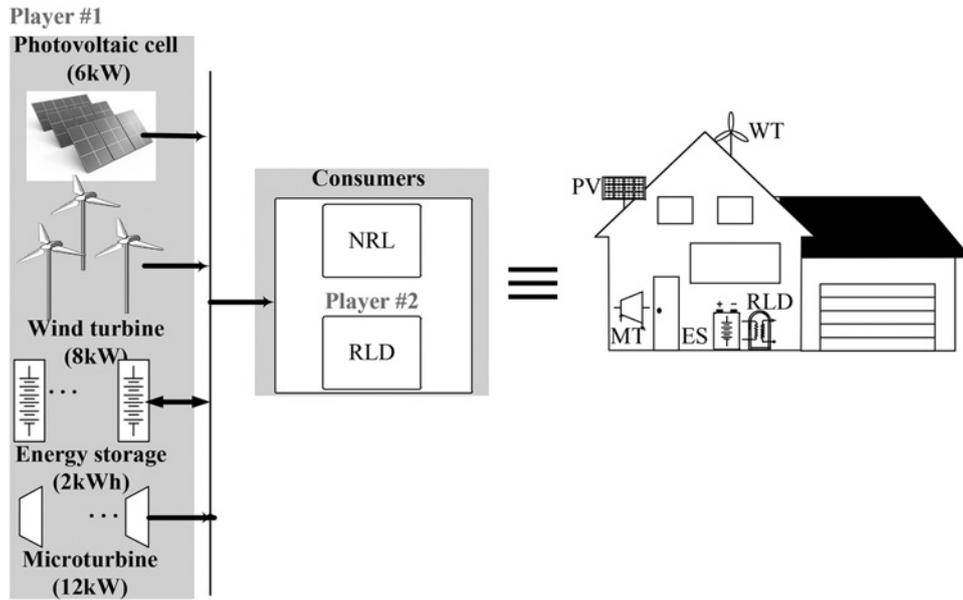


Fig. 2 Schematic of the system under study

Stage 4: The optimum capacity for market participation is obtained by using relaxation algorithm and Nikaido–Isoda function for a set of players in line with reaching maximum profit. During this stage by noting the optimum Nash equilibrium obtained previously, the pay-off function of each player can be computed.

### 3 ED system under study

For testing the proposed method, an isolated ED is developed in MATLAB software. The specifications of the system and the control plans for the DERs are detailed in [18–21]. Fig. 2 illustrates the system under study.

The ED under study includes a collection of production resources (WT, PV and MT), ES and consumers as NRL and RLD. The predicted data for load demand, WT and PV taken from [22]. The collection of production resources is defined as player #1 and RLD as player #2. The cost coefficients ( $a$ ,  $b$  and  $c$ ), retail electricity price ( $\theta$  and  $\beta$ ), the NRL load ( $\zeta$ ) and supply price ( $\pi^{\text{ES-}}$  and  $\pi^{\text{ES+}}$ ) are summarised in Table 1.

The cost coefficients ( $a$ ,  $b$ ,  $c$ ), profit and the price of power supply of players #1 and #2 have been summarised in Table 1.

For evaluating the performance of the retail energy market (REM) based on the Nikaido–Isoda/relaxation (REM-NIRA) algorithm, a sensitivity analysis is performed by applying the following scenarios:

*Scenario #1:* Sudden increase (10%) the power generated by renewable resources.

*Scenario #2:* The initial value of state-of-charge (SOC) set at maximum value.

*Scenario #3 (normal condition):* The initial value of SOC at average value.

*Scenario #4:* The initial value of SOC set at minimum value.

*Scenario #5:* Sudden decrease (10%) the power generated by renewable resources.

The value of SOC in the initial conditions is adjusted equal to its average value (i.e. 50%) in scenarios #1, #3 and #5.

Table 1 Cost coefficients and the profits of players #1 and #2

$a$	$b$	$c$	$\theta$	$\beta$	$\zeta$	$\pi^{\text{ES-}}$	$\pi^{\text{ES+}}$
\$/kW <sup>2</sup> h	\$/kWh	\$/h	\$/kWh	\$/h		\$	\$
$6 \times 10^{-6}$	0.01	0	0.001	3.4	60	0.125	0.145

### 4 Problem formulation

In this section, the mathematical formulation of the problem is presented by considering the key components of participation in the retail market. The general framework is easily expandable and adjustable with other electricity distribution systems with high levels of customers' participation.

#### 4.1 Objective functions and problem constraints

The proposed market key components include two players: the generation collective (Player #1) and consumption collective (Player #2) in ED. Each one presents its own objective function, as presented in the following.

**4.1.1 Objective functions:** *Player #1:* The generation resources in the grid under study is made up of MT, WT, PV and ES during discharging mode. The profit obtained from production collections (Player #1) can be calculated at  $t$ th hour by

$$\max J_t^{\text{Player-1}} = \mathbb{R}_t^{\text{Player-1}} - \mathbb{C}_t^{\text{Player-1}} \quad (1)$$

$$t \in \{1, 2, \dots, 24\}$$

where  $\mathbb{R}_t^{\text{Player-1}}$  and  $\mathbb{C}_t^{\text{Player-1}}$  are, respectively, the revenue and cost of Player #1 at time  $t$ . The revenue (i.e.  $\mathbb{R}_t^{\text{Player-1}}$ ) can be defined as:

$$\mathbb{R}_t^{\text{Player-1}} = \lambda(P_n) \times [P_t^{\text{MT}} + P_t^{\text{WT}} + P_t^{\text{PV}} + P_t^{\text{ES-}} - P_t^n] \quad (2)$$

where  $P_t^{\text{MT}}$  is the MT output power (kW),  $P_t^{\text{WT}}$  WT output power (kW),  $P_t^{\text{PV}}$  is the PV system output power (kW) and  $P_t^{\text{ES-}}$  is the ES discharge power at  $t$ th hour (kW).

For sake of simplicity, the retail electricity price is considered the same for all the players following a singular distribution system [23]

$$\lambda(P_n) = (-\theta \times P_t^n) + \beta, \quad \theta \geq 0 \quad (3)$$

where  $\theta$  and  $\beta$  are load demand curve coefficient,  $\lambda$  the identical electricity price in \$/kWh and  $P_t^n$  total ED system given load demand in kW at the  $t$ th hour. The cost function (i.e.  $\mathbb{C}_t^{\text{Player-1}}$ ) is expressed in \$ by

$$\mathbb{C}_t^{\text{Player-1}} = \mathbb{C}_t^{\text{MT}} + \mathbb{C}_t^{\text{WT}} + \mathbb{C}_t^{\text{PV}} + \mathbb{C}_t^{\text{ES-}} + \mathbb{C}_t^{\text{ES+}} \quad (4)$$

The generators WT and PV are as non-distributable units and their output power is dependent upon climatic conditions. The cost of renewable resources generation is negligible compared to other resources; as a result,  $C_t^{WT} = 0$  and  $C_t^{PV} = 0$  have been considered. The MT generation cost has been formulated as a quadratic function as follows

$$C_t^{MT} = a \cdot (P_t^{MT})^2 + b \cdot P_t^{MT} + c, \quad a \geq 0 \quad (5)$$

where  $a$ ,  $b$  and  $c$  are the coefficients of cost function for MT. The ES charge and discharge costs are also computed through the following relations [24, 25].

$$C_t^{ES-} = \pi^{ES-} \times P_t^{ES-} \quad (6)$$

$$C_t^{ES+} = \pi^{ES+} \times P_t^{ES+} \quad (7)$$

where  $\pi^{ES-}$  and  $\pi^{ES+}$  are, respectively, the ES supply price in the charge and discharge modes and  $P_t^{ES+}$  and  $P_t^{ES-}$  are also the powers of ES charge and discharge modes.

