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A review on non-classical continuum mechanics with applications in marine engineering

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ABSTRACT
Marine structures are advanced material and structural assemblies that span over different length scales. The classical structural design approach is to separate these length scales. The used structural models are based on classical continuum mechanics. There are multiple situations where the classical theory breaks down. Non-classical effects tend arise when the size of the smallest repeating unit of a periodic structure is of the same order as the full structure itself. The aim of the present paper is to discuss representative problems from different length scales of ship structural design.

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1. Introduction

Modern marine structures are optimized, advanced material and structural assemblies that bridge over numerous length scales. The relative density of a hull girder of a cargo ship can be of the order $\rho\ /\rho_{\text{steel}} = 10^{-2}$, which means that the construction materials are effectively positioned to form a lightweight structure. These thin-walled structures are exposed to random physical environments with load effects arising from waves, wind and ice and the operational life can be decades, requiring the assessment of numerous load cases during the design process by using quasi-static or dynamic structural analyses. Nowadays there is also an expectation for adequate strength against human-caused load effects such as collisions and groundings, and fires and explosions involving time-consuming non-linear structural assessments. Typically, these structural assessments are performed via finite element analyses. The constraints for design optimization are often technical but, increasingly, also economical, societal and experiential. This sets challenges and opportunities for the structural design, making maritime applications a fruitful venue to exploit and develop further the theories and computational approaches developed in different areas of engineering. This paper focuses on the structural assessment of ships.

The classical ship structural design approach is to consider the length scales for the primary (i.e. hull girder), secondary (i.e. bulkhead spacing) and tertiary (i.e. panel) responses separately, see Figure 1 [2]. This multi-scale structural modeling accounts for displacement compatibility and (stress resultant) equilibrium in the coupling between the consecutive length scales. Usually the structural models at larger length scales are based on classical continuum mechanics, e.g. Euler-Bernoulli beam modeling of the ship hull girder. The classical beam kinematics are assumed to be valid (e.g. curvature) and the resulting stress resultants (e.g. bending moment) are balanced by loads arising from the environment. The relation between the curvature and bending moment can be non-linear, with sources of non-linearity arising from buckling, plasticity or fracture of the structural elements (e.g. [3–12]), see Figure 2.

As mentioned, the used structural theories are based on classical continuum mechanics and there are numerous challenges that have been tackled based on "engineering judgement." First of all, classical continuum mechanics assumes that any two consecutive length scales are far apart. In a classical continuum, the stress at a point is defined fully if we know the strain at that point along with the constitutive (generally non-linear) behavior. This is not true in ship structures. When modeling a large ship structure, the material point is often, in fact, a unit cell (or Representative Volume Element, RVE) that describes the smallest repeating unit of a larger structural element. Moreover, often the unit cell kinematics and the interaction between neighboring unit cells cannot be modeled accurately using a classical approach. Thus, the engineering remedy has been to model, for example, the zone of interest for collapse analysis together with surrounding structure, see Figure 2. This enables proper modeling of higher-order effects in the zone of interest.

However, the problem is to define the extent of the surrounding structure. For instance, in bulk-carriers and tankers following Euler-Bernoulli beam kinematics, so-called three-, two- or one-cargo hold models are used, see Figure 2. The main motivation for this is to ensure proper structural behavior at the middle cargo hold where the failure needs to be accurately modeled. As the boundary

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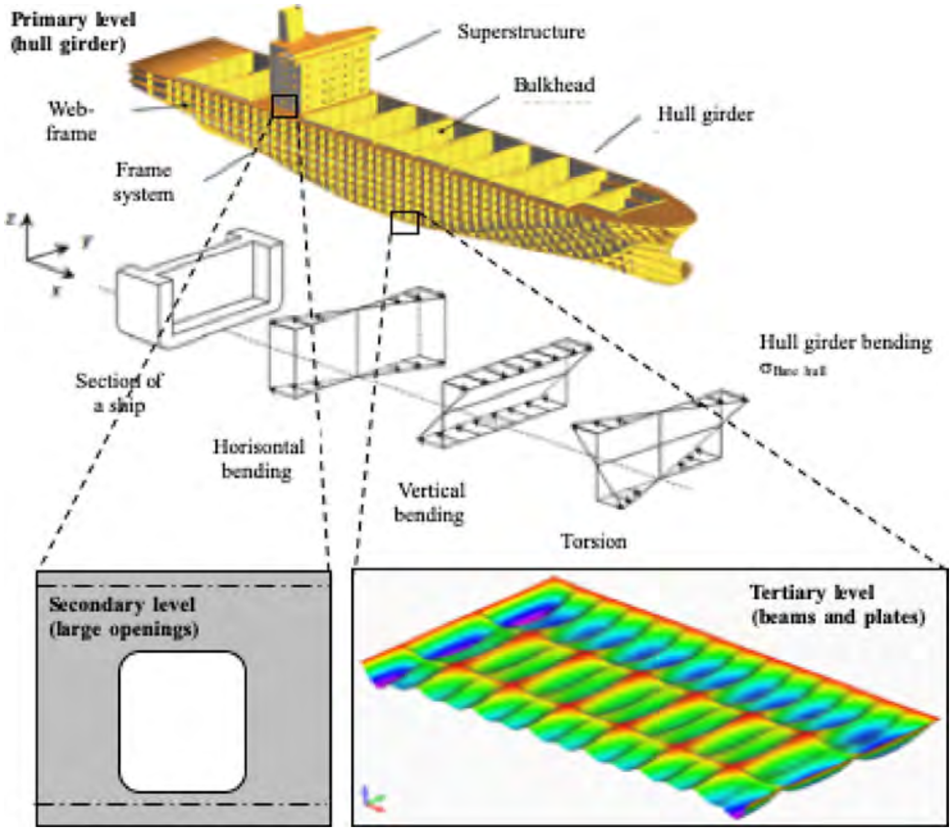

Figure 1. Structural responses on primary (hull-girder), secondary (double bottom of cargo-hold) and tertiary (e.g. deck plating) length scales in ship structures. Modified from [1].

