An Inexact Probabilistic–Possibilistic Optimization Framework for Flood Management in a Hybrid Uncertain Environment

Shuo Wang, Guohe Huang, and Brian W. Baetz

***Abstract*—Flooding is one of the leading causes of loss due to natural catastrophes, and at least one third of all losses due to natural forces can be attributed to flooding. Flood management systems involve a variety of complexities, such as multiple uncertainties, dynamic variations, and policy implications. This paper presents an inexact probabilistic–possibilistic programming with fuzzy random coefficients (IPP-FRC) model for flood management in a hybrid uncertain environment. IPP-FRC is capable not only of tackling multiple uncertainties in the form of intervals with fuzzy random boundaries but of addressing the dynamic complexity through capacity expansion planning within a multi-region, multi-flood-level, and multi-option context. The possibility and necessity measures used in IPP-FRC are suitable for risk-seeking and risk-averse decision making, respectively. A case study is used to demonstrate the applicability of the proposed methodology for facilitating flood management. The results indicate that the inexact degrees of possibility and necessity would decrease with increased probabilities of occurrence, implying a potential tradeoff between fulfillment of objectives and associated risks. A number of decision alternatives can be obtained under different policy scenarios. They are helpful for decision makers to formulate the appropriate flood management policy according to practical situations. The performance of IPP-FRC is analyzed and compared with a possibility-based fractile model.**

***Index Terms*—Floods, optimization, possibility theory, random variables, uncertainty.**

NOMENCLATURE

- f Total system cost ($).
- i Flood diversion region, $i = 1, 2, \ldots, u$.
- j Level of flood flow, $j = 1, 2, \ldots, v$.
- m Expansion option of diversion capacity, $m = 1, 2, \ldots, w$.
- W_i Amount of allowable flood diversion to region i (m^3).
- T_{ij} Amount of increased allowance to region i when its diversion capacity is expanded under flood flow q_j with probability p_j (m^3).
- S_{ij} Amount of excess flood to region i when allowable diversion level is exceeded under flood flow q_j with probability p_j (m^3).
- y_{ijm} Binary decision variable for region i with capacity-expansion option m under flow level j.
- C_i Regular cost per m^3 of allowable flood diversion to region i ($/$m^3$).
- D_i Penalty per m^3 of excess flood diversion to region i ($/$m^3$).
- E_{im} Capital cost of capacity expansion for region i with option m ($/$m^3$).
- p_j Probability of flood flow occurring with level j.
- q_j Amount of random flood flow to be diverted (m^3).
- R_i Existing diversion capacity of region i (m^3).
- $R_{i\max}$ Existing maximum diversion capacity of region i (m^3).
- O_{im} Expanded capacity of option m for region i (m^3).

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I. INTRODUCTION

FLOODING is the most frequent and expensive natural catastrophe in Canada. In June 2013, massive rainfall brought catastrophic flooding across Alberta in Western Canada, affecting tens of thousands of families throughout the region, resulting in the loss of four lives and displacing over 100 000 people from their homes [1]. It is estimated that the total damage will exceed $5 billion, which is the costliest disaster in Canadian history [2]. Floods are typically caused by heavy rainfall, rapid melting of a thick snow pack, and ice jams. Due to the increasing utilization of floodplains for mitigating flood disasters, ecosystems within watersheds are deteriorating, leading to increased soil loss, decreased flood retention capacity, and varied runoff pattern [3]. These effects may result in more frequent and severe flood disasters. Since losses cannot be avoided when a major flood occurs, a sound flood mitigation plan is of vital importance for helping reduce flood-related losses.

Optimization techniques play a crucial role in helping decision makers identify effective flood control strategies, and they have been used extensively for flood management problems over the past decades [4]–[9]. For instance, Unver and Mays [10] formulated a nonlinear programming model to address the real-time reservoir operation problem under flood conditions. Needham *et al.* [11] developed a mixed-integer linear programming model to assist with the U.S. Army Corps of Engineers' flood management studies on the Iowa and Des Moines rivers. Wei and Hsu [12] proposed mixed-integer linear programming models to solve the problem of real-time flood control for a multireservoir operation system. Ding and Wang [13] developed a nonlinear optimization approach to determine the optimal flood control operation and mitigate flood water stages in the channel network of a watershed. These methods were useful for identifying optimal flood mitigation schemes and reducing the risk of flood damage. In flood management systems, however, uncertainty is unavoidable for a variety of reasons, such as randomness from natural variability of the observed phenomenon, incomplete information, insufficient knowledge, and diversity in subjective judgments. Thus, decisions have to be made in the face of an uncertain future. As a result, conventional optimization methods would fail in describing the inherent uncertainty appropriately.

In the past decade, a number of optimization techniques were proposed for coping with uncertainties in different ways [14]–[27]. Most of these methods can be classified into stochastic, interval, and fuzzy mathematical programming. Among them, two-stage stochastic programming (TSP) is capable of taking corrective (or recourse) actions after a random event occurs. In a typical TSP, decisions are made at the first stage in an uncertain environment. Once uncertainties are realized, the optimal second-stage decisions or recourse decisions are taken [28]. Such a method is suitable for addressing the flood management problem. For example, decision makers need to determine an allowable flood diversion level according to the existing capacity of the floodplain before the flood season, and then, they may want to carry out corrective action when floods occur. TSP is effective in making decisions in a two-stage fashion. It was extensively studied in the past [29]–[33]. However, TSP requires exact specifications of probability distributions of the underlying uncertainties, resulting in difficulties when the sample size is too small to obtain distributional information in practical problems. Thus, inexact two-stage stochastic programming (ITSP) was proposed for dealing with interval uncertainties without the requirement of known distribution functions [34]. ITSP is able to simply describe uncertain information in the format of intervals with lower and upper bounds. However, ITSP can hardly deal with ambiguity and vagueness of human judgments in the decision-making process.

Subjective judgments from the knowledge of experts are often involved in optimization problems, and they are influential in reaching a decision. Fuzzy mathematical programming is recognized as a powerful tool for treating imprecision and vagueness of subjective estimates. Possibilistic programming is one of the fuzzy mathematical programming methods. It can be used to quantify imprecision by means of fuzzy sets that represent the continuous possibility distributions for ambiguous parameters [35]–[39]. In flood management problems, however, the related economic data such as regular costs of flood diversion and capital costs of floodplain expansions are often highly uncertain and can be estimated based on both subjective judgments and objective evaluations. Thus, fuzziness and randomness may need to be taken into account simultaneously for expressing these uncertain data that commonly act as the coefficients of the objective function in a cost minimization model. For example, a certain parameter can be described as fuzzy set; meanwhile, a number of fuzzy sets can be obtained from a group of decision makers or stakeholders with different subjective estimates for this particular parameter, leading to a fuzzy random phenomenon. From this point of view, it is important to realize the simultaneous consideration of fuzziness and randomness in the coefficients of the objective function [40]–[43]. When multiple uncertainties exist in the form of probability distributions, intervals, and possibility distributions, as well as their combinations, optimization models that address uncertainty in a single format would neglect a lot of valuable information during the decision-making process, resulting in unreliable solutions.

Therefore, the objective of this study is to develop an inexact probabilistic–possibilistic programming with fuzzy random coefficients (IPP-FRC) model for flood management in a hybrid uncertain environment. A case study on flood management will be used to demonstrate the applicability of IPP-FRC. The performance of IPP-FRC will then be analyzed and compared with a possibility-based fractile model.