*Player #2:* This player includes RLD load. The aim is to minimise the cost of exploitation through managing its distributable loads, for which the obtained profit at the  $t$ th hour is defined as follows

$$\min J_t^{\text{Player-2}} = \lambda(P_n) \times P_t^{\text{RLD}} \quad (8)$$

Each one of the above mentioned players make their decisions depending on local and global constraints.

**4.1.2 Local and global constraints:** Equilibrium constraint: the equilibrium of the power generated by Player #1 must be equivalent to the total power consumed by load demand satisfying:

$$P_t^{MT} + P_t^{WT} + P_t^{PV} + P_t^{ES-} = P_t^n + P_t^{ES+} + P_t^{\text{RLD}} \quad (9)$$

*Player #1:* MT is exploited during the given time  $t$  under the following conditions.

$$\underline{P}^{MT} \leq P_t^{MT} \leq \bar{P}^{MT} \quad (10)$$

where  $\underline{P}^{MT}$  and  $\bar{P}^{MT}$  are, respectively, the minimum and maximum output power when MT is active (kW). The ED system, exploits its WT and PV by noting the expected mathematical value resulting from uncertainty scenarios dependent on the following constraints.

$$0 \leq P_t^{WT} \leq EV_t^{WT} \quad (11)$$

$$0 \leq P_t^{PV} \leq EV_t^{PV} \quad (12)$$

where  $EV_t^{WT}$  and  $EV_t^{PV}$  are, respectively, the WT and PV

mathematical expected value at the  $t$ th hour in kW which is calculated from the product of the probability of the occurrence of each one of the uncertainty scenarios by the value of that scenario according to the following relations

$$EV_t^{WT} = \sum_{s=1}^{N_s} \rho_t^{WT,s} \times P_t^{WT,s} \quad (13)$$

$$EV_t^{PV} = \sum_{s=1}^{N_s} \rho_t^{PV,s} \times P_t^{PV,s} \quad (14)$$

where  $N_s$  is the number of uncertainty scenarios and  $\rho_t^{WT,s}$  and  $\rho_t^{PV,s}$  are the possibility of occurrence of each one of the scenarios WT and PV at the  $t$ th hour and  $P_t^{WT,s}$  and  $P_t^{PV,s}$  are the WT and PV power values in the related scenario. The proposed algorithm uses ES in  $t$  by noting the charge or discharge mode by following the constraints as follows [23]

$$0 \leq P_t^{ES-} \leq \bar{P}^{ES-} \quad (15)$$

$$0 \leq P_t^{ES+} \leq \bar{P}^{ES+} \quad (16)$$

where  $\bar{P}^{ES-}$  and  $\bar{P}^{ES+}$  are, respectively, the ES maximum power in kW at ES discharge or charge mode. For preventing excessive charge and discharge when ES SOC reaches its highest and lowest mode, ES charges its state to stand-by mode.

$$\underline{\text{SOC}} \leq \text{SOC}_t \leq \overline{\text{SOC}} \quad (17)$$

$$\text{SOC}_{t+1} - \text{SOC}_t = (P_t^{ES+} - P_t^{ES-}) \times \Delta t / ES_{\text{Tot}} \quad (18)$$

where  $\text{SOC}_t$  is the state of charge of ES.  $\underline{\text{SOC}}$  and  $\overline{\text{SOC}}$  are, respectively, the minimum and maximum state of ES in the charge and discharge mode.  $ES_{\text{Tot}}$  is ES capacity in kWh and  $\Delta t$  is the time interval.

*Player #2* is the grid made up of RLD and NRL loads. The relation between RLD and NRL during each time interval satisfies the following relation.

$$0 \leq P_t^{\text{RLD}} \leq \zeta \times P_t^n \quad (19)$$

$\zeta$  is a coefficient of NRL load.

## 4.2 Mathematical modelling of PV, WT and load demand uncertainty

Since the market is based on predicted data and generation units are variable, uncertainty must be considered. For conforming the predicted data with reality, probabilistic models are used.

**4.2.1 Consumed load demand uncertainty modelling:** Load uncertainty can be modelled using normal distribution curve and by using average parameters and standard deviation. The average value in the load normal curve distribution is equal to the predicted load during each time interval. The standard deviation is obtained from the load prediction method based on experience and previous consumption pattern. To simplify mathematical calculations, the normal distribution can be divided into several sections showing the load occurrence probability with the value equal to the mean value of that section. In this study the normal probability distribution curve according to Fig. 3 is used [26, 27].

**4.2.2 WT uncertainty modelling:** Noting that the wind speed is a random variable, calculation of wind speed variability during 24 h Weibull distribution is used. The average value of this distribution, is the wind speed prediction datum. The Weibull distribution curve can also be divided into several separate sections. The possibility of occurrence of each interval is determined through the corresponding wind speed with the mode of each section. The

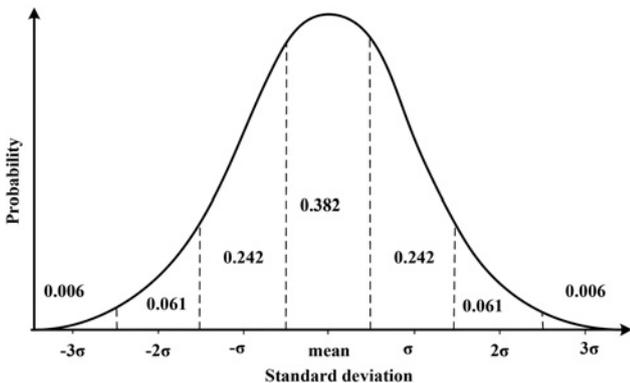


Fig. 3 Seven pieces normal probability distribution curve

wind speed probability distribution curve in this study is divided in the five pieces distribution density function as shown in Fig. 4 [28, 29].

Wind output power is determined from power function based on wind speed according to the following relation

$$P_t^{WT}(v) = \begin{cases} \left(\frac{P_r}{V_r - V_{ci}}\right)(v - V_{ci}) & \text{if } V_{ci} \leq v \leq V_r \\ P_r & \text{if } V_r \leq v \leq V_{co} \\ 0 & \text{others} \end{cases} \quad (20)$$

where  $P_t^{WT}(v)$  is the WT output power,  $v$  is the wind speed,  $P_r$  WT nominal power,  $V_r$  wind nominal speed and  $V_{ci}$  turbine cut-in speed and  $V_{co}$  is the turbine cut-off speed. If the turbine generation starts at the speed  $V_{ci}$ ; the output power will increase proportional to speed increase from  $V_{ci}$  to  $V_r$  and the nominal power  $P_r$  is generated when the wind speed is variable between  $V_r$  and  $V_{co}$ . Because of security reasons, the turbine will turn off at speed  $V_{co}$  and the output power will be zero at a speed outside the mentioned limits.

**4.2.3 PV uncertainty modelling:** The amount of solar radiation which reaches earth, in addition to the external daily and annual motion of the sun, depends on the geographic position (length, width and height) and climatic conditions (e.g. cloud cover). The PV surfaces output power is dependent on the amount of solar radiation of the PV panel surface. The solar radiation hourly distribution can be divided into five sections similar to Weibull distribution model for wind speed which is illustrated in Fig. 5 [30]. PV system power distribution is obtained based on radiation distribution. The PV system output power is calculated as follows:

$$P_t^{PV} = A_C \cdot \eta \cdot I_t^\beta \quad (21)$$

where  $A_C$  is the area of array surface [ $m^2$ ],  $I_t^\beta$  is the amount of solar radiation over a surface with  $\beta$  slop relation to the horizon surface [ $kWm^{-2}$ ],  $\eta$  is the efficiency of PV system at the realistic reporting conditions.