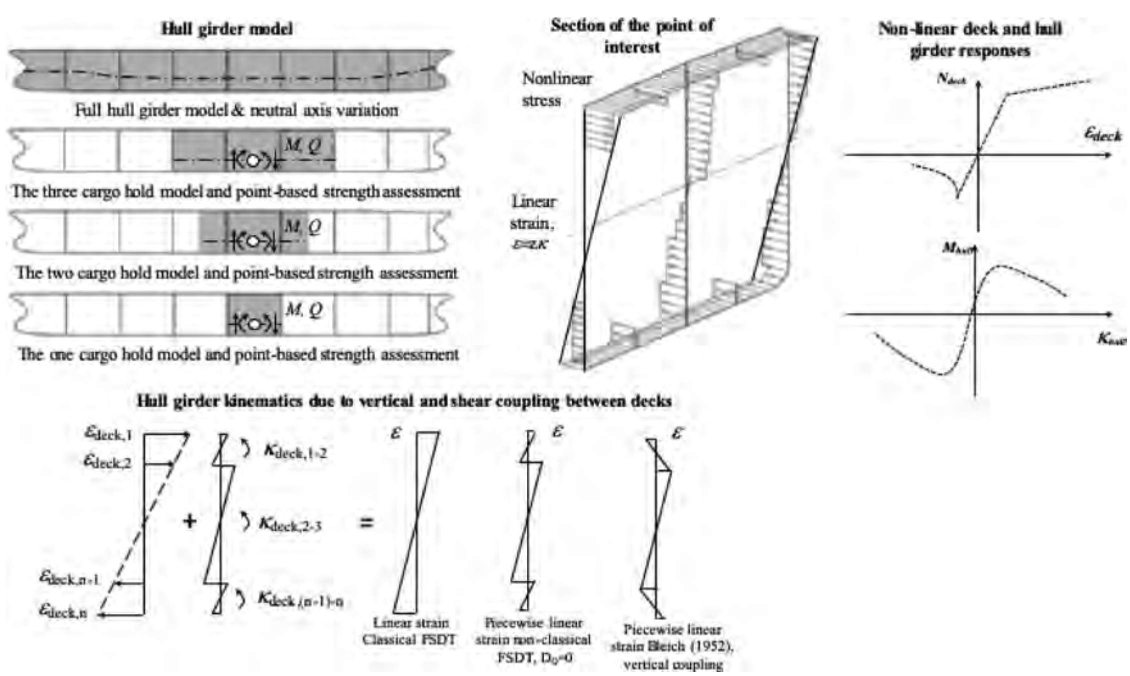

Figure 2. Modeling of bending strain and stress of hull girder by using Euler-Bernoulli theory and different extents of beam modeling to account the effects of structural discontinuities. Modified from [13] and [8].

reactions interact with the process-zone of failure, the extent of the model cannot be uniquely defined for all ship types with different structural elements present (e.g. a very stiff double bottom). Another challenge is large, multi-deck passenger ships, see Figure 2, where the deviation of strain from linear distribution can occur due to weak shear or vertical coupling between the decks (e.g. [13–16]). Similar interactions can also be found at the smaller length scales, i.e. between secondary and tertiary, which affect the structural modeling and the accuracy of the predicted results.

It should also be noted that as material technology develops, our materials become more structured, which basically

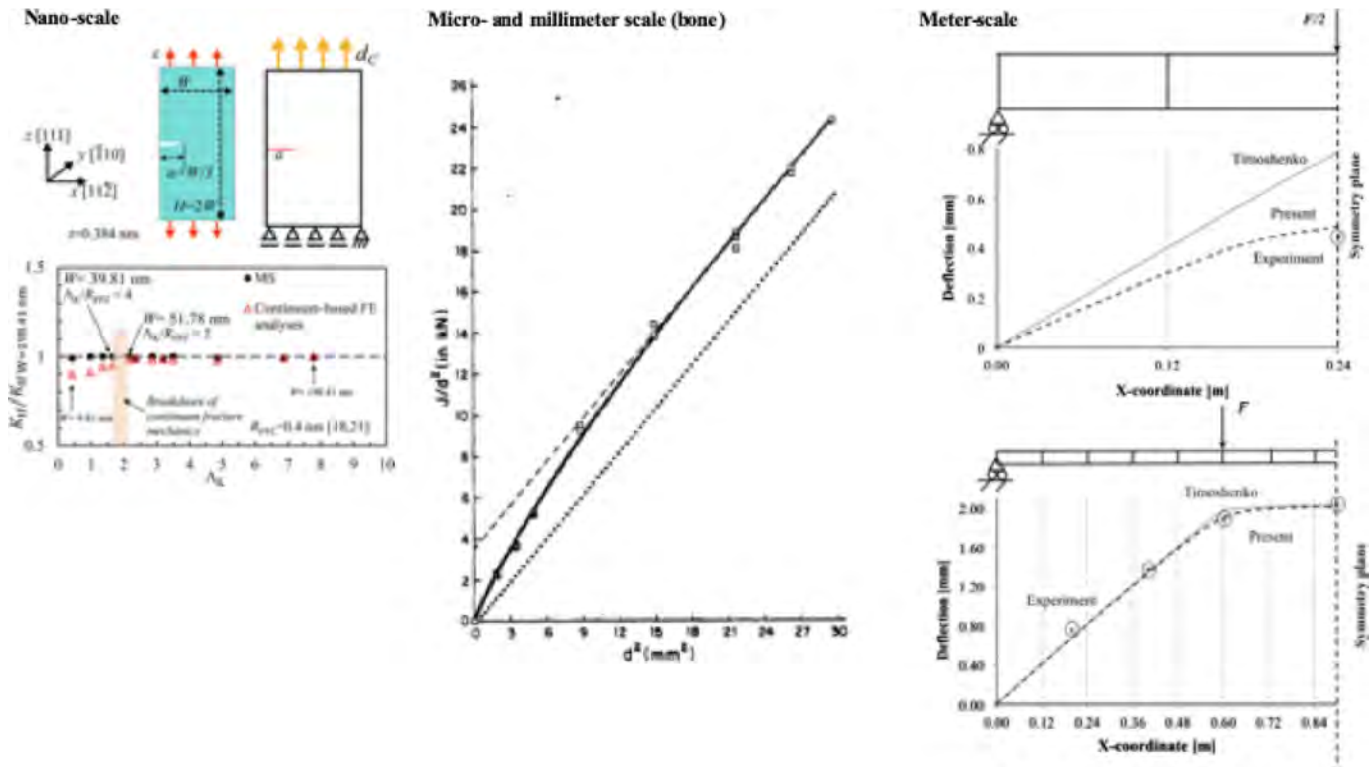

Figure 3. Examples of experiments performed close to the continuum limit; atomistic simulations of fracture [22], biological tissue [23], and bending of web-core sandwich beams [24].

adds structural length scales to our designs [17]. Each added length scale makes the numerical analysis based on finite element analyses more costly as the mesh size is set by the smallest structural scale. A grand challenge is the modeling of damage through all these length scales accurately and the structural models utilized have an effect on this process [9–12, 18–20]. As most of structural models (e.g. beams, plates and shells) are based classical continuum mechanics, we cannot accurately predict the impact of these new materials on our full designs, except by using costly 3-D finite element models. Thus, there is a need for better methods.