II. Model Development

A. Inexact Two-Stage Mixed-Integer Programming

Consider a watershed system wherein floodwater can be diverted from a river channel to multiple water diversion regions (i.e., flood retention areas) in a flood season. The river has a limited water conveyance capacity and may overflow when a flood occurs. According to the local flood management policy, a flood warning level of the river should be determined and several projected flood diversion regions should be assigned. If the water level in the river exceeds the warning level, water will be diverted to adjacent floodplains.

A flood management system should contain both a specification of allowable levels of flood diversions and a decision scheme for efficiently utilizing flood diversion capacities [44]. Developing effective policies for diverting floods under limited diversion capacities is critical for minimizing flooding to densely populated communities and industries located in the lower reaches. Thus, an allowable flood diversion level can be predetermined according to the existing diversion capacity of each floodplain. If this allowance is not exceeded, a regular cost will be applied to flood diversion; otherwise, it will result in a surplus flood diversion along with economic penalties and/or expansion costs. Penalties can be expressed in terms of raised operating costs for flood diversion and/or destruction of land-based infrastructure. Expansions of floodplains will help increase allowable flood diversion capacities and thus reduce penalties. The total amount of flood diversion will be the sum of the allowable flood diversion level, the incremental quota, and the probabilistic excess flow. Consequently, it is desirable to identify sound decision schemes for flood diversion and capacity expansion with minimized total cost and maximized system safety.

In this problem, a first-stage decision on the amounts of allowable flood diversion must be made in the face of uncertain flood flows. When a flood occurs, a second-stage decision (i.e., recourse action) can be taken to compensate for adverse effects as a result of the first-stage decision. The problem under consideration can, thus, be formulated as a two-stage mixed-integer programming model with the following objective function:

$$\text{Min } f = \sum_{i=1}^{u} C_i W_i + \sum_{i=1}^{u} \sum_{j=1}^{v} \Bigg[p_j \left(C_i T_{ij} + D_i S_{ij} \right) + \sum_{m=1}^{w} E_{im} O_{im} y_{ijm} \Bigg]. \tag{1a}$$

The objective of this problem is to minimize the total system cost subject to several constraints related to flood diversion capacity, floodplain management, and capacity expansion. These constraints are shown as

$$W_i \le R_i \quad \forall i \tag{1b}$$

$$W_i + S_{ij} \le R_{i\max} \quad \forall i, j \tag{1c}$$

$$T_{ij} \le \sum_{m=1}^{w} O_{im} y_{ijm} \quad \forall i, j \tag{1d}$$

$$\sum_{i=1}^{u} (W_i + S_{ij} + T_{ij}) \le \sum_{i=1}^{u} R_{i\max} + \sum_{i=1}^{u}\sum_{m=1}^{w} O_{im} y_{ijm} \quad \forall j \tag{1e}$$

$$\sum_{i=1}^{u} (W_i + S_{ij} + T_{ij}) \ge q_j \quad \forall j \tag{1f}$$

$$W_i \ge 0 \quad \forall i \tag{1g}$$

$$S_{ij} \ge 0 \quad \forall i, j \tag{1h}$$

$$T_{ij} \ge 0 \quad \forall i, j \tag{1i}$$

$$y_{ijm} = \begin{cases} 1, & \text{if capacity expansion is undertaken} \\ 0, & \text{if otherwise} \end{cases} \quad \forall i, j, m \tag{1j}$$

$$\sum_{m=1}^{w} y_{ijm} \le 1 \quad \forall i, j. \tag{1k}$$

The meanings of the aforementioned constraints are explained as follows.

Constraint	*Meaning*
(1b)	Amount of allowable flood diversion should not be greater than the existing diversion capacity for each region.
(1c)	Total amount of allowable and excess flood diversion should not be greater than the existing maximum diversion capacity for each region.
(1d)	Increased allowance to each region should not be greater than its expanded capacity.
(1e)	Total flood flow diverted should not be greater than the total diversion capacity.
(1f)	Total flood flow should be fully diverted to different regions.
(1g)–(i)	All decision variables should be nonnegative.
(1j)	These are the binary decision variables for capacity expansion.
(1k)	Diversion capacity of each region can be expanded only once under each flow level.

All parameters and variables of model (1) are presented in the Nomenclature.

Model (1) is able to tackle the right-hand-side random parameters in the flood flow constraints and address the flood control problem in a two-stage fashion. However, uncertainties may exist in other parameters such as existing diversion capacities, maximum diversion capacities, and related economic data. In consideration of the limited sample size, all parameters can be easily expressed as intervals with lower and upper bounds. Thus, an inexact two-stage mixed-integer programming model can be formulated as

$$\text{Min } f^{\pm} = \sum_{i=1}^{u} C_i^{\pm} W_i^{\pm} + \sum_{i=1}^{u}\sum_{j=1}^{v}\left[p_j \left(C_i^{\pm} T_{ij}^{\pm} + D_i^{\pm} S_{ij}^{\pm}\right) + \sum_{m=1}^{w} E_{im}^{\pm} O_{im}^{\pm} y_{ijm}^{\pm}\right] \tag{2a}$$

subject to

$$W_i^{\pm} \le R_i^{\pm} \quad \forall i \tag{2b}$$

$$W_i^{\pm} + S_{ij}^{\pm} \le R_{i\max}^{\pm} \quad \forall i, j \tag{2c}$$

$$T_{ij}^{\pm} \le \sum_{m=1}^{w} O_{im}^{\pm} y_{ijm}^{\pm} \quad \forall i, j \tag{2d}$$

$$\sum_{i=1}^{u} (W_i^{\pm} + S_{ij}^{\pm} + T_{ij}^{\pm}) \le \sum_{i=1}^{u} R_{i\max}^{\pm} + \sum_{i=1}^{u}\sum_{m=1}^{w} O_{im}^{\pm} y_{ijm}^{\pm} \quad \forall j \tag{2e}$$

$$\sum_{i=1}^{u} (W_i^{\pm} + S_{ij}^{\pm} + T_{ij}^{\pm}) \ge q_j^{\pm} \quad \forall j \tag{2f}$$

$$W_i^{\pm} \ge 0 \quad \forall i \tag{2g}$$

$$S_{ij}^{\pm} \ge 0 \quad \forall i, j \tag{2h}$$

$$T_{ij}^{\pm} \ge 0 \quad \forall i, j \tag{2i}$$

$$y_{ijm}^{\pm} = \begin{cases} 1, & \text{if capacity expansion is undertaken} \\ 0, & \text{if otherwise} \end{cases} \quad \forall i, j, m \tag{2j}$$

$$\sum_{m=1}^{w} y_{ijm}^{\pm} \le 1 \quad \forall i, j \tag{2k}$$

where "±" denotes an interval that is defined as a number with known lower and upper bounds but unknown distributional information [45]. For example, letting W_i^- and W_i^+ be lower and upper bounds of $W_i^{\pm}$, respectively. We have $W_i^{\pm} = [W_i^-, W_i^+]$. When $W_i^- = W_i^+$, then $W_i^{\pm}$ becomes a deterministic number.

Acquisition of the economic data such as benefits and costs is crucial for optimization problems. Probabilistic and ambiguous information may exist simultaneously when collecting the economic data. Fuzzy random variables can thus be introduced to address randomness and fuzziness.