### 4.3 Nikaido–Isoda function

In the proposed electricity market, all the players compete with each other regulating their output power and load demand. The nature of the electricity market fits into game theory with  $n$ -person in a non-cooperative game in which each player is after maximising its own profit through a distributed decision-making process. This non-cooperative game with  $n$  players can be solved repeatedly by using the game theory special method (Nikaido–Isoda function and Relaxation algorithm). In a non-cooperative game with  $n$  players, each player may have conflict of interests with others [23]. All of them try to maximise their welfare by regulating their strategies. In a mathematical form, a game with  $n$  players is defined with tertiary structure  $i \in N$ ,  $N$ ,  $X_i$ ,  $\Theta_i$ . Each  $i$ th player

belongs to a set of  $N = 1, 2, \dots, n$  players.  $X_i$  is the strategy space of the  $i$ th player. The set of collective strategies is defined as  $X = X_1 \times X_2 \times \dots \times X_n$ . Each player noting its special strategy, selects an action through a vector  $x_i$ . The set of collective action is defined as  $x = (x_1, x_2, \dots, x_n)$ .  $\Theta_i$  is the  $i$ th player pay-off function who calculates the obtainable profit of that player through regulating its strategy by noting the others strategy space. The expression  $y_i|x$  shows the element  $(x_1, \dots, x_{i-1}, y_i, x_{i+1}, \dots, x_n)$  and states that the  $(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$  player has adopted the behaviour while the other players  $i$ th are present in the game. The Nash equilibrium for each  $i$ th player is defined as  $x^* = (x_1^*, \dots, x_n^*)$ .

$$\Theta_i(x^*) = \max_{(x_i|x) \in X} \Theta_i(x_i|x) \quad (22)$$

In other words, a Nash equilibrium exists if always for all the players and all the  $x_i$  holds  $i \in N$ ,  $\Theta_i(x_i^*|x^*) \geq \Theta_i(x_i|x^*)$ . Noting that the strategies of other players is constant, no player can improve its payoff function by the unilateral changes of its strategy. Relations (1) and (8) can, respectively, be considered as pay-off function in the form of relation (23). The Nikaido–Isoda function is defined as follows

$$\psi(x, y) = \sum_{i=1}^n [\Theta_i(y_i|x) - \Theta_i(x_i)] \quad (23)$$

The expression  $[\Theta_i(y_i|x) - \Theta_i(x_i)]$  at the right-hand side of the above relation shows improvement in the  $i$ th pay-off function when the player changes its behaviour from  $x_i$  to  $y_i$ , while the strategies of other players stays without change.  $x^*$  is a Nash normalised equilibrium if

$$\max_{(x^*, y) \in X} \psi(x^*, y) = 0 \quad (24)$$

When a Nikaido–Isoda function reaches zero, no player can increase its pay-offs by the unilateral changes of its strategy. One Nash normalised equilibrium is also a Nash equilibrium point if

$$X = X_1 \times X_2 \times X_n \quad (25)$$

Therefore, the Nikaido–Isoda function can approximately approach the Nash equilibrium point. Because of this, optimum response function is stated as Nikaido–Isoda function maximum result

$$Z(x) = \arg \max_{y \in X} \psi(x, y), Z(x) \in X \quad (26)$$

$\arg \max$  is for maximum argument and states a set of  $x$  variables for which  $\psi(x, y)$  reaches its maximum value. Each player tries unilaterally to maximise its profit.

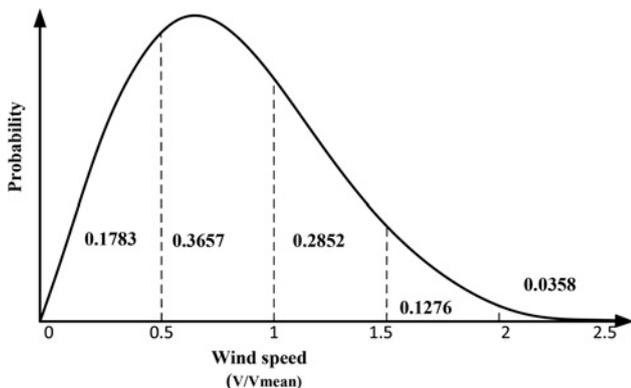


Fig. 4 Wind speed probability distribution

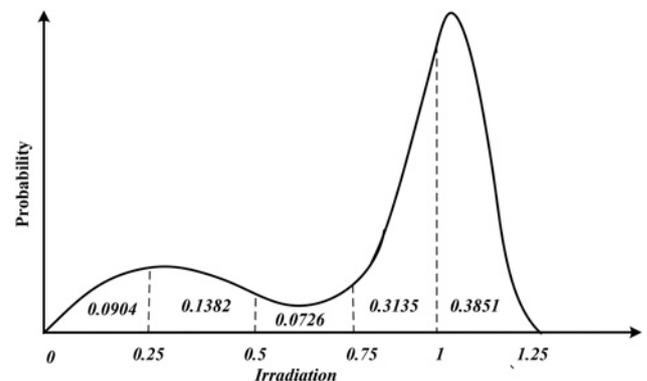


Fig. 5 Solar radiation probabilistic distribution

#### 4.4 Relaxation algorithm

Relaxation algorithm improves the optimum response function, respectively, until when it becomes convergent to a Nash equilibrium point  $x^*$ . The initial guess  $x^0$  is selected based on the information obtained from uncertainty scenarios and the defined planning of units participation. In this paper,  $x^0$  is the vector of the expected mathematical values obtained from uncertainty scenarios  $EV_t^{MT}$ ,  $EV_t^{WT}$ ,  $EV_t^{PV}$ ,  $EV_t^{ES}$  and  $EV_t^{RLD}$  which the values  $EV_t^{WT}$  and  $EV_t^{PV}$  are calculated through relations (13) and (14) and noting the relations below the other components of vector  $x^0$  can be determined

$$EV_t^{MT} = \sum_{s=1}^{N_s} \rho_t^{MT,s} \times P_t^{MT,s} \quad (27)$$

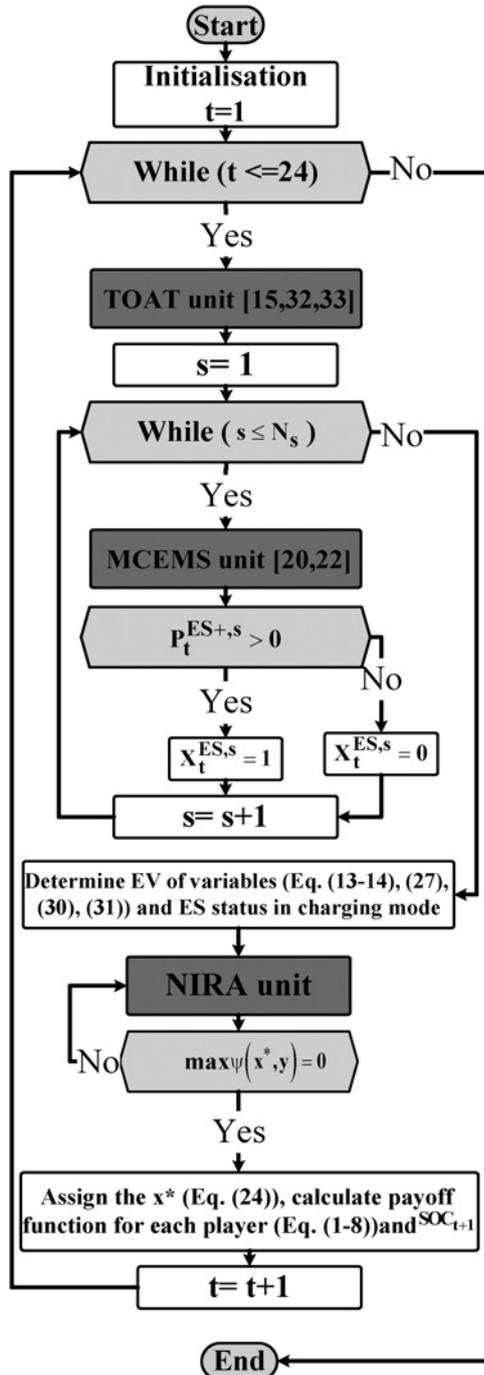


Fig. 6 Proposed algorithm for implementing REM-NIRA

where  $EV_t^{MT}$ ,  $\rho_t^{MT,s}$  and  $P_t^{MT,s}$  are, respectively, the values of mathematical expected value, possibility of the occurrence of scenario and MT power in the related scenario

