Improving Transformers' Utilization Under Single Contingency Policy and Customer Reliability Requirements

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***Abstract*—During the past two decades, utilities have saved tremendous capital investments in new transformers by improving utilization of existing transformers in distribution systems. Meanwhile, in these highly-utilized systems, deteriorated reliability has been observed at the end-user level. This is mainly because utilities determine transformer utilization conventionally through the N-1 approach, which only guarantees reliability at the system level but not reliability requirements of customers. This paper presents an innovative method that balances the financial interest of utilities and service quality of customers by maximizing transformers' utilization while assuring reliability at the end-user level. The proposed method's advantages are enhanced by its capability of handling customer reliability requirements of a whole distribution system and/or of designated locations with critical loads. In addition to determining the maximum utilization of transformers, the proposed method can be extensively applied to other power delivery equipment in distribution systems.**

***Index Terms*—Power distribution planning, power system reliability, power transformers.**

I. Nomenclature

General Variables:

u_{in} : Utilization rate of transformer n in substation i.

u_i : Utilization rate of substation $i, u_i = \sum_n u_{in}$.

p_{ij} : Power transferred from substation j to i, or load transferred from substation i to j under a contingency at $i, p_{ij} = \sum_n p_{ijn}$.

p_{ijn} : Power transferred from Transformer n in Substation j to i, or load transferred from substation i to transformer n in substation j under a contingency at i.

e_i : Annual Energy Not Served (ENS) of loads under substation i.

η_{ti} : Rate of failure to serve loads under substation i due to loss of its *Contingency Support Neighborhood* (CSN) transformers.

η_{fi} : Rate of failure to serve loads under substation i due to changeover failure of its CSN feeders.

Logical Variables:

k_{ij} : Binary indicator of substation j's membership to substation i's CSN, $= \begin{cases} 1, & if\ j \in \tilde{\mathcal{N}}_\imath \\ 0, & otherwise \end{cases}.$

q_{ijn} : Binary indicator of the membership of substation j's transformer n to substation i's CSN, $= \begin{cases} 1, & if\ n \in \tilde{\mathcal{N}}_j\ and\ j \in \widetilde{\mathcal{N}_\imath} \\ 0, & otherwise \end{cases}.$

Sets:

Δ_i : Substations that are connected to substation i by tie feeders.

$\mathcal{T}_i$: Transformers in substation i.

T_i : Number of transformers in substation i's CSN.

$\tilde{\mathcal{T}}_\imath$: Transformer units in substation i's CSN.

$\tilde{\mathcal{T}}_\imath$: Number of transformer in substation i's CSN.

F $\tilde{\mathcal{F}}_\imath$: Substation j and feeders connecting j to i in substation i's CSN.

$\tilde{\mathcal{F}}_\imath$: Number of feeders in substation i's CSN.

System Parameters:

R_{in} : Normal rating of Transformer n in substation i.

D_i : Peak load under substation i.

F_{ij} : Transmission limit of tie feeders connecting substation i and j.

E_i : Maximal allowed ESN for substation i.

E : Maximal allowed ENS for whole distribution system.

A: Contingency overloading factor of transformers.

H_t : Failure rate of a single transformer.

H_f : Changeover failure rate of a single feeder.

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II. Introduction

DURING THE past two decades, utilities have increased transformer utilization by about 30% in North American power distribution systems. Table I[1] shows utilization averages obtained from a comprehensive survey of loading prac-

[1]The table is created in [1] with data from [6].

TABLE I
DESIGN LOADING GUIDELINES FOR NORMAL AND CONTINGENCY LOADING[1]

Average maximum peak load as planned, under normal conditions, as a percent of nameplate thermal rating.	Percent of Nameplate			
	1998	1988	1979	1970
Among utilities with > 1,000,000 customers	85	75	70	64
Among utilities with 500,000 and 1,000,000	85	75	70	65
Among utilities with < 500,000 customers	75	70	70	65
In rural areas	70	65	65	62
In suburban areas	85	80	75	67
In urban areas	90	82	75	72
Maximum planned peak load expected under contingency conditions (at least four hours duration)	**Percent of Nameplate**			
	1998	1988	1979	1970
Among utilities with > 1,000,000 customers	145	140	135	133
Among utilities with 500,000 and 1,000,000	140	135	130	127
Among utilities with < 500,000 customers	135	130	130	127