In non-classical continuum mechanics, one of the main ideas is to relax the assumption that consecutive length scales are far apart which makes the application of these theories interesting in ship structural design. The separate length scales are essentially bridged by non-local material models that have intrinsic length scales in them (in the broad sense, "non-local" refers to all constitutive models that involve a characteristic length). There are numerous continuum theories developed around this idea as reviewed by [21]. Figure 3 shows some examples on the experiments of non-classical continuum mechanics at different length-scales with applications on MEMS (e.g. [25]), biological tissue (e.g. [23]), atomistic simulations of fracture (e.g. [22]) and bending of web-core sandwich beams [24]. Based on the non-classical theories, much better performing structural theories with both analytical and finite element solutions have been formulated for applications where the size of the microstructure is of the same order as that of the macrostructure.

This paper presents an overview on the present and prospective applications of non-classical continuum mechanics theories in marine engineering. This is done in order to bridge the two scientific communities of solid mechanics and naval architects with aim to create mutual benefits. Naval architects need better tools to handle the complex problems they face daily when developing new designs utilizing better materials. For the solid mechanics community, the problems provide an opportunity to see the newly developed theories applied to practice and to develop something new. Although the paper focuses on maritime applications, the problems presented can also be found in other applications of thin-walled structures in civil, aeronautical and mechanical engineering.

2. Modeling the response of periodic structures – the tradeoff

Any structure with discrete geometry can be analyzed in its discrete form using a mesh (e.g. analytical frame mesh, finite element mesh) or in homogenized (averaged) form in which the discreteness is not modeled explicitly, see Figure 4.

While the discrete form leads to large computational models with high accuracy for both global and detailed responses, the homogenized solution produces smooth, averaged, responses with low computational cost, and often lower accuracy. In practice, the homogenized approach leads to a computational (finite element) mesh as well but the mesh will not be as dense as for the fully discrete approach. As an example of this paradigm, the following simple beam example is shown.

The accurate discrete solution to periodic beam bending (Figure 4) can be derived based on discontinuity functions.

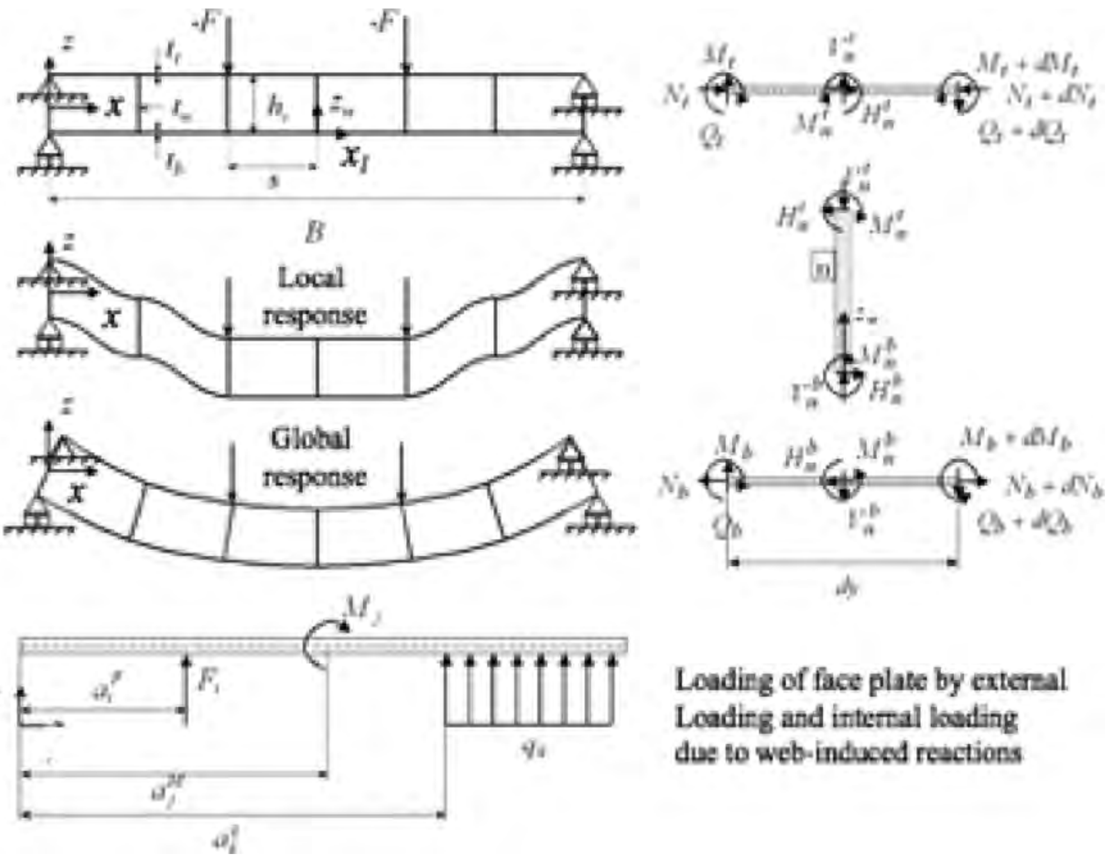

Figure 4. Discrete solution for a discrete web-core beam by use of discontinuity functions and classical Euler-Bernoulli beam theory locally at the microstructure [26].