$$EV_t^{ES+} = \sum_{s=1}^{N_s} \rho_t^{ES+,s} \times P_t^{ES+,s} \quad (28)$$

$$EV_t^{ES-} = \sum_{s=1}^{N_s} \rho_t^{ES-,s} \times P_t^{ES-,s} \quad (29)$$

$$EV_t^{ES} = EV_t^{ES+} - EV_t^{ES-} \quad (30)$$

where,  $EV_t^{ES+}$ ,  $\rho_t^{ES+,s}$  and  $P_t^{ES+,s}$  are the values of mathematical expected value, probability of occurrence of scenario and scenario power in the ES charging mode and in this order  $EV_t^{ES-}$ ,  $\rho_t^{ES-,s}$  and  $P_t^{ES-,s}$  are the values of expected mathematical value, possibility of occurrence of scenario and scenario power in the ES discharging mode.  $EV_t^{ES}$  is the value of mathematical expected value of the sum of charging and discharging modes of ES

$$EV_t^{RLD} = \sum_{s=1}^{N_s} \rho_t^{RLD,s} \times P_t^{RLD,s} \quad (31)$$

where  $EV_t^{RLD}$ ,  $\rho_t^{RLD,s}$  and  $P_t^{RLD,s}$  are, respectively, the values of RLD mathematical expected value, probability of scenario occurrence and RLD consumed power in the related scenario. The next step, is the respected improvement of optimum response function by using the relaxation algorithm

$$\begin{cases} x^{k+1} = (1 - \alpha_k)x^k + \alpha_k Z(x^k) \\ k = 0, 1, 2, \dots \end{cases} \quad (32)$$

where  $k$ , is the iteration step and  $\alpha_k$  is the weight allocated to  $x^k$  and  $Z(x^k)$  is for obtaining new  $x^{k+1}$  in the  $k$  next step. The optimum response function, as long as shutdown condition is satisfied as follows, is updated repeatedly

$$\max_{(x^k, y) \in X} \psi(x^k, y) \leq \epsilon \quad (33)$$

$\epsilon$  is a small value defined by the user for controlling the convergence rate. The convergence of relaxation algorithm is guaranteed with a constant  $\alpha_k$  for a convex-concave function, when  $\alpha_k$  satisfies the following conditions

$$\sum_{k=0}^{\infty} \alpha_k = \infty, \quad 0 \leq \alpha_k \leq 1, \quad \alpha_k \rightarrow 0, k \rightarrow \infty \quad (34)$$

During each iteration  $k$ , an optimum value  $\alpha_k$  is found through solving the following optimum response function

$$\alpha_k^* = \arg \min_{0 \leq \alpha_k \leq 1} \left[ \max_{y^k \in X} \psi(x^{k+1}(\alpha_k), y^k) \right] \quad (35)$$

After several iterations, the tranquility algorithm converges to an  $x^*$  Nash equilibrium point. That is  $x$  is a compact convex set of Euclidean space. Optimum response function  $Z(x^k)$  is a singular and continues value over  $x^s$ . In addition to this, the Nikaido-Isoda function (i.e.  $\psi(x, y)$ ) is a weak convex-concave and quadratic continues differentiable function. In the other hand, a unique equilibrium point exist such that the relaxation algorithm can become convergent in it [31].

## 5 REM-NIRA algorithm

The flowchart for implementing the REM-NIRA algorithm is presented in Fig. 6. As observed in this figure, the algorithm is made up of three main units including uncertainty, modified conventional energy management system (MCEMS) unit and NIRA unit. The proposed REM-NIRA algorithm is illustrated by a Pseudo-code in Algorithm 1 (see Fig. 7). The method for implementing them is explained in the following sections.

### 5.1 Taguchi's orthogonal array testing (TOAT) unit

This unit is proposed for including uncertainty related to load demand and the effect of the change of environmental conditions on the power generated by renewable resources by using the TOAT method. The TOAT method in [15] is produced a robust solutions for consideration of uncertainties within multiple electrically coupled EDs. The TOAT method ensures that the testing scenarios provide good statistical information with a minimum scenario number in the uncertain operating space, which significantly reduces the testing burden [32]. For additive and quadratic models, TOAT has been proven to have the ability to select optimally representative

scenarios for testing from all possible combinations [33]. Compared with Monte Carlo simulation, TOAT provides much smaller testing scenarios and leads to shorter computing time. Despite of its ability, it has only been adopted as an optimisation algorithm to solve the economic dispatch problem in the power system field [34, 35].

The existing uncertainties in the discussed problem have been implemented with the scenarios formed according to Fig. 8. The proposed structure include three main stages as follows:

*Stage 1:* Selecting the orthogonal matrix noting the number of uncertainties existing in the system.

*Stage 2:* Create  $n$  values for load demand using a normal distribution, for WT using a Weibull distribution and for PV using a Weibull distribution and the radiation equation.

*Stage 3:* Calculate the possibility of the occurrence of created scenario by using the distribution functions related to load, WT and the radiation function related to PV.

### 5.2 MCEMS unit

For determining the set-point of the power related to generation resources and consumption MCEMS algorithm presented in [20] is

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#### Algorithm 1

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**Require:** Initialisation  $\triangleright$  Hourly prediction data of MG,  $\overline{SOC}$ ,  $\underline{SOC}$ ,  $\overline{P}^{MT}$  and  $\underline{P}^{MT}$ .

**while**  $t \leq 24$  **do**

$\triangleright$  REM-NIRA

1. TOAT unit

Generate power scenarios randomly for WT, PV and NRL with the probability of occurrence [15, 32, 33].

2. MCEMS unit

Determine  $P_t^{MT,s}$ ,  $P_t^{ES-,s}$ ,  $P_t^{ES+,s}$ ,  $P_t^{RLD,s}$ ,  $X_t^{ES,s}$  [20, 22];

3. Calculate  $EV_t^{WT}$  (Eq. 13),  $EV_t^{PV}$  (Eq. 14),  $EV_t^{MT}$  (Eq. 27),  $EV_t^{ES}$  (Eq. 30),  $EV_t^{RLD}$  (Eq. 31), in addition to determining  $X_t^{ES}$ .

4. Define  $x^0$  using the results obtained in step 3;

5. NIRA unit

(a) Determine the parameters required by the REM-NIRA algorithm (the dimension vector of players, upper/lower bound players, maximum number of iterations, selecting the  $\alpha$  optimization step, tolerance)

(b) Define  $\Phi_i$  (Eq. 1-8);

(c) Define the players constraints (Eq. 9-19);

(d) Form  $\Psi(x, y)$  (Eq. 23);

(e) Calculate  $x^*$  (Eq. 24);

(f) Maximize  $\Psi(x, y)$  and form  $Z(x)$  (Eq. 26)

(g) Apply the relaxation algorithm and improving  $Z(x)$  by Eqs. 32 and 35 until the stooping condition (Eq. 24);

(h) Determine output data;

(i) Determine  $x^*$ ;

(j) Calculate the values of the players profit based on  $x^*$  (Eq. 1-8);

(k) Determine  $SOC_{t+1}$ ;

**end while**

**return** Determine the optimum capacity and profit of the all players.

---

Fig. 7 REM-NIRA algorithm

used. In fact this unit determines the initial values for all the variables defined based on technical and economic constraints related to each element for the NIRA unit. Details about the method of implementation and the quality of the performance of these units has been explained completely in [20] and is outside the scope of this paper.