B. Probabilistic–Possibilistic Programming With Fuzzy Random Coefficients

First, let us consider the following linear programming problem involving fuzzy random coefficients in the objective function:

$$\text{Min } f = \sum_{j=1}^{n} \tilde{c}_j(\omega) x_j \tag{3a}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i = 1, 2, \ldots, m \tag{3b}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{3c}$$

where f denotes the objective function, $\tilde{c}_j(\omega)$ denotes the coefficient of the objective function that is expressed as an n-dimensional fuzzy random vector, x_j is an n-dimensional decision vector, a_{ij} is an $m \times n$ matrix, and b_i is an m-dimensional parameter vector. In this study, each coefficient $\tilde{c}_j(\omega)$ was given as a triangular fuzzy number characterized by the following membership function [46]:

$$\mu_{\tilde{C}_j(\omega)}(\gamma) = \begin{cases} \max\left(1 - \dfrac{d_j(\omega) - \gamma}{\alpha_j}, 0\right), & \text{if } \gamma \leq d_j(\omega) \\ \max\left(1 - \dfrac{\gamma - d_j(\omega)}{\beta_j}, 0\right), & \text{otherwise.} \end{cases} \tag{4}$$

where $\alpha_j > 0$ and $\beta_j > 0$ denote the left and right spreads of a fuzzy number, respectively. When $\alpha_j = \beta_j$, the fuzzy number is a symmetric triangular fuzzy number. $d_j(\omega)$ represents the center of a fuzzy number and is assumed to be a Gaussian random vector with expected value μ_j and standard deviation σ_j. Since each coefficient $\tilde{c}_j(\omega)$ is a fuzzy number, the objective function can also be expressed as a fuzzy number characterized by the following membership function:

$$\mu_{\sum_{j=1}^{n} \tilde{C}_j(\omega) x_j}(\varphi) = \begin{cases} \max\left(1 - \dfrac{\sum_{j=1}^{n} d_j(\omega) x_j - \varphi}{\sum_{j=1}^{n} \alpha_j x_j}, 0\right), & \text{if } \varphi \leq \displaystyle\sum_{j=1}^{n} d_j(\omega) x_j \\ \max\left(1 - \dfrac{\varphi - \sum_{j=1}^{n} d_j(\omega) x_j}{\sum_{j=1}^{n} \beta_j x_j}, 0\right), & \text{otherwise.} \end{cases} \tag{5}$$

In the case that decision makers consider minimizing the total system cost, they generally set a goal for the expected objective function value. Obviously, decision makers will be totally satisfied if the expected total cost is less than the goal. In real-world decision-making problems, the goal is often vague and flexible and can be quantified by the following fuzzy membership function:

$$\mu_{\tilde{G}}(z) = \begin{cases} 1, & \text{if } z < z_1 \\ \dfrac{z_0 - z}{z_0 - z_1}, & \text{if } z_1 \leq z \leq z_0 \\ 0, & \text{if } z > z_0 \end{cases} \tag{6}$$

where z_1 and z_0 represent the minimum (most satisfactory) and maximum (least satisfactory) total costs that decision makers wish to achieve, respectively. Their values can be determined based on decision makers' subjective estimates of the goal.

To address the fuzziness of the objective function, the concept of possibility and necessity measures, which plays a key role in possibility theory [47], [48], is adopted to reveal the degree that the objective function fulfills the fuzzy goal. Decision making using the possibility measure is suitable for optimistic decision makers; instead, the necessity measure is appropriate when decision makers are risk-averse. The degrees of possibility and necessity can be, respectively, defined as [41]

$$\text{Pos}_{\sum_{j=1}^{n} \tilde{C}_j(\omega) x_j}(\tilde{G}) = \sup_{\ell} \min\left\{\mu_{\sum_{j=1}^{n} \tilde{C}_j(\omega) x_j}(\ell), \mu_{\tilde{G}}(\ell)\right\} \tag{7}$$

$$\text{Nec}_{\sum_{j=1}^{n} \tilde{C}_j(\omega) x_j}(\tilde{G}) = \inf_{\ell} \max\left\{1 - \mu_{\sum_{j=1}^{n} \tilde{C}_j(\omega) x_j}(\ell), \mu_{\tilde{G}}(\ell)\right\} \tag{8}$$

where $\text{Pos}_{\sum_{j=1}^{n} \tilde{C}_j(\omega) x_j}(\tilde{G})$ and $\text{Nec}_{\sum_{j=1}^{n} \tilde{C}_j(\omega) x_j}(\tilde{G})$, respectively, represent the degrees of possibility and necessity that the fuzzy goal $\tilde{G}$ is fulfilled under the possibility distribution $\mu_{\sum_{j=1}^{n} \tilde{C}_j(\omega) x_j}(\cdot)$ of the objective function.

To address both fuzziness and randomness in the coefficients of the objective function, the concept of possibility and necessity measures can be incorporated into the fractile criterion optimization model [43], leading to a probabilistic–possibilistic programming with fuzzy random coefficients problem. Under the possibility case, model (3) can thus be reformulated as

$$\text{Max } h \tag{9a}$$

subject to

$$\Pr\{\omega | \text{Pos}_{\sum_{j=1}^{n} \tilde{c}_j(\omega) x_j}(\tilde{G}) \geq h\} \geq \theta \tag{9b}$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i = 1, 2, \ldots, m \tag{9c}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{9d}$$

where h denotes a permissible level that the degree of possibility is greater than or equal to, and θ represents a probability level predetermined by decision makers for the degree of possibility being greater than or equal to h. In model (9), since the center of the fuzzy number $\tilde{c}_j(\omega)$, which is denoted as $d_j(\omega)$, is assumed to be a Gaussian random vector with expected value μ_j and standard deviation σ_j, the stochastic chance constraints can be transformed as

$$\Pr\left\{\frac{\sum_{j=1}^{n} d_j(\omega) x_j - \sum_{j=1}^{n} \mu_j x_j}{\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2}} \leq \frac{\sum_{j=1}^{n} (1-h)\alpha_j x_j + (z_1 - z_0)h + z_0 - \sum_{j=1}^{n} \mu_j x_j}{\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2}}\right\} \geq \theta. \tag{10}$$

Since $(\sum_{j=1}^{n} d_j(\omega) x_j - \sum_{j=1}^{n} \mu_j x_j)/\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2}$ is a standard normal random variable with mean 0 and variance 1, the above inequality can be equivalently transformed as

$$\Phi\left\{\frac{\sum_{j=1}^{n} (1-h)\alpha_j x_j + (z_1 - z_0)h + z_0 - \sum_{j=1}^{n} \mu_j x_j}{\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2}}\right\} \geq \theta \tag{11}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal random variable. Thus, we have

$$\theta = \Phi(\Phi^{-1}(\theta)) \tag{12}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of $\Phi(\cdot)$. As $\Phi(\cdot)$ is a monotonically increasing function, we have

$$\frac{\sum_{j=1}^{n} (\alpha_j - \mu_j)x_j - \Phi^{-1}(\theta)\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2} + z_0}{\sum_{j=1}^{n} \alpha_j x_j - z_1 + z_0} \geq h. \tag{13}$$

Thus, model (9) can be transformed into the following problem:

$$\text{Max } h \tag{14a}$$

subject to

$$\frac{\sum_{j=1}^{n} (\alpha_j - \mu_j)x_j - \Phi^{-1}(\theta)\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2} + z_0}{\sum_{j=1}^{n} \alpha_j x_j - z_1 + z_0} \geq h \tag{14b}$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = 1, 2, \ldots, m \tag{14c}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n. \tag{14d}$$

Since $h = (\sum_{j=1}^{n} (\alpha_j - \mu_j)x_j - \Phi^{-1}(\theta)\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2} + z_0)/(\sum_{j=1}^{n} \alpha_j x_j - z_1 + z_0)$ holds if h is maximum, model (14) can be equivalently rewritten as

$$\text{Max } f = \frac{\sum_{j=1}^{n} (\alpha_j - \mu_j)x_j - \Phi^{-1}(\theta)\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2} + z_0}{\sum_{j=1}^{n} \alpha_j x_j - z_1 + z_0} \tag{15a}$$

subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = 1, 2, \ldots, m \tag{15b}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n. \tag{15c}$$

When the necessity measure is used, model (3) can be reformulated as

$$\text{Max } h \tag{16a}$$

subject to

$$\Pr\{\omega | \text{Nec}_{\sum_{j=1}^{n} \tilde{c}_j(\omega)x_j}(\tilde{G}) \geq h\} \geq \theta \tag{16b}$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = 1, 2, \ldots, m \tag{16c}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n. \tag{16d}$$

Similar to the possibility case, model (16) can be equivalently transformed as

$$\text{Max } f = \frac{-\sum_{j=1}^{n} \mu_j x_j - \Phi^{-1}(\theta)\sqrt{\sum_{j=1}^{n} (\sigma_j x_j)^2} + z_0}{\sum_{j=1}^{n} \beta_j x_j - z_1 + z_0} \tag{17a}$$

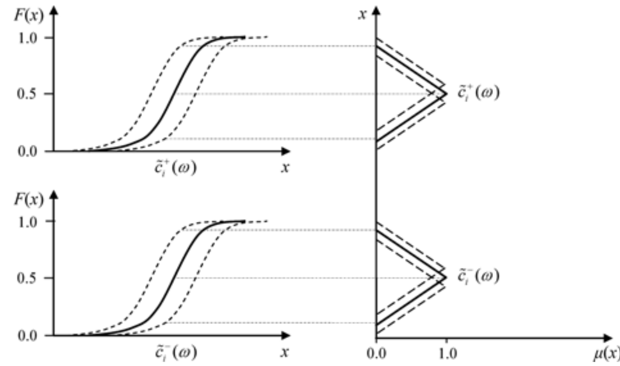

Fig. 1. Interval with fuzzy random boundaries.

subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = 1, 2, \ldots, m \tag{17b}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n. \tag{17c}$$

The probabilistic–possibilistic programming model is capable of tackling fuzzy random coefficients in the objective function. A number of decision alternatives can be obtained with degrees of possibility and necessity under different probabilities of occurrence. Nevertheless, the probabilistic–possibilistic programming model is unable to address uncertainties in modeling constraints or the resulting solutions, posing a major obstacle to practical applications.

C. Inexact Probabilistic–Possibilistic Programming With Fuzzy Random Coefficients

In real-world problems, a variety of uncertainties such as intervals and fuzzy random variables often exist within the objective function, constraints, and the resulting solutions. Optimization techniques that address uncertainty in a single format are not satisfactory enough to adequately reflect all uncertain information in the decision-making process. Such a simplification may result in unreliable and misleading solutions. Moreover, certain parameters may be highly uncertain, such as the related economic data in flood management problems. They can be expressed as intervals with imprecise boundaries. In other words, it may be difficult to acquire deterministic values for the lower and upper bounds of an interval; instead, the two bounds of an interval can be given with fuzziness and randomness due to the estimates from a number of decision makers or stakeholders. Thus, the concept of intervals with fuzzy random boundaries is proposed to address such multiple uncertainties (as shown in Fig. 1). By incorporating inexact two-stage mixed-integer programming, probabilistic–possibilistic programming with fuzzy random coefficients, and the concept of intervals with fuzzy random boundaries within a general framework, an IPP-FRC model can be formulated as

$$\text{Min } f^{\pm} = \sum_{i=1}^{u} \tilde{C}_i^{\pm}(\omega)W_i^{\pm} + \sum_{i=1}^{u}\sum_{j=1}^{v} \Big[p_j(\tilde{C}_i^{\pm}(\omega)T_{ij}^{\pm} + \tilde{D}_i^{\pm}(\omega)S_{ij}^{\pm}) + \sum_{m=1}^{w} \tilde{E}_{im}^{\pm}(\omega)O_{im}^{\pm}y_{ijm}^{\pm}\Big] \tag{18}$$

subject to a number of constraints that are as the same as those in model (2). $\tilde{C}_i^{\pm}(\omega)$, $\tilde{D}_i^{\pm}(\omega)$, and $\tilde{E}_{im}^{\pm}(\omega)$ are intervals with fuzzy random boundaries.

D. Solution Algorithm

To solve the IPP-FRC model, a robust two-step method can be used to convert the inexact programming problem into two submodels that correspond to upper and lower bounds of the objective function value [49]. Since the first-stage decision variables ($W_i^{\pm}$) in model (18) are considered as intervals, it is difficult to determine whether its lower bound (W_i^-) or upper bound (W_i^+) corresponds to the lower bound of the total system cost. Therefore, an optimized set of $W_i^{\pm}$ will be identified by having z_i being decision variables, which is helpful for achieving a minimized total cost. Accordingly, let $W_i^{\pm} = W_i^- + \Delta W_i z_i$, where $\Delta W_i = W_i^+ - W_i^-$, and $z_i (0 \le z_i \le 1)$ are decision variables used for identifying the optimized set of $W_i^{\pm}$. Since the objective is to minimize the total system cost, the submodel corresponding to the upper bound of the objective function value (f^+) should be first formulated as

$$\text{Min } f^+ = \sum_{i=1}^{u} \tilde{C}_i^+(\omega)(W_i^- + \Delta W_i z_i) + \sum_{i=1}^{u}\sum_{j=1}^{v}\left[p_j(\tilde{C}_i^+(\omega)T_{ij}^+ + \tilde{D}_i^+(\omega)S_{ij}^+) + \sum_{m=1}^{w} \tilde{E}_{im}^+(\omega)O_{im}^+ y_{ijm}^+\right] \quad (19a)$$

subject to

$$0 \le W_i^- + \Delta W_i z_i \le R_i^- \quad \forall i \quad (19b)$$

$$W_i^- + \Delta W_i z_i + S_{ij}^+ \le R_{i\max}^- \quad \forall i,j \quad (19c)$$

$$T_{ij}^+ \le \sum_{m=1}^{w} O_{im}^+ y_{ijm}^+ \quad \forall i,j \quad (19d)$$

$$\sum_{i=1}^{u} (W_i^- + \Delta W_i z_i + S_{ij}^+ + T_{ij}^+) \le \sum_{i=1}^{u} R_{i\max}^- + \sum_{i=1}^{u}\sum_{m=1}^{w} O_{im}^+ y_{ijm}^+ \quad \forall j \quad (19e)$$

$$\sum_{i=1}^{u} (W_i^- + \Delta W_i z_i + S_{ij}^+ + T_{ij}^+) \ge q_j^+ \quad \forall j \quad (19f)$$

$$S_{ij}^+ \ge 0 \quad \forall i,j \quad (19g)$$

$$T_{ij}^+ \ge 0 \quad \forall i,j \quad (19h)$$

$$0 \le z_i \le 1 \quad \forall i \quad (19i)$$

$$y_{ijm}^+ = \begin{cases} 1, & \text{if capacity expansion is undertaken} \\ 0, & \text{if otherwise} \end{cases} \quad \forall i,j,m \quad (19j)$$

$$\sum_{m=1}^{w} y_{ijm}^+ \le 1 \quad \forall i,j \quad (19k)$$

where T_{ij}^+, S_{ij}^+, and z_i are continuous decision variables, and y_{ijm}^+ are binary decision variables. Their solutions of $T_{ij\text{opt}}^+$, $S_{ij\text{opt}}^+$, $z_{i\text{opt}}$, and $y_{ijm\text{opt}}^+$ can be obtained through solving submodel (19). Then, the optimized set of the first-stage decision variables ($W_i^{\pm}$) can be determined by calculating $W_{i\text{opt}}^{\pm} = W_i^- + \Delta W_i z_{i\text{opt}}$. Based on the solutions of submodel (19), the submodel corresponding to the lower bound of the objective function value (f^-) can be formulated as

$$\text{Min } f^- = \sum_{i=1}^{u} \tilde{C}_i^-(\omega)(W_i^- + \Delta W_i z_{i\text{opt}}) + \sum_{i=1}^{u}\sum_{j=1}^{v}\left[p_j(\tilde{C}_i^-(\omega)T_{ij}^- + \tilde{D}_i^-(\omega)S_{ij}^-) + \sum_{m=1}^{w} \tilde{E}_{im}^-(\omega)O_{im}^- y_{ijm}^-\right] \quad (20a)$$

subject to

$$W_i^- + \Delta W_i z_{i\text{opt}} \le R_i^+ \quad \forall i \quad (20b)$$

$$W_i^- + \Delta W_i z_{i\text{opt}} + S_{ij}^- \le R_{i\max}^+ \quad \forall i,j \quad (20c)$$

$$T_{ij}^- \le \sum_{m=1}^{w} O_{im}^- y_{ijm}^- \quad \forall i,j \quad (20d)$$

$$\sum_{i=1}^{u} (W_i^- + \Delta W_i z_{i\text{opt}} + S_{ij}^- + T_{ij}^-) \le \sum_{i=1}^{u} R_{i\max}^+ + \sum_{i=1}^{u}\sum_{m=1}^{w} O_{im}^- y_{ijm}^- \quad \forall j \quad (20e)$$

$$\sum_{i=1}^{u} (W_i^- + \Delta W_i z_{i\text{opt}} + S_{ij}^- + T_{ij}^-) \ge q_j^- \quad \forall j \quad (20f)$$

$$S_{ij\text{opt}}^+ \ge S_{ij}^- \ge 0 \quad \forall i,j \quad (20g)$$

$$T_{ij\text{opt}}^+ \ge T_{ij}^- \ge 0 \quad \forall i,j \quad (20h)$$

$$y_{ijm}^- = \begin{cases} 1, & \text{if capacity expansion is undertaken} \\ 0, & \text{if otherwise} \end{cases} \quad \forall i,j,m \quad (20i)$$

$$\sum_{m=1}^{w} y_{ijm}^- \le 1 \quad \forall i,j \quad (20j)$$

$$y_{ijm\text{opt}}^+ \ge y_{ijm}^- \quad \forall i,j,m \quad (20k)$$

where T_{ij}^-, S_{ij}^-, and y_{ijm}^- are decision variables. Their solutions can be obtained through solving submodel (20). By integrating the solutions of submodels (19) and (20), interval solutions can be obtained as follows:

$$T_{ij\text{opt}}^{\pm} = [T_{ij\text{opt}}^-, T_{ij\text{opt}}^+] \quad \forall i,j \quad (21a)$$

$$S_{ij\text{opt}}^{\pm} = [S_{ij\text{opt}}^-, S_{ij\text{opt}}^+] \quad \forall i,j \quad (21b)$$

$$y_{ijm\text{opt}}^{\pm} = [y_{ijm\text{opt}}^-, y_{ijm\text{opt}}^+] \quad \forall i,j,m \quad (21c)$$

$$f_{\text{opt}}^{\pm} = [f_{\text{opt}}^-, f_{\text{opt}}^+]. \quad (21d)$$

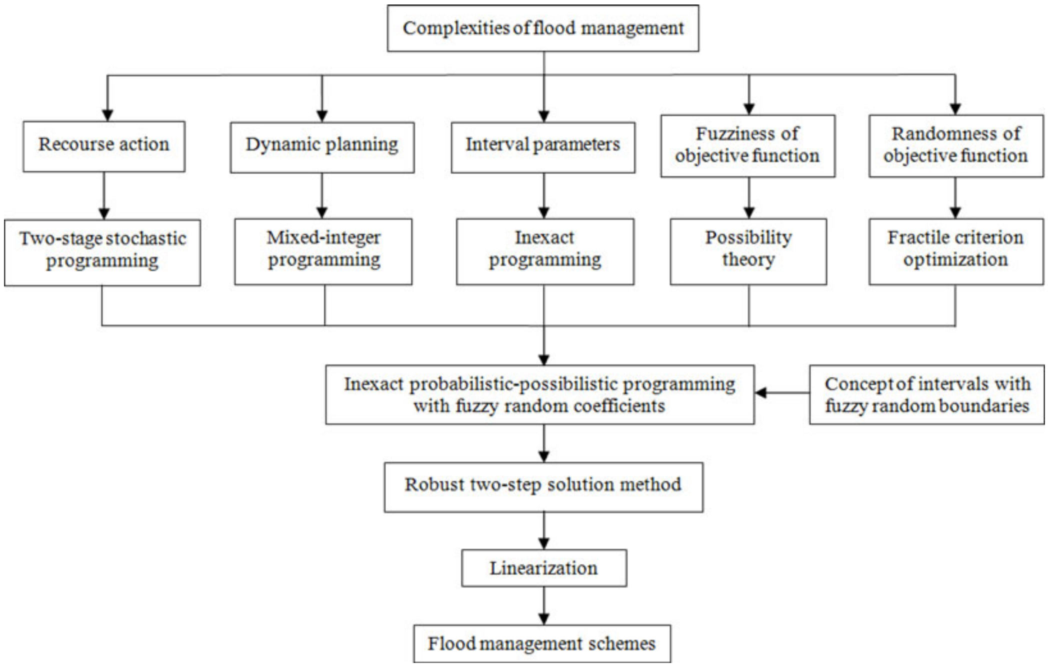

Fig. 2. Framework of the IPP-FRC approach.

The optimized flood diversion schemes would then be

$$A^{\pm}_{ij\text{opt}} = W^{\pm}_{i\text{opt}} + T^{\pm}_{ij\text{opt}} + S^{\pm}_{ij\text{opt}} \quad \forall i, j \tag{22}$$

where $A^{\pm}_{ij\text{opt}}$ denotes the total diverted flow, which is a sum of the allowable flow, the incremental quota, and the probabilistic excess flow.

The framework of the IPP-FRC model is presented in Fig. 2. The detailed solution procedure of IPP-FRC can be summarized as follows.

Step 1: Formulate the IPP-FRC model.

Step 2: Acquire model parameters in the forms of intervals, probability distributions, possibility distributions, and intervals with fuzzy random boundaries.

Step 3: Reformulate the IPP-FRC model by introducing $W^{\pm}_i = W^-_i + \Delta W_i z_i$, where $\Delta W_i = W^+_i - W^-_i$ and $z_i \in [0, 1]$.

Step 4: Transform the IPP-FRC model into two submodels by using the robust two-step algorithm.

Step 5: Solve each submodel through transforming the corresponding objective function with fuzzy random coefficients into the deterministic equivalent function.

Step 6: Combine solutions from two submodels and final solutions can be obtained as $f^{\pm}_{\text{opt}} = [f^-_{\text{opt}}, f^+_{\text{opt}}]$, $T^{\pm}_{ij\text{opt}} = [T^-_{ij\text{opt}}, T^+_{ij\text{opt}}]$, and $S^{\pm}_{ij\text{opt}} = [S^-_{ij\text{opt}}, S^+_{ij\text{opt}}]$.

III. Application to Flood Management

A. Statement of Problems

A flood management system is composed of multiple interconnected components related to various socioeconomic and environmental concerns. For instance, a flood management system may involve several flood diversion regions for flood mitigation. These regions are interrelated to each other. Any changes (e.g., floodplain expansions) in one region would bring a series of consequences to the others, resulting in variations in economic costs. Such interrelationships lead to a variety of complexities, such as uncertainties in economic and technical data, dynamic variations in system components, randomness in flood flows, and policy implications in flood management. These complexities can become further intensified not only by potential interactions among uncertainties but through their economic effects as well.

In flood management systems, inherent uncertainties exist due to the unavailability of system information, modeling inaccuracies, randomness of natural processes, and diversity in subjective judgments. Thus, rough estimations of the related parameters have to be made in the decision-making process. A large portion of available information can be qualitative, such as implicit knowledge from decision makers or stakeholders. Such complexities often exist in factors related to socioeconomic conditions, flood characteristics, and geographical conditions. They have to be addressed through stochastic, fuzzy, and interval-analysis methods. A typical example is capital costs of expansion in floodplains. This uncertainty can be expressed as an interval, indicating that its true value is between the lower and upper bounds of a given range. However, in some cases, even the lower and upper bounds can hardly be known with certainty; instead, they may be given as subjective judgments from a large number of decision makers or stakeholders, which can be expressed as fuzzy random variables. This leads to multiple uncertainties (i.e., intervals with fuzzy random boundaries), as shown in Fig. 1.

Generally, flood management systems are subject to effects of extensive complexities, particularly in terms of the existence of multiple uncertainties and their interdependences, as well as multiregion and dynamic features. It is thus essential to develop advanced methodologies for dealing with such complexities.

TABLE I
EXISTING AND MAXIMUM DIVERSION CAPACITIES, CAPACITY EXPANSION OPTIONS, AND THE RELATED ECONOMIC DATA

Activity	Flood diversion region		
	$i = 1$	$i = 2$	$i = 3$
Existing capacity, $R_i^{\pm}$ (10^6 m^3)	[3.0, 4.0]	[4.0, 5.0]	[3.5, 4.5]
Maximum diversion capacity, $R_{i\max}^{\pm}$ (10^6 m^3)	[4.0, 5.0]	[7.0, 8.0]	[6.0, 7.0]
Regular cost for allowable diversion, $\tilde{C}_i^{\pm}(\omega)$ ($/$m^3$)	[($\overline{80}$, 6, 6), ($\overline{100}$, 6, 6)][a]	[($\overline{90}$, 6, 6), ($\overline{110}$, 6, 6)]	[($\overline{100}$, 6, 6), ($\overline{130}$, 6, 6)]
Penalty cost for excess diversion, $\tilde{D}_i^{\pm}(\omega)$ ($/$m^3$)	[($\overline{200}$, 9, 9), ($\overline{250}$, 9, 9)]	[($\overline{150}$, 9, 9), ($\overline{180}$, 9, 9)]	[($\overline{180}$, 9, 9), ($\overline{210}$, 9, 9)]
Capacity expansion option (10^6 m^3):			
$O_{i1}^{\pm}$ (option 1)	[3, 4]	[5, 7]	0
$O_{i2}^{\pm}$ (option 2)	[4, 5]	[6, 8]	0
$O_{i3}^{\pm}$ (option 3)	[5, 6]	[7, 9]	0
Capital cost of expansion ($/$m^3$):			
$\tilde{E}_{i1}^{\pm}(\omega)$ (option 1)	[($\overline{50}$, 6, 6), ($\overline{70}$, 6, 6)]	[($\overline{80}$, 6, 6), ($\overline{100}$, 6, 6)]	0
$\tilde{E}_{i2}^{\pm}(\omega)$ (option 2)	[($\overline{60}$, 6, 6), ($\overline{80}$, 6, 6)]	[($\overline{90}$, 6, 6), ($\overline{110}$, 6, 6)]	0
$\tilde{E}_{i3}^{\pm}(\omega)$ (option 3)	[($\overline{70}$, 6, 6), ($\overline{90}$, 6, 6)]	[($\overline{100}$, 6, 6), ($\overline{120}$, 6, 6)]	0

[a] random variable $\bar{\delta} \sim N(\delta, \sigma^2)$, where $\sigma^2 = 0.1$, indicating that a random variable follows a normal distribution with an expected value of δ and a standard deviation of σ. For example, $\bar{\delta} = \overline{80} \sim N(80, 0.1)$, where $\delta = 80$ and $\sigma^2 = 0.1$.

TABLE II
STREAM FLOWS WITH GIVEN PROBABILITIES

Flow level (j)	Flow (q_j) (10^6 m^3)	Probability (p_j) (%)
Low ($j = 1$)	[5.5, 7.5]	10
Low-medium ($j = 2$)	[8.5, 11.5]	20
Medium ($j = 3$)	[12.5, 15.5]	40
Medium-high ($j = 4$)	[17.5, 21.0]	20
High ($j = 5$)	[23.0, 27.0]	10

B. Overview of the Study System

The following flood management problem will be used to demonstrate the applicability of the developed IPP-FRC model. In a watershed system, floodwater needs to be diverted from a river channel to three flood diversion regions during a flood season. Table I shows the existing and maximum diversion capacities for three floodplains, the capacity expansion options, as well as the related economic data. According to the flood management policy, regions 1 and 2 can be expanded once by any of the given expansion options, but no expansion is undertaken for region 3 due to intensive human activities within this region. Table II presents different flow levels with given probabilities of occurrence.

The problems under consideration include 1) how to divert floodwater to three flood diversion regions in an effective way when a flood occurs; 2) how to identify optimal capacity expansion schemes; 3) how to achieve the minimized total system cost; and 4) how to formulate the appropriate flood management policy. A variety of uncertainties exist in these problems, increasing the complexity in the decision-making process. IPP-FRC is thus considered to be a promising approach for dealing with this flood management problem.

C. Result Analysis

To address randomness and fuzziness in the coefficients of the objective function, the cost minimization problem in this study was transformed into a maximization problem with the concept of possibility and necessity measures. Due to the vagueness in human judgments, it was assumed that decision makers had a fuzzy goal for each objective function, and they preferred to maximize the degrees of possibility and necessity that each objective function value fulfilled toward the corresponding fuzzy goal. Thus, the goals for lower and upper bounds of the objective function were, respectively, quantified by means of a fuzzy set whose membership function was shown as

$$\mu_{\tilde{G}^-}(z) = \begin{cases} 1, & \text{if } z < 2000 \\ \dfrac{2100 - z}{100}, & \text{if } 2000 \le z \le 2100 \\ 0, & \text{if } z > 2100 \end{cases} \quad (23a)$$

$$\mu_{\tilde{G}^+}(z) = \begin{cases} 1, & \text{if } z < 3400 \\ \dfrac{3500 - z}{100}, & \text{if } 3400 \le z \le 3500 \\ 0, & \text{if } z > 3500. \end{cases} \quad (23b)$$

Due to the randomness involved in the objective function, the concept of possibility and necessity measures was incorporated into the fractile criterion optimization model. It was assumed that the probability for the resulting degrees of possibility and necessity was greater than or equal to a predefined level. Fig. 3 presents a comparison of degrees of possibility and necessity with different probabilities of occurrence. It is revealed that the degrees of possibility and necessity would be obtained with lower and upper bounds. This is because intervals with fuzzy random boundaries exist in the coefficients of the objective function. These coefficients represent the regular costs for allowable diversion, penalty costs for excess diversion, and capital costs of expansion. The results indicate that the degrees of possibility and necessity would be decreasing with increasing probability levels, implying a potential tradeoff between fulfillment of objectives and associated risks. The degree of necessity would be much lower than the degree of possibility. This is because decision making using possibility would be suitable for optimistic decision makers; contrarily, decision making using necessity would be appropriate for risk-averse decision makers. Thus, decision makers are able to choose either possibility or necessity

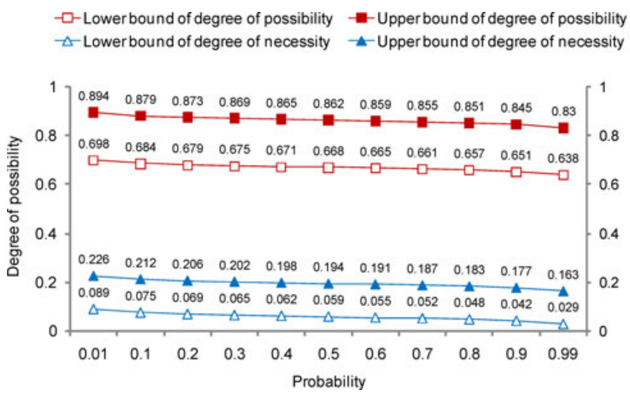

Fig. 3. Comparison of degrees of possibility and necessity with different probabilities.

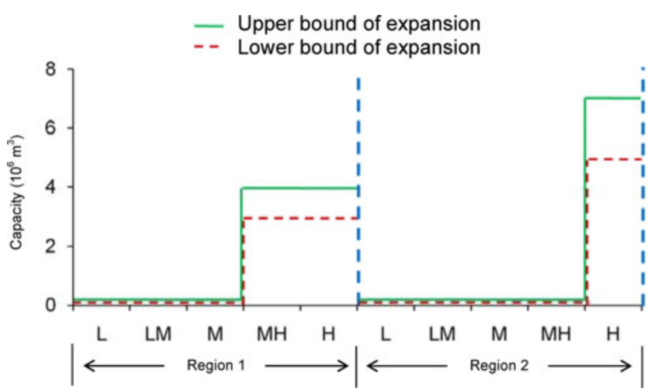

Fig. 4. Solutions of capacity expansion for two regions under different flow levels.

measures based on their risk preferences in the decision-making process.

Fig. 4 shows the solutions of capacity expansion for two regions under different flow levels. The results were obtained in the form of intervals using the expected values of fuzzy random variables. It is indicated that region 1 would be expanded with an increment of [3.0, 4.0] $\times$ 10^6 m^3 under medium-high and high flow levels, while region 2 would be expanded with an incremental capacity of [5.0, 7.0] $\times$ 10^6 m^3 under the high flow level. Expansions of regions 1 and 2 would lead to increased allowable flows. In comparison, the allowable flow to region 3 would not be increased because no expansion plan is considered.

Fig. 5 presents the flood diversion patterns for three regions under different flow levels. Generally, the allowable flow is determined according to the existing flood diversion capacity, the increased allowance is related to the expanded capacity, and the excess flow is confined by the maximum capacity. When a flooding event occurs, the allowable flows would be first diverted to the assigned regions. If the remaining floodwater in the river still exceeds the flood warning level, the flow would continue to spill over the river banks to adjacent regions, resulting in excess flow and/or increased allowance. Thus, the total diverted flow is a sum of the allowable flow, the incremental quota, and the probabilistic excess flow.

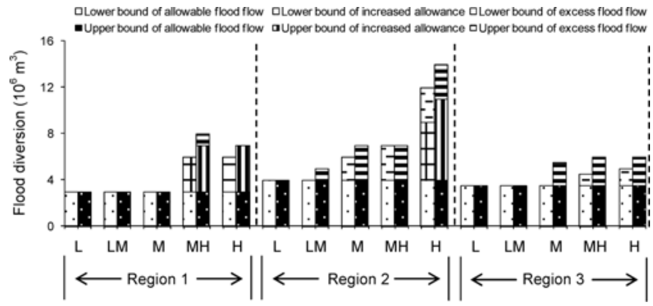

Fig. 5. Flood diversion patterns for three regions under different flow levels.

As shown in Fig. 5, the allowable flow to region 1 would be 3.0 $\times$ 10^6 m^3. Region 1 would be expanded with an incremental capacity of [3.0, 4.0] $\times$ 10^6 m^3 under medium-high and high flow levels. There would be an excess flow of [0, 1.0] $\times$ 10^6 m^3 when the flow level is medium-high with a probability of 20%. For region 2, the allowable flow would be 4.0 $\times$ 10^6 m^3. Region 2 would not be expanded unless the flow level is high with a probability of 10%. There would thus be much more surplus flows diverted to region 2 compared with those to region 1. This is because the capital cost of expansion for region 2 is higher than that for region 1, and region 2 has the lower penalty cost for excess diversion. For region 3, the allowable flow would be 3.5 $\times$ 10^6 m^3. There would be zero increase in the allowable flow to region 3 since no expansion plan is considered for this region. The excess flows diverted to region 3 would be [0, 2.0] $\times$ 10^6 m^3 under the medium flow level, [1.0, 2.5] $\times$ 10^6 m^3 under the medium-high level, and [1.5, 2.5] $\times$ 10^6 m^3 under the high flow level. In the case of flooding events, the excess flow should first be diverted to region 2, second to region 3, and, finally, to region 1. This is because region 2 is subject to the largest expansion capacity and the lowest penalty for excess diversion. In comparison, region 1 is confined by the highest penalty for excess diversion, resulting in the smallest proportion of excess flows being diverted to region 1.

The expected total system cost would be \$[2085.0, 3441.5] $\times$ 10^6 due to the existence of intervals with fuzzy random boundaries. It is composed of the regular cost for allowable diversion, the penalty for excess diversion, and the capital cost for capacity expansion. The lower and upper bounds of the total cost represent advantageous and disadvantageous conditions, respectively.

Variations in allowable levels of flood diversion reflect different flood management policies, which play an important role in flood mitigation planning. The proposed IPP-FRC model is capable of establishing a relationship among flood management policies, diversion schemes, and economic implications. Solutions of the IPP-FRC model under different scenarios of allowable levels of flood diversion can be obtained by letting $W_i^{\pm}$ (expressed as intervals) have different deterministic values. If all $W_i^{\pm}$ reach their lower bounds ($W_i^{\pm} = W_i^{-}$, $i = 1, 2, 3$), such an optimistic policy would cause high penalties when the amounts of allowable flood diversion are exceeded under high flow conditions. Conversely, if all $W_i^{\pm}$ reach their upper bounds ($W_i^{\pm} = W_i^{+}$, $i = 1, 2, 3$), such a conservative policy would

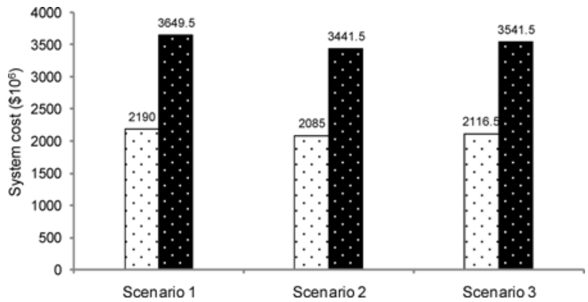

Fig. 6. Comparison of total system costs under different scenarios of allowable levels of flood diversion.

result in a waste of resources under low flow conditions but, at the same time, a low risk of system failure when the allowances are violated due to a large amount of flood water. If all $W_i^{\pm}$ reach their mid-values $[W_i^{\pm} = (W_i^- + W_i^+)/2,\ i = 1, 2, 3]$, a risk-neutral flood management policy would be made.

Fig. 6 presents a comparison of total system costs under different policy scenarios. From the economic point of view, the conservative policy under scenario 2 would bring the lowest total system cost of \$[2085.0, 3441.5] $\times$ 10^6. Contrarily, the optimistic policy under scenario 1 would lead to the highest total system cost of \$[2190.0, 3649.5] $\times$ 10^6. The results indicate that flood management policies are directly related to economic efficiency. To avoid the problem of over- or underestimation for the amount of allowable flood diversion at the first stage, each $W_i^{\pm}$ was considered as a decision variable rather than a deterministic value in this study. The optimized amount of allowable flood diversion ($W_{i\text{opt}}$) for regions 1, 2, and 3 would thus be 3.0, 4.0, and 3.5 $\times$ 10^6 m^3, respectively. The results imply that the conservative policy when all $W_i^{\pm}$ reach their upper bounds would be optimal to achieve the lowest system cost in this case. Consequently, policy analysis is crucial for flood diversion planning in an uncertain and complex environment.

IV. Discussion

The flood management problem was also solved through a possibility-based fractile model [43]. The fuzzy goal for the objective function was quantified by a fuzzy set with the following membership function:

$$\mu_{\tilde{G}^-}(z) = \begin{cases} 1, & \text{if } z < 2700 \\ \dfrac{2800 - z}{100}, & \text{if } 2700 \le z \le 2800 \\ 0, & \text{if } z > 2800. \end{cases} \tag{24}$$

Fig. 7 presents the degrees of possibility and necessity obtained from the possibility-based fractile model. It is revealed that the deterministic degrees of possibility and necessity would be obtained under different probabilities without reflecting the fluctuations in model parameters and the resulting solutions. This is because the possibility-based fractile model is only able to address uncertainty in the coefficients of the objective function based on an assumption that all the other parameters are deterministic. In comparison, the proposed IPP-FRC model is capable of generating inexact degrees of possibility and ne-

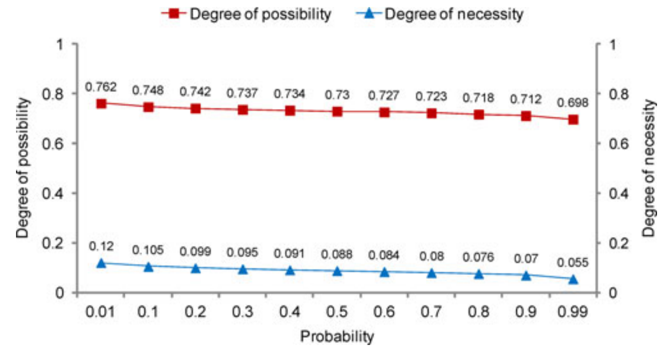

Fig. 7. Degrees of possibility and necessity obtained from the possibility-based fractile model.

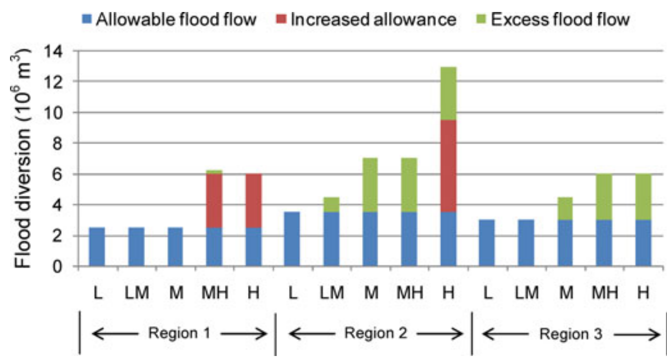

Fig. 8. Flood diversion patterns obtained from the possibility-based fractile model.

cessity due to intervals existing in the objective function and constraints, thus addressing multiple uncertainties in the flood management problem. Fig. 8 shows the flood diversion patterns obtained from the possibility-based fractile model. The results indicate that one set of deterministic solutions would be obtained. In comparison, IPP-FRC is able to generate solutions in the form of intervals, and a variety of decision alternatives can, thus, be obtained through adjusting the values of continuous variables within their lower and upper bounds, promoting diversity of solutions. Consequently, IPP-FRC improves upon the possibility-based fractile model by not only taking into account interval uncertainties that exist in the objective function and constraints, as well as the resulting solutions, but also reflecting two layers of uncertain information (i.e., intervals with fuzzy random boundaries) in one single coefficient of the objective function.

Generally, the proposed IPP-FRC approach has the following contributions: 1) IPP-FRC tackles multiple uncertainties in the forms of intervals, probability distributions, and possibility distributions, as well as their combinations; 2) IPP-FRC addresses the complexity in terms of intervals with fuzzy random boundaries; 3) IPP-FRC is capable of generating inexact degrees of possibility and necessity under different probabilities of occurrence; 4) IPP-FRC provides the possibility and necessity measures that are suitable for risk-seeking and risk-averse decision making, respectively; 5) IPP-FRC is capable of conducting a policy analysis by generating a number of decision alternatives under different policy scenarios; and 6) IPP-FRC addresses the dynamic complexity through capacity expansion planning for flood diversion within a multi-region, multi-flood-level, and multi-option context.

IPP-FRC is based on a normal distribution assumption that all random variables are independent and normally distributed with known means and variances. The stochastic objective in the IPP-FRC model cannot be transformed into the deterministic equivalent function when random variables follow nonnormal distributions. Moreover, the simplest triangular fuzzy sets were used in this study to reflect the fuzziness in lower and upper bounds of intervals because they were widely used in practical problems due to the roughness of subjective fuzzy information. However, nonlinear membership functions may also be used for dealing with real-world problems that involve finer information. As an extension to nonlinear membership functions, piecewise linear approximations can be employed to convert the nonlinear functions into a series of linear segments so that the problem can be solved by conventional linear programming algorithms in a computationally efficient manner. However, our proposed methodology would become much more complicated with nonlinear membership functions when dealing simultaneously with random variables. Thus, one potential extension of this research is to integrate IPP-FRC with other techniques to enhance its applicability to practical situations.

V. Conclusion

In this study, an IPP-FRC model has been developed for not only addressing multiple uncertainties in the forms of intervals, probability distributions, and possibility distributions but reflecting highly uncertain information (i.e., intervals with fuzzy random boundaries) in one single coefficient of the objective function as well. IPP-FRC took advantage of the concept of possibility and necessity measures to represent risk-seeking and risk-averse decision making, respectively. It was useful for decision makers to adopt either possibility or necessity measure based on their risk preferences in practical problems.

The performance of IPP-FRC was analyzed and compared with the possibility-based fractile model through a case study of flood diversion planning under uncertainty. The results indicated that inexact degrees of possibility and necessity were obtained under different probabilities of occurrence. These decreased with increasing probability levels, implying a potential tradeoff between the fulfillment of objectives and associated risks. A number of decision alternatives were obtained under different policy scenarios, which were helpful for decision makers to formulate the appropriate flood management policy. IPP-FRC improved upon the possibility-based fractile model through addressing interval uncertainties in the objective function and constraints, as well as the resulting solutions. This study was a first attempt on the flood management problem. The proposed methodology would be applicable to various problems in a hybrid uncertain environment.

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