Romanoff and Varsta [26] split the behavior of a web-core beam into two parts, i.e., the local response due to local bending of the face sheets and the web-plates bending around their own neutral axes without mid-plane elongation for the face plate deflection, and the global response due to the mid-plane (membrane) stretching/compression of the two face plates to opposite directions. Euler-Bernoulli beam theory was assumed to be valid at the level of web and face plates. The local response results mainly in a local deformation wave-length with a period equal to the web-plate spacing (s), $l_{\text{micro}} = s$, whereas the global deformation produces a wave-length with a period equal to the beam length (L), $l_{\text{macro}} = L$. In case of weak rotation coupling between the webs and face plates, the deformation length-scale between these two extremes activates. It was shown by Romanoff and Varsta [26] that the global and local deflections are

$$
\begin{aligned}
w_g &= w_0^g + \theta_0^g x - \frac{1}{D_g}\left(\frac{\left(M_n^t + M_n^b\right)H\left(x - x_n^w\right)\left(x - x_n^w\right)^2}{2!}\right)\\
w_{f,i} &= w_0^{l,i} + \theta_0^{l,i} x - \frac{1}{D_{f,i}}\sum_{n1=1}^{N1}\frac{H\left(x - a_{n1}^M\right)M_{n1}\left(x - a_{n1}^M\right)^2}{2!}\\
&\quad + \sum_{n2=1}^{N2}\frac{H\left(x - a_{n2}^F\right)F_{n2}\left(x - a_{n2}^F\right)^3}{3!} + \sum_{n3=1}^{N3}\frac{H\left(x - a_{n3}^q\right)q_{n3}\left(x - a_{n3}^q\right)^4}{4!}\\
w_n^w &= \frac{M_n^t d^2}{6D_w}\left(-\frac{z_w}{d} + \frac{z_w^3}{d^3}\right) + \frac{M_n^b d^2}{D_w}\left(2\frac{z_w}{d} - 3\frac{z_w^2}{d^2} + \frac{z_w^3}{d^3}\right)
\end{aligned}
\tag{1}
$$

where H denotes the Heaviside-operator with its 1st and 2nd derivatives being Dirac's delta and unit doublet function, respectively. Symbols q, F and M denote the pressure applied at face i and the external and internal forces and moments located at the web-plates with superscripts t and b denoting the top and bottom faces, see Figure 4. These equations can be solved by setting the deflections and slopes equal at the location of web-plates and at the boundaries of the beam and this results 3 equations per location [26]. In this solution it is important to notice the following Strong Conditions:

1. The displacement continuity of global deflection, w_g, is satisfied between deflection, w, and slopes, dw_g/dx, at the location of the webs. The curvature, d^2w_g/dx^2, is constant between the webs and a step is made once moving from one unit cell to another. Thus, the global bending moments and curvatures are piecewise continuous.
2. In the same way, the local behavior, $w_{f,i}$, is smooth in terms of deflections and slopes at the locations of the web-plates. The higher-order derivatives are piecewise constant (M), linear (F) or parabolic (q) with steps at the locations of webs.

It has been shown by Romanoff and Varsta [26] that by adding the piecewise global and local bending moments results in the bending moment diagram of the entire beam, see Figure 5A. This proves that the stress resultants of the beam are in equilibrium with the loading of the beam.

We could approximate locally this accurate solution by Taylor series expansion around point a, by writing

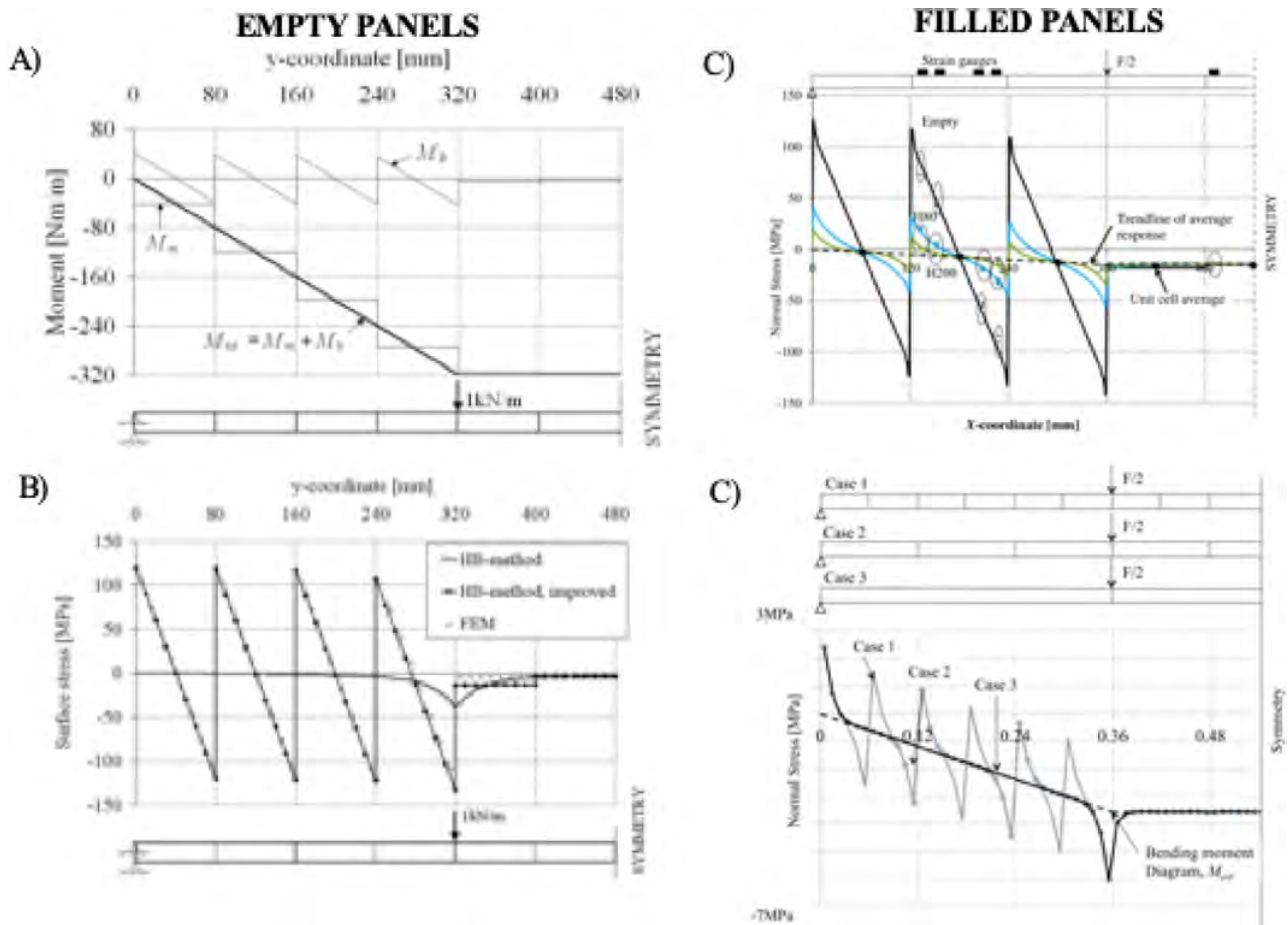

Figure 5. A) Discrete local (M_b) and global (M_m) solution for a discrete web-core beam by use of discontinuity functions [26]; B) periodic top surface stresses computed by 3D-FEA, and by using Homogenized Beam (HB) approach [27]; C&D simulated and experimentally measured strains and their averages from top surface of foam filled beams with different grades of Divinycell and different unit cell lengths [28].

$$w(x) = w^0(a) + \frac{w^1(a)}{1!}(x-a) + \frac{w^2(a)}{2!}(x-a)^2 + \ldots \quad (2)$$

thus, the accurate description in the close neighborhood is obtained including only few terms to the series, but the solution over the entire beam would require as many unknows to be solved as in case of the accurate solution so this method does not introduce any computational savings.