### 5.3 NIRA unit

This unit has two important duties with two sub-problems including (i) Maximising Nikiado–Isoda function (ii) Applying the relaxation algorithm and improving the optimum response function. Both of these duties are repeatedly followed by the NIRA unit when the difference of the optimum response function in relation (26) between two adjacent iterations becomes small enough. After initial value giving, first relation (23) must be maximised for finding  $y_i$  by noting the local and global constraints (9–19). Then, gradually, the solution obtained from relation (23) in the first sub-problem, converges to a new stable state which shows the desired result in solving the first sub-problem. After the intermediate solution found in the first sub-problem, we go to

the second sub-problem hence the relaxation algorithm is executed. In the second sub-problem, the relaxation algorithm is executed in relation (32) such that it improves the solution space  $x^k$ . Then for updating  $x^k$ , the relaxation algorithm is executed through relations (32) and (35). If the values Nikaido–Isoda reaches zero, no player can unilaterally improve its pay-off function. On the other hand, a balanced (approximate) response for electricity market settlement by following the global and local constraints (relations (9–19)) is found. By the repeated

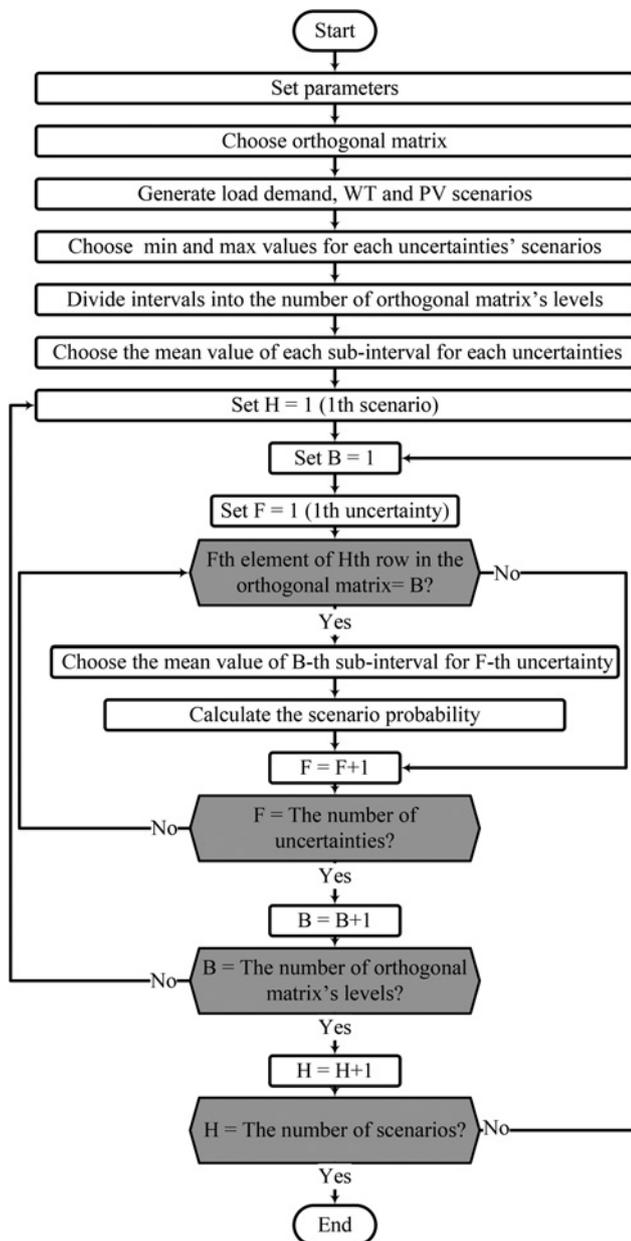


Fig. 8 TOAT unit

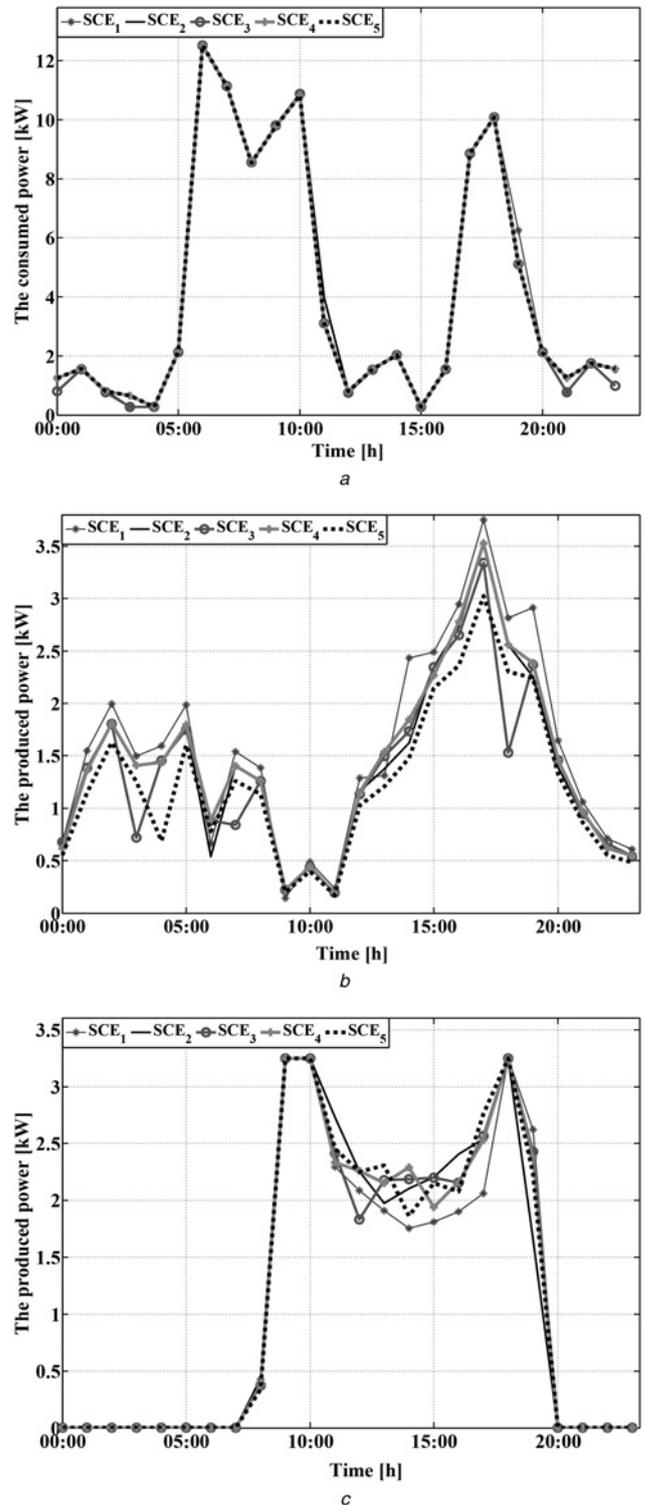


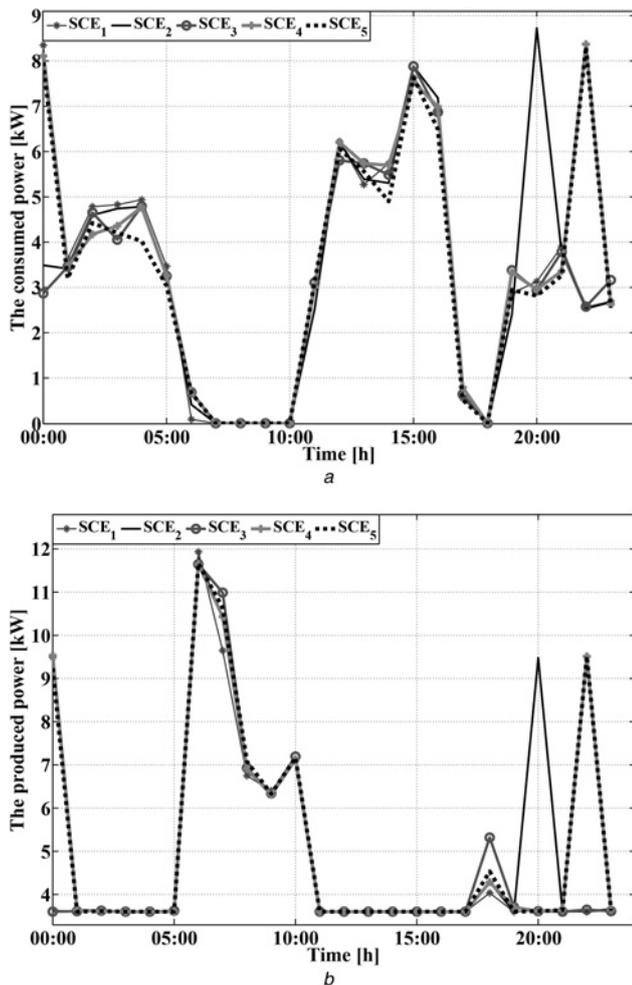
Fig. 9 Results obtained by REM-NIRA algorithm

a Load demand profile  
b WT profile  
c PV profile

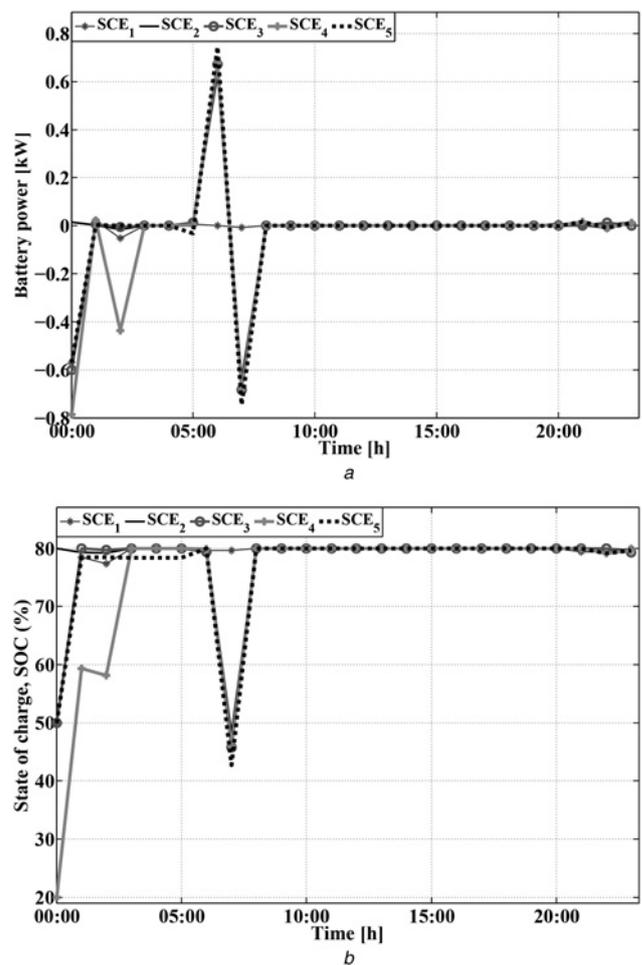
improvement of the optimum response function in relation (32), gradually the values of the pay-off function of all the players converges to an approximate equilibrium point. Noting the found balanced response through continuous iterating rings, the electricity market price for an ED system with several customers can be settled.

## 6 Result and discussion

Figs. 9a–c show data resulting from applying uncertainty (TOAT unit) for load demand inputs, WT and PV during 24 h time interval. As observed, significant changes over the value of generated power by renewable resources by applying uncertainty under different scenarios is observable. Especially in scenario #1 the value of the increase of power generated by WT in some of time intervals is more than 18% of its value in other scenarios. As observed in the Fig. 10a, despite the application of different scenarios the value of load demand power remains constant during the whole system operation. The real life experimental data carried out from [18, 19] are used to WT, PV and NRL profiles. The main effect of system conditions change shows itself over the value of the power consumed by DR. As it is observed from Fig. 10a, during the early hours of the day very small changes in the value of DR power is observed and during the day until 15:00 clock. Significant change in its value has not been observed. After this time, if the SOC initial value is set at minimum value (scenario #3), the system will observe sudden decrease with the value of 20% from its maximum value in this time interval. The reason for

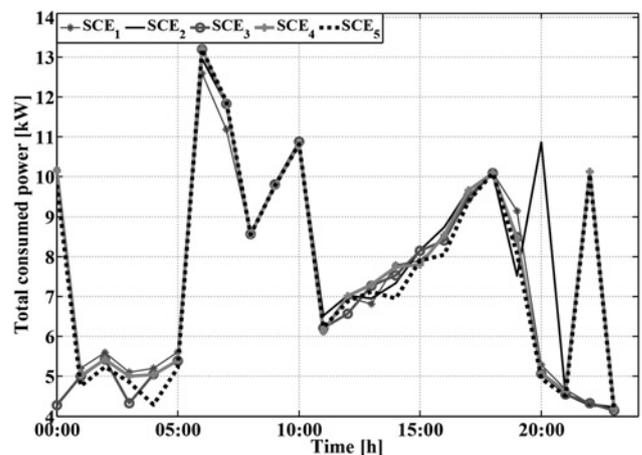


**Fig. 10** Results obtained by REM-NIRA algorithm  
a DR profile  
b MT profile



**Fig. 11** Results obtained by REM-NIRA algorithm  
a ES profile  
b SOC profile

that is the REM-NIRA algorithm tries to reduce 30% of maximum power generated by MT resulting in much less excess power is generated by generation resources compared with other scenarios, as shown in Fig. 10b. During the remaining hours of the day, algorithm in all of the applied scenarios puts MT in service with more capacity of generation. In scenario #1 with the increase of the value of the power generated by renewable resources, the value of SOC about 54% of time intervals has increased relative to normal conditions. Despite the increase of its value during the



**Fig. 12** TCP profile

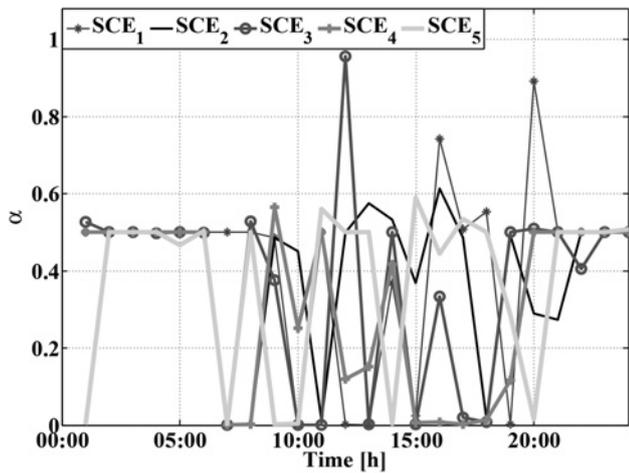


Fig. 13 Optimum value of  $\alpha_k$

system daily performance, and it reaching its maximum value among applied scenarios (about 70%), its value at the end of the day has reached its minimum value. Its reason is that the value of generated excess power is spent supplying DR power, and during 92% of time intervals its value is more than scenario #3. In scenario #2, only 4% of time intervals the value of SOC is more than its value during normal conditions; however 42% of time

intervals the value of DR power has reduced relative to normal conditions. This fact shows that after complete discharge during the system performance initial time, the ES SOC value during most time intervals remains constant at minimum value and the excess power generated is used mainly for supplying DR until the end of system operation, during which the proposed algorithm has decided to change ES in all of the scenarios except scenario #1, as depicted in Fig. 11a. Regarding the total consumed power (TCP) generated by the grid under study as it is observed from Fig. 12, in more than 96% of the time intervals its value has become more than its value in scenario #3 which the difference of the consumed power in fact is spent supplying DR and ES in more intervals. This condition is scenario #2 has reached about 54% of scenario #3 and its reason can be pointed out as more ES discharging times (10%) (according to Fig. 11a) under this scenario relative to scenario #3 despite the 42% increase in supplying DR.

For reaching convergence in the algorithm, each sequence of the measures of step (i.e.  $\alpha_k$ ) may be selected between each iteration for converging to the Nash equilibrium point. The measure of adequate steps may improve by trial and error while its value changes between the interval  $0 \leq \alpha \leq 1$ .

In many cases, it has been observed that by using constant step  $\alpha_k = 0.5$  it is quickly guided to the convergence value. Although the size of the steps can also be selected by the user, however in the proposed REM-NIRA algorithm the possibility of finding  $\alpha_k$  optimum value during the simulation operations with the aid of a mechanism exists. For this purpose by defining the optimum value control parameter,  $\alpha_k$  is also determined by the algorithm for the next step. The optimum value of  $\alpha_k$  during the time intervals has been shown in Fig. 13.

Figs. 14a and b show the values of converged pay-off function for consuming and generating players in the best iteration during each time interval. As it is observed from the figures, the values of pay-off functions of the players in all time intervals have become the same. The values of pay-off functions of all of the players gradually converge to the Nash equilibrium point by periodically updating the optimum response function (relation (26)). In the Nash equilibrium point, noting that the strategies of other players are constant, no player can improve its pay-off function by the unilateral changes of its strategy.

## 7 Conclusions

In this paper, a method has been proposed for implementing a market based on a competitive framework by noting obtaining maximum profit by DER units and minimum MCP by the consumers. The method presented has applied a relaxation algorithm, Nikaido–Isoda function and non-cooperative game theory. Both generation and the adequate distribution of renewable energy resources for consumers of electricity distribution system are main advantages of REM-NIRA algorithm. The customers in the proposed structure play an important role through exploiting and managing DERs, ESs and RLD loads locally, in contrast with conventional market structures. This algorithm provides the possibility of combining grid and other renewable energy resources constraints as well as presenting the ability to implement generation and demand prediction curves. The proposed ED including DER, ES and responsive loads resources and by considering the effect of load demand uncertainties and the variations of DERs output power. It is observed that in the presented structure, effective motivations for DER units to supply load and cooperate with each other for obtaining more profit and, in the case of consumers, for cooperating with each other for reducing electricity cost. The developed model can also be used as a useful tool for analysing market power and also for evaluating the strategic behaviour of effective agents in a competitive electricity market. Moreover, it can be used by the system independent operator for increasing market efficiency. For continuing the work, the proposed structure for systems with multi-ownership is under development in which each owner has several DERs, ES and separate responsive loads. In the proposed structure, the existing DER resources in each grid

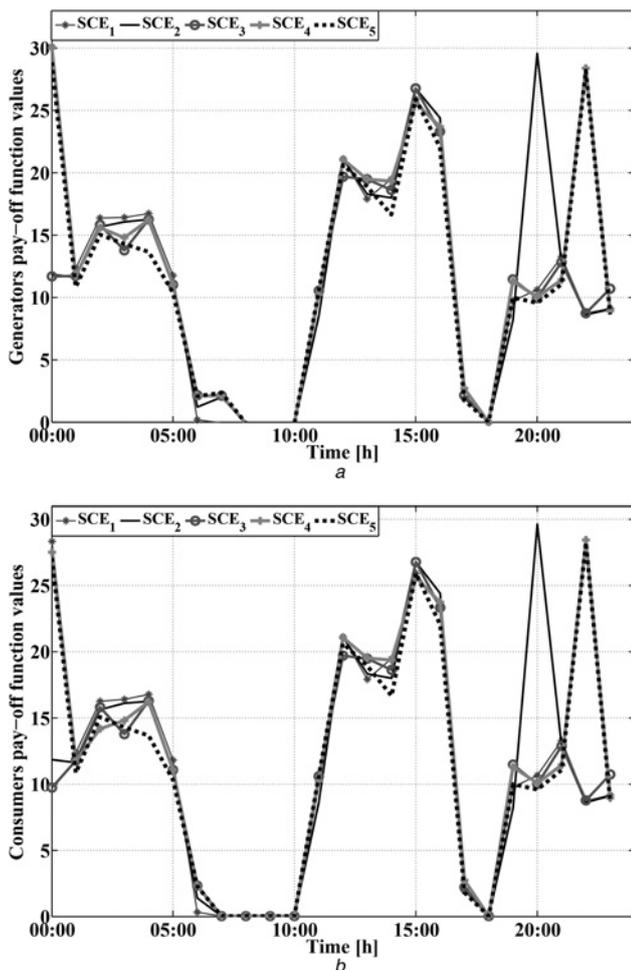


Fig. 14 Profile of the pay-off function of consumers and generation resources

a Pay-off function of generation resources

b Pay-off function of consumers

enter game independently with the consumers and follow their goal for obtaining more profit while satisfying all the constraints defined. Furthermore, a market structure based on non-cooperative game theory is also developed among different owners for increasing their obtained profit relative to independent optimisation. Therefore, the contributions and aims of the presented paper can be summarised as follows:

- (i) Presenting a survey relevant to retail competition model in electricity market.
- (ii) Proposing a market model for trading distributed renewable energy by guaranteeing fairness among non-cooperative players.
- (iii) Using a static non-cooperate game theory to model the interaction among different DERs, ESSs, and loads.
- (iv) Studying the competition of DER and RLD in a deregulated market.
- (v) Applying the NIRA methodology to find an electricity market equilibrium in order to clearing the retail electricity market price.
- (vi) Analysing the players's behaviour by using the concept of Nash equilibrium as a solution in the multi-agent interaction problems.
- (vii) Implementing a REM with full prosumers' participation through probabilistic methodology.
- (viii) Implementing demand-side energy management in order to use as a method for further balancing the supply and demand of power systems with high levels of renewable energy generation and controlling a residential energy storage system.
- (ix) Trying to increase the contribution of green energy in electricity supply, by high-performance green buildings.
- (x) Promoting DER such as energy efficiency, DER, energy storage devices, and renewable energy resources, increasing the number of DER installations through encouraging the costumers.
- (xi) Encouraging demand side participation in the electricity market.
- (xii) Decreasing problems caused by DG and increase the value of intermittent energy in the market by combining DG with energy storage and demand response.
- (xiii) Employing the supply function bidding mechanism to model electricity market.
- (xiv) Applying the developed RLD scheme as an energy resource of electricity retailers.

## 8 References

- 1 Shahidehpour, M., Yamin, H., Li, Z.: 'Market operations in electric power systems: forecasting, scheduling, and risk management' (John Wiley & Sons, 2002), vol. 9
- 2 Kopaei, M.A., Manbachi, M., Khezeli, H., et al.: 'Netpms: an internet-based power market simulator for educational purposes', *IET Gener. Transm. Distrib.*, 2012, **6**, pp. 472–481
- 3 Nafisi, H., Farahani, V., Abyaneh, H.A., et al.: 'Optimal daily scheduling of reconfiguration based on minimisation of the cost of energy losses and switching operations in microgrids', *IET Gener. Transm. Distrib.*, 2015, **9**, pp. 513–522
- 4 Ruiz, J., Wang, J., Liu, C., et al.: 'An outer-approximation method for security constrained unit commitment', *IET Gener. Transm. Distrib.*, 2013, **7**, pp. 1210–1218
- 5 Tesfatsion, L.: 'Agent-based computational economics: a constructive approach to economic theory', in Tesfatsion, L., Judd, K.L. (Eds.) *Handbook of Computational Economics Vol. 2*, (North-Holland, Amsterdam, 2006), pp. 831–880
- 6 Carvalho, M., Pedroso, J.P., Saraiva, J.: 'Electricity day-ahead markets: Computation of nash equilibria', *J. Ind. Manage. Optim.*, 2015, **11**, (3), pp. 985–998
- 7 Lee, K.: 'Strategy equilibrium in stackelberg model with transmission congestion in electricity market', *J. Electr. Eng. Technol.*, 2014, **9**, (1), pp. 90–97
- 8 Molina, J., Zolezzi, J., Contreras, J., et al.: 'Nash-cournot equilibria in hydrothermal electricity markets', *IEEE Trans. Power Syst.*, 2011, **26**, (3), pp. 1089–1101
- 9 Krawczyk, J.: 'Numerical solutions to coupled-constraint (or generalised nash) equilibrium problems', *Comput. Manage. Sci.*, 2007, **4**, (2), pp. 183–204
- 10 Uryasév, S., Rubinstein, R.: 'On relaxation algorithms in computation of noncooperative equilibria', *IEEE Trans. Autom. Control*, 1994, **39**, (6), pp. 1263–1267
- 11 Tesser, M., Pagès, A., Nabona, N.: 'An oligopoly model for medium-term power planning in a liberalized electricity market', *IEEE Trans. Power Syst.*, 2009, **24**, (1), pp. 67–77
- 12 Wen, F., David, A.: 'Optimal bidding strategies for competitive generators and large consumers', *Int. J. Electr. Power Energy Syst.*, 2001, **23**, (1), pp. 37–43
- 13 Contreras, J., Klusch, M., Krawczyk, J.: 'Numerical solutions to nash-cournot equilibria in coupled constraint electricity markets', *IEEE Trans. Power Syst.*, 2004, **19**, (1), pp. 195–206
- 14 Sanjari, M., Gharehpetian, G.: 'Game-theoretic approach to cooperative control of distributed energy resources in islanded microgrid considering voltage and frequency stability', *Neural Comput. Appl.*, 2014, **25**, (2), pp. 343–351
- 15 Marzband, M., Parhizi, N., Savaghebi, M., et al.: 'Distributed smart decision-making for a multi-microgrid system based on a hierarchical inter-active architecture', *IEEE Trans. Energy Convers.*, 2016, **31**, (2), pp. 637–648
- 16 Marzband, M., Parhizi, N., Adabi, J.: 'Optimal energy management for stand-alone microgrids based on multi-period imperialist competition algorithm considering uncertainties: experimental validation', *Int. Trans. Electr. Energy Syst.*, 2016, **26**, (6), pp. 1358–1372
- 17 Marzband, M., Yousefnejad, E., Sumper, A., et al.: 'Real time experimental implementation of optimum energy management system in standalone microgrid by using multi-layer ant colony optimization', *Int. J. Electr. Power Energy Syst.*, 2016, **75**, pp. 265–274
- 18 Marzband, M., Azarnejadian, F., Savaghebi, M., et al.: 'An optimal energy management system for islanded microgrids based on multiperiod artificial bee colony combined with Markov chain', *IEEE Syst. J.*, 2015, **100**, (99), pp. 1–11
- 19 Marzband, M., Ghadimi, M., Sumper, A., et al.: 'Experimental validation of a real-time energy management system using multi-period gravitational search algorithm for microgrids in islanded mode', *Appl. Energy*, 2014, **128**, pp. 164–174
- 20 Marzband, M., Sumper, A., Ruiz-Álvarez, A., et al.: 'Experimental evaluation of a real time energy management system for stand-alone microgrids in day-ahead markets', *Appl. Energy*, 2013, **106**, pp. 365–376
- 21 Marzband, M., Sumper, A., Domínguez-García, J.L., et al.: 'Experimental validation of a real time energy management system for microgrids in islanded mode using a local day-ahead electricity market and MINLP', *Energy Convers. Manage.*, 2013, **76**, pp. 314–322
- 22 Marzband, M.: 'Experimental validation of optimal real-time energy management system for microgrids'. Phd thesis, Departament d'Enginyeria Elèctrica, EU d'Enginyeria Tècnica Industrial de Barcelona, Universitat Politècnica de Catalunya, 2013
- 23 Su, W., Huang, A.Q.: 'A game theoretic framework for a next-generation retail electricity market with high penetration of distributed residential electricity suppliers', *Appl. Energy*, 2014, **119**, pp. 341–350
- 24 Marzband, M., Sumper, A.: 'Implementation of an optimal energy management within islanded microgrid'. Int. Conf. on Renewable Energies and Power Quality (ICREPPQ), Cordoba, Spain, 2014
- 25 Marzband, M., Sumper, A., Chindriş, M., et al.: 'Energy management system of hybrid microgrid with energy storage'. The Int. Word Energy System Conf. (WESC), Suceava, Romania, 2012
- 26 Billinton, R., Bonaert, A., Koivo, A.: 'Power system reliability evaluation' (Gordon and Breach, New York, 1971), 5th edn., vol. 1
- 27 Fotuhi-Firuzabad, M., Rastegar, M., Safdarian, A., et al.: 'Probabilistic home load controlling considering plug-in hybrid electric vehicle uncertainties'. Reliability Modeling and Analysis of Smart Power Systems, Springer India, 2014, pp. 117–132
- 28 Daniel, A., Chen, A.: 'Stochastic simulation and forecasting of hourly average wind speed sequences in Jamaica', *Sol. Energy*, 1991, **46**, (1), pp. 1–11
- 29 Borowy, B., Salameh, Z.: 'Optimum photovoltaic array size for a hybrid wind/PV system', *IEEE Trans. Energy Convers.*, 1994, **9**, (3), pp. 482–488
- 30 Tina, G., Gagliano, S., Raiti, S.: 'Hybrid solar/wind power system probabilistic modelling for long-term performance assessment', *Sol. Energy*, 2006, **80**, (5), pp. 578–588
- 31 Jin, B.S.Y.: 'Trade-off between performance and robustness: An evolutionary multiobjective approach' (Springer Berlin Heidelberg, 2003), pp. 237–251
- 32 Chatthaworn, R., Chaitusaney, S.: 'Transmission network expansion planning considering renewable energy target with Taguchi's orthogonal array testing', *IEEE Trans. Electr. Electron. Eng.*, 2014, **9**, pp. 588–599
- 33 Yu, H., Chung, C., Wong, K.: 'Robust transmission network expansion planning method with Taguchi's orthogonal array testing', *IEEE Trans. Power Syst.*, 2011, **26**, (3), pp. 1573–1580
- 34 Bounou, M., Lefebvre, S., Do, X.: 'Improving the quality of an optimal power flow solution by Taguchi method', *Int. J. Electr. Power Energy Syst.*, 1995, **17**, (2), pp. 113–118
- 35 Klusch, M., Gerber, A.: 'Dynamic coalition formation among rational agents', *Intell. Syst.*, 2002, **17**, (3), pp. 42–47