[1] The table is created in [1] with data from [2]

tices across the industry [1], [2]. This is mainly because of the recent fast growing demand for electricity service and difficulties, arising from economic and political consideration, of augmenting equipment to accommodate the demand in a timely fashion [3]. The engineers in charge of these higher-utilization-rate power systems make use of, and in fact planned for, meshed structure and reconfiguration capability of their systems, which enables the provision of alternative sources and power flow paths to back up the outage of any one of its major components (a single contingency) [1], [4]–[6]. In a conventional radial distribution system, each substation is often equipped with two transformers and each of them is loaded to 66% of its normal rating at peak (given their single contingency loading is 133%). Therefore, during a single contingency, each transformer requires its neighboring unit to "stand by" to pick up its outage. In contrast, a modern distribution system, which is planned to be of meshed structure and operates in radial in normal conditions, is able to reconfigure to connect the substation of contingent transformers to other substations by closing normally opened switches. This provides more backup resources in the vicinity for contingency support and justifies higher utilization rate. As a result, each transformer in the distribution system can be loaded to 88% at peak if it has two other units for its contingency support.

High transformer utilization makes utility companies more financially efficient, which is potentially beneficial to both stockholders and customers. Each transformer unit usually costs in a range of \$100 000 to \$800 000. Great monetary savings are attainable by making more use of their capacity. However, reduced customer service reliability is often detected in highly utilized distribution systems [1], [2]. This can be attributed to the incompatibility between the supply-side reliability planning approach and customer-side reliability requirements. Conventionally, utilization of distribution systems is estimated through the *Single Contingency Policy* (SCP or $N-1$ criteria), which requires a power system to be fully functional even if any one of its major components is out of service [1], [4]–[6].

Customer reliability, on the other hand, put emphasis on the service seen by the customer, often measured by measures of system unavailability, such as System Average Interruption Duration Index (SAIDI), System Average Interruption Frequency Index (SAIFI), and Energy Not Served (ENS) [2], [7]. In a highly-utilized system, each transformer needs more contingency support units in its vicinity, which implies a proportionally larger *Contingency Support Neighborhood* (CSN). This large CSN will have a higher exposure to multiple simultaneous outages, which is not constrained by the single contingency policy but will very likely lead to interruption of service to some customers [1]. Therefore, a system planned to satisfy the $N-1$ criteria will not necessarily meet a designated requirement of customer service reliability.

It is important to understand that high utilization rates are not, in and of themselves, a cause of poor reliability. A power system that operates at 83% or 90% or even 100% utilization of transformers at peak can be designed to operate reliably [1], [6], but something beyond the $N-1$ approach is needed to assure required customer service quality.

For this reason, this paper proposes a method to estimate the allowed maximum transformers' utilization under given customer reliability requirement. The proposed method also identifies the CSN associated with the maximum utilization, which is not necessarily defined by the physical connectivity of tie feeder in the region. The proposed method is formulated as a mixed-binary-integer optimization problem. The formula enables the specification of customer reliability requirements both at designated locations and at the system level, which can result in high performance at critical loads or uniform performance throughout the system.

To introduce the proposed method, Section II presents in detail the problem of conventional planning of transformers' utilization in distribution systems; Section III proposes the method and derives its formula; Section IV demonstrates the proposed method on a simple numerical example; finally, conclusions are drawn in Section V.

III. ESTIMATING TRANSFORMER MAXIMUM UTILIZATION UNDER SINGLE CONTINGENCY POLICY: METHODOLOGY AND PROBLEMS

The traditional distribution system planning method that assures reliability at the substation level is the *Single Contingency Policy* (SCP or $N-1$ criterion). This method requires that a power system must be able to operate and fully meet expectations for amount (kW) and quality of power (voltage, power factor, etc.) even if any one of its major components is out of service [1]. As distribution networks are operated radially, a failure always involves an outage. Because the radial operation is obtained with a meshed physical network in which some normally open (NO) switches define a radial configuration, service can be restored by closing NO switches that feed the outage area. Likewise, for a contingency of a transformer failure, NO switches may be closed to reconfigure the distribution system providing alternative power delivery paths from neighborhood substations to the area under the faulted transformer.

A critical concept associated with $N-1$ criterion is *Contingency Support Neighborhood* (CSN): the portion of the system including all equipment that is part of the planned contingency support for a unit's outage. For a substation power transformer, this might include at least one neighboring transformer (at the same substation or a unit that can be connected to in the vicinity) which would provide capacity margin during its outage [1], [7]–[10]. Recent development of communication, sensor and switching technology enables fast and accurate reconfiguration of distribution systems and consequently easy access to more neighboring units to a contingent transformer, which can be potentially in its CSN. Considering this possibility of enlarged CSN, some studies and