An alternative to the described method is homogenization, i.e., working with averaged responses. In homogenization, the idea is to use, for example, an equivalent single-layer (ESL) structural model for which the equivalent stiffnesses are determined from unit cell (RVE) analyses (for beams see [27] and for plates [26]). In homogenization, the focus is on the periodic response, i.e. condition $f(x)=f(x+l_{\text{micro}})$, which states that the function values are equal at the edges of the unit cell. If in addition to this, the function is odd within the unit cell, the volume average of the function will be zero. This way we can get rid of the solution of microfluctuations in the response analysis, which accelerates the solution considerably. If localization is carried out based on correct kinematics, the stresses can be recovered with very good accuracy, see Figure 5B and for example [27]. These assumptions are feasible for for both empty and filled web-core panels as shown by [28]; see Figure 5A,C,D. Therefore, this logic can be applied for example to the displacement of the face sheet in Eq. (1) by splitting the local response into two parts, and forming Approximating Conditions, i.e.

1. Constant slope within the unit cell and undeforming face plate. Thus, the response is simply the slope times the distance. This allows the modeling of deformation through an equivalent shear angle used for example in First-Order Shear Deformation theory.
2. Varying periodic, odd, slope within the unit cell, i.e. $f(x)=f(x+l_{\text{micro}})$. This secures the fact that the function itself and the derivatives of different order result in zero average if the local response is mathematically described by odd functions with respect to mid-point of unit cell.

If the Approximating conditions are valid, the approximation for the homogenized deflection can be written as:

$$w^k(x) = w^0(x,y) + k^1 w^1(x,y) + k^2 w^2(x,y) + \ldots$$
$$k = \frac{l_{micro}}{l_{macro}} \ll 1, \;\; w^k(x,y) = w^k(x, y + l_{micro}) \quad (3)$$

where the first term gives the classical solution. The additional terms can be used to enhance the solution and

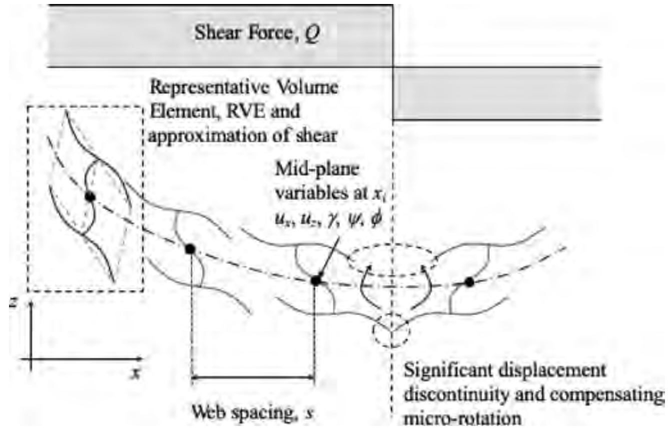

Figure 6. Continuum modeling of a periodic beam with microrotation ψ provided by the micropolar theory of elasticity that describes the local bending of the face sheets.

compensate for some of the broken strong and approximating conditions given above.

Figure 6 shows that at the location of the kink in the shear force, the shear-induced warping deformation changes sign and, for example, a compensating microrotation provided by the non-classical micropolar continuum theory, is needed to ensure the compatibility for the local face deflections (see for example [19, 20, 29]). Depending on the type of additional variables or gradients used, various non-classical continuum, and structural, theories can be derived to model structures like the one shown in Figure 6, as summarized by [21]. Also, the finite elements can be derived (e.g. [30]).

In conclusion, as demonstrated by the example above, there is always a tradeoff between accuracy and computational efficiency in engineering. With non-local models, we can include additional information in homogenized continuum descriptions that lead to computationally efficient models, and this way we can also get also closer to the accurate discrete cases in terms of displacements, stresses and eigenvalue vibrations and buckling (e.g. [19, 20, 27, 29, 31]). This is important as homogenization based on stiffness only solves only partially the engineering problem, but with these required extensions we can enrich the solution to account also the micro-fluctuations and interactions between micro- and macroscales.

3. Selected examples of length scale interaction in ship structural design

3.1. Design process overview, focus on optimization

Ship structural design is governed by the definition of environmental and accidental loads, definition of structural response for these loads and by the checking of the strength against limit states of yielding, serviceability, ultimate strength and accidental limit state [2, 8]. A successful design process is a result of the idealizations made in load, response and strength modeling. A numerical design process example is given in [32] and Figure 7 where finite elements are coupled with evolutionary optimization algortihm.

The problem with this classical process is that the non-local effects are excluded from the beam and shell element formulations used to assess the structural response and, thus, the deformations of the periodic structures may not be modeled in sufficient detail. During optimization, we may visit any regions of design space in which response and strength predictions are false due to assumptions in beam and plate element formulations. Such an example is shown in Figure 8, where two optimal solutions (alternatives are due to different constraint relaxation) are compared with each other and in relation to the linear Euler-Bernoulli beam theory. It can be seen that the normal stress distribution of the hull girder is different for different optimal designs and the stresses arising from classical solution significantly overestimate the load due to hull-girder bending at most decks. Due to this reason, the optimization must be performed with 3-D FEA in passenger ships, while in tanker and bulk carriers beam theory can be used. The reasons are explained in the next section, where some selected examples from different length scales where these problems may emerge are shown. We start from the smallest of length scales, tertiary, where the investigations have been already performed and published.

3.2. Tertiary and emerging length scale: Strength of panels, girders, frames

Ship structures are assemblies of beams, plates and shells. Plates are used mainly to carry global loads in membrane action, while stiffeners (beams) are used to reduce the bending actions arising from local pressures from cargo and environment. The most commonly used structures are single-sided stiffened panels, while also composite laminates and sandwich structures are used, but to a lesser extent. The orthotropic plate theory and offset beams are often used in the modeling. When properly formulated, a single plate model can handle different materials, structural topologies and the combination of these within a finite element framework as shown in the previous example. In ultra-lightweight structures, the volume fractions for structural materials become very low. This means that the microstructure of the material or structural unit becomes visible at the macroscopic length scale. [20] and [18] have shown that sandwich plates with high orthotropy ratio in shear, D_{Qx}/D_{Qy}, have an ever-decreasing eigenvalue buckling strength in the weak direction when classical continuum models are used. The same is found in biaxial buckling, while in strong direction, the classical models predict the lowest buckling eigenmodes accurately. In turn, when a micropolar plate formulation [20] is utilized, the plate model predicts correct buckling loads, see the red 2-D curve in Figure 9 which follows the 3-D solution. The challenge with this buckling case is that the differential equations based on classical continuum mechanics lose their ellipticity with high orthotropy ratio, see [33] for details. Due to this 2-D solutions tend to produce false results when compared to 3-D finite element analyses. When numerical solutions are considered, an additional mistake is introduced as the eigenvalues become mesh-size dependent. Thus, as the case demonstrates, there is a need for a micropolar plate element that can model the size-dependency in all relevant regions of the design space.

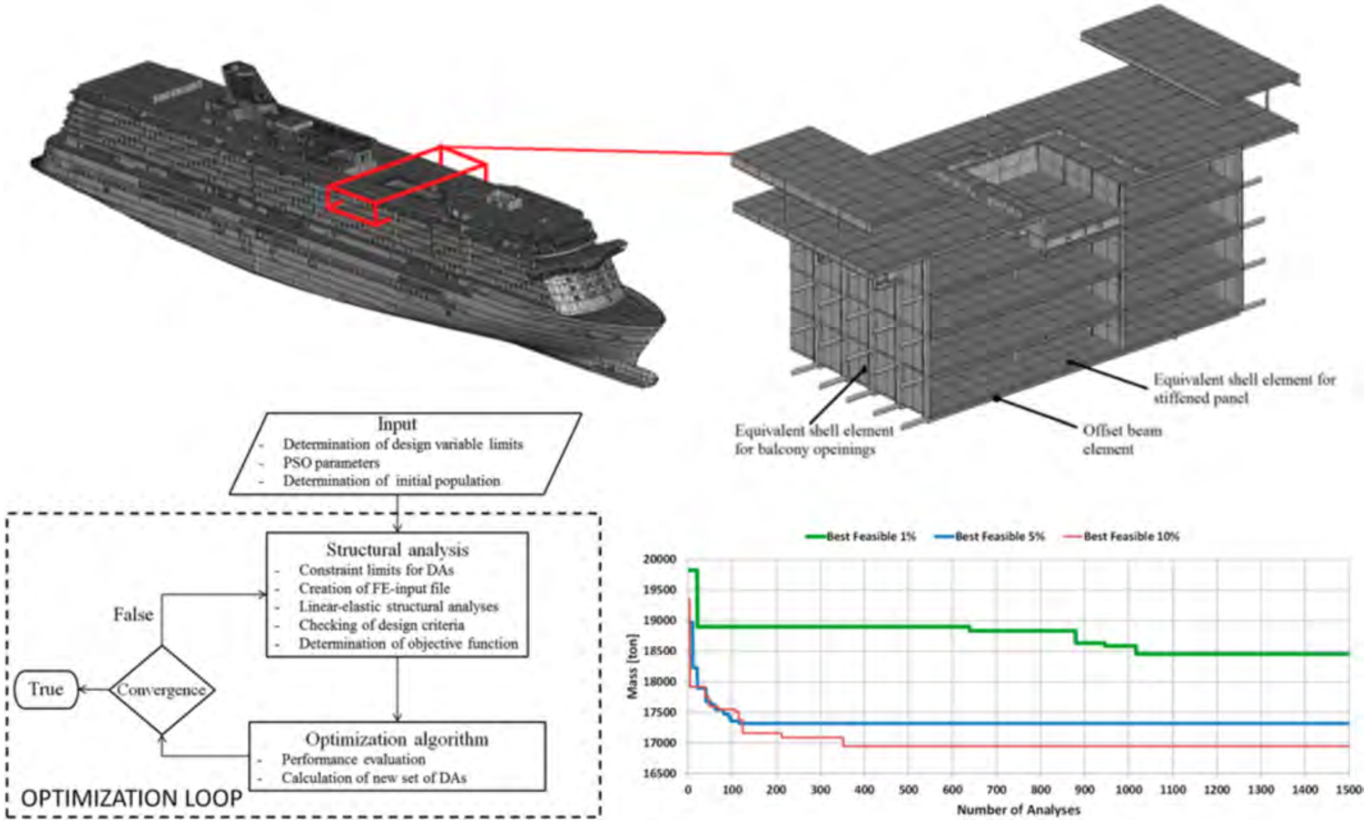

Figure 7. Optimization of a large passenger ship by using beam and shell elements based on classical continuum mechanics [32].

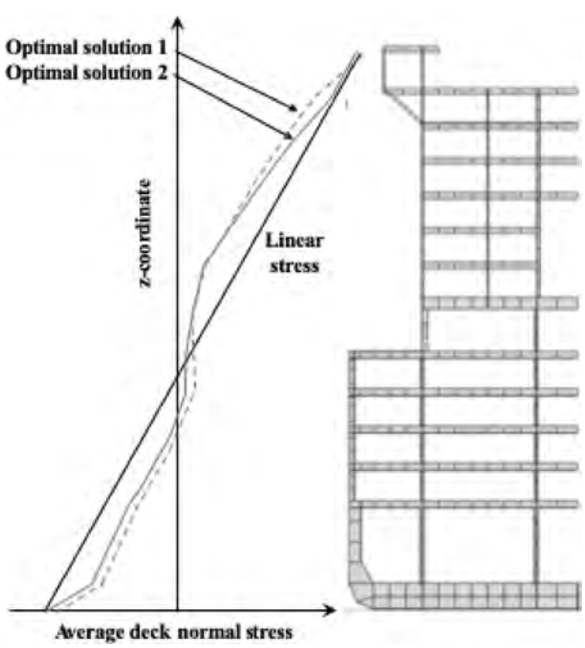

Figure 8. Comparison of the load-carrying mechanism of two optimal design in relation to the classical Euler-Bernoulli beam theory (modified from [32]).

3.3. Secondary length scale: Strength assessment of main structural elements

The secondary length scale corresponds to the intermediate scale between the primary (hull-girder) and tertiary (panels) length-scales. It is also the reason for the deviation of the through-thickness stress distribution from the classical Euler-Bernoulli theory as seen in Figure 8. Examples of this are the side shells of passenger ships that act as vertical shear-walls between the decks. Traditionally, these have been full of holes for windows. In continuum sense, the windows are modeled at the center of the shear wall meaning that the in-plane stretch and shear are decoupled and do not introduce bending to the shear wall. In recent years, however, the openings have increased in size and they have been positioned off-center due to introduction of balconies into cruise ships. In these ships, the structural element size is defined by the positioning of the decks, while the opening is typically non-symmetric with respect to the vertical center of the element. This introduces in-plane membrane-bending coupling to the equilibrium equations that current membrane elements are uncapable of treating, see Figure 10 and [34]. Thus, there is an urgent need for membrane elements that possess this coupling. Such element could be based on the work performed at masonry structures (see for example [35, 36]).

3.4. Primary length scale: Hull girder strength assessment

When ships operate under wave actions, they deform as a whole. The behavior can be described with classical beam theories based on assumptions of Euler-Bernoulli, Timoshenko or Vlasov if the ship hull girder is able to fulfill the stiffness requirements required by different kinematic assumptions. For slender closed-cell primary structures such as tankers and bulk carriers, the Euler-Bernoulli beam

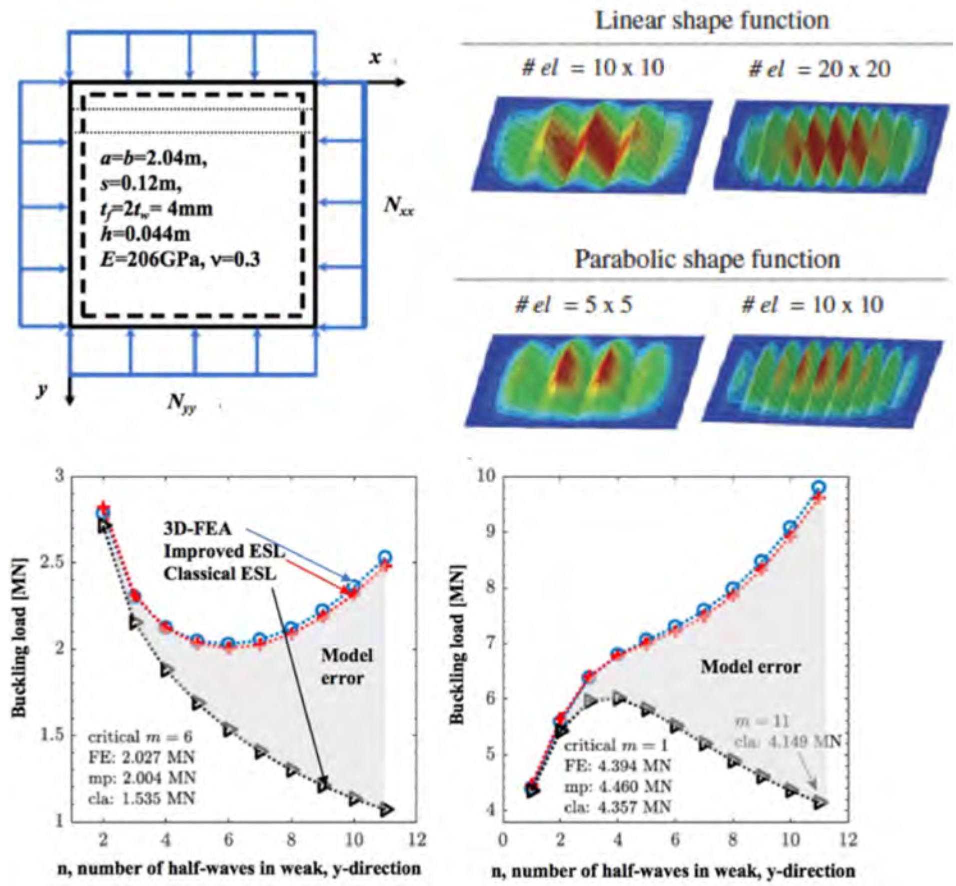

Figure 9. Modeling error for bifurcation buckling of a square plate. Top. Case description and mesh-size dependency of the finite element solution based on classical continuum mechanics. Bottom. The modeling error of a classical solution (black) in comparison to an improved micropolar solution (red) and 3-D FEA (blue). (Modified from [18] and [19, 20]).

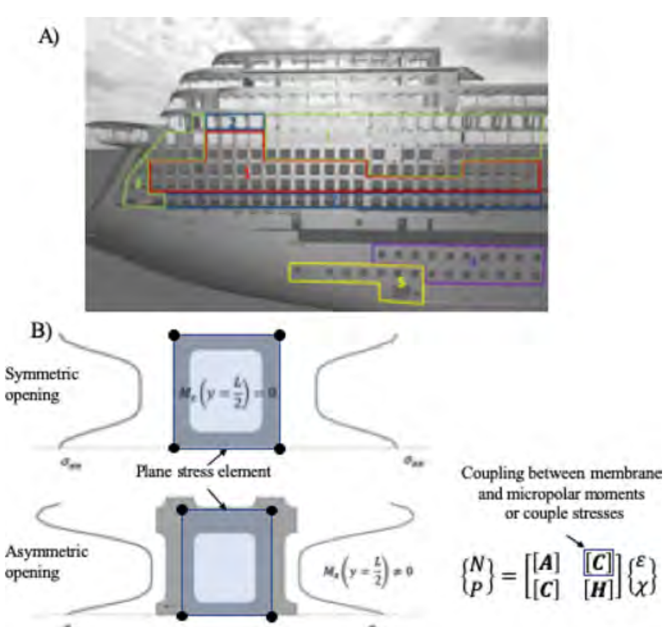

Figure 10. Different 2-D plane stress problems in hull girder modeling: A) different continuity conditions in the side shell of a cruise ship and B) occurrence of couple stresses in asymmetric side-shell openings due to balconies undergoing in-plane stretch and modeled with single plane stress finite element (modified from [34]).

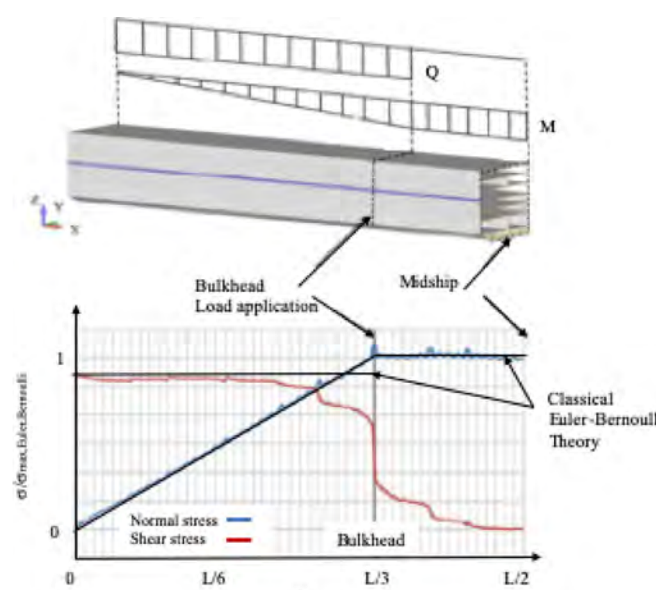

Figure 11. Classical continuum mechanics and hull girder bending in 4-point bending with resulting normal stress in decks and shear stress in neutral axis. (modified from [16]).

theory based on classical continuum mechanics is sufficient. In case of container ships with large deck openings, the shear stiffness is reduced and instead the theories of Timoshenko or Vlasov give better results. However, as the previous examples from Figures 8 and 10 show, there is a need for non-local formulations in passenger ships due to their complex geometry.

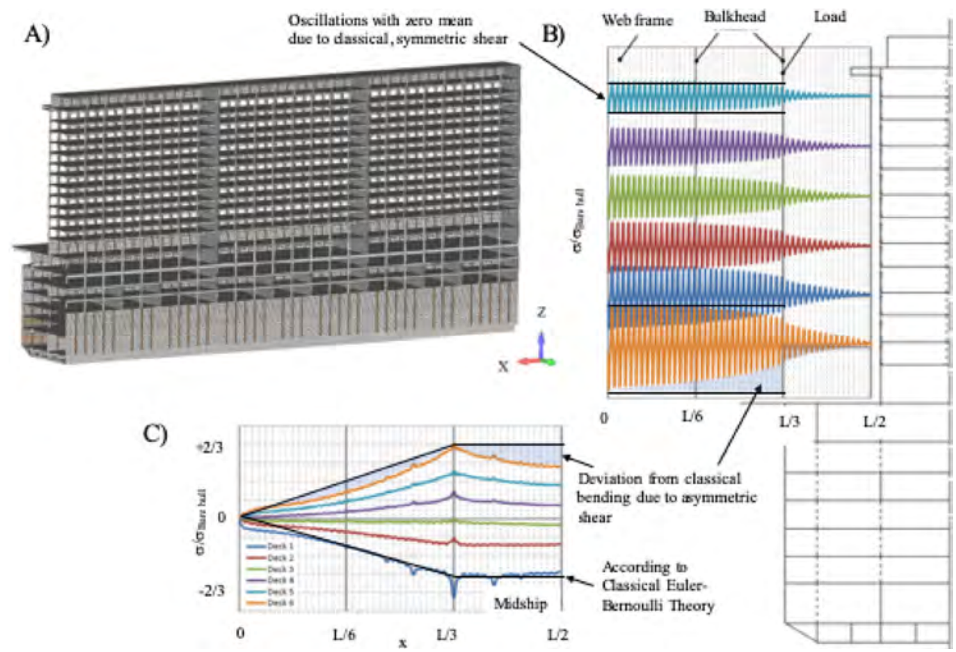

Figure 12. Deviation from classical continuum mechanics in case of a passenger ship (modified from [16]).

The selected demonstration case, based on 3-D FEA, shows the effect of adding shear-weak superstructure of equal length to the hull girder of a ship. Figure 11 shows the behavior of the hull girder alone in 4-point bending. It is clear that the top-fibre normal stress and the vertical shear stress at neutral axis, follow the distributions of classical shear and bending moments. Figure 12 shows the situation after the superstructure is added to the top of the hull girder.

Figure 12 shows that the normal stress at the hull deviates from the classical bending moment diagram when superstructure is included to the structural model. It also shows that the response of shear weak superstructure does not follow the classical shear force diagram. The deviation is largest at the interface between the superstructure and hull and also close to the step in the shear force diagram, where the shear angle changes rapidly. These effects can only be modeled if the shear strain can be divided into symmetric and antisymmetric parts (e.g. [19, 20, 29]) as in the micropolar continuum mechanics and if the microrotation along beam height can vary. Thus, there is a need for microrotation correction factor for Timoshenko beam, which could analogous to shear-correction factor, model the distribution of micropolar moments over the thickness of the beam. The alternative could be layer wise formulation for the beam. The first steps on development of such element have been presented in [14] for linear and in [15] for non-linear case, where the asymmetric shear is modeled by use of shear and vertical springs between classical Timoshenko beams representing the decks of the ships.

4. Conclusions

Modern marine structures are highly-optimized, advanced material and structural assemblies that bridge over numerous length scales. They are exposed to random environments and need to be designed with computational approaches for adequate safety against failure. The classical structural design approach is to consider the length scales for primary (i.e. hull girder), secondary (i.e. bulkhead spacing) and tertiary (i.e. panel) responses separately. In essence, the behavior at all these scales is similar to that seen in architected lattice materials. Usually, the structural models at the larger length scales are based on classical continuum mechanics, in which the asymptotic approach to decoupled length scale interaction is assumed. However, there are multiple situations where this assumption is violated.

The present paper gave an overview the representative problems from different length scales of ship structural design. We showed a representative case from each length scale seen in ships where non-classical continuum mechanics can make a significant improvement to the ship structural design. It is clear that there is an urgent need especially for micropolar beam, plate and shell elements that can be used to assess the structural response of different structural and material alternatives during optimization without a fear of modeling error. It is also clear that the micropolar elements should be able to model the coupling between in-plane stretching and bending, coupling between global and micropolar moments and shear, and non-uniform distribution of micropolar moments over the thickness of the structure. Incorporation of these effects to micropolar finite elements would have a great impact on structural design of advanced marine structures.

Although the paper focused on maritime applications, the problems presented can be found also from other applications of thin-walled structures such as civil engineering, aeronautical engineering and mechanical engineering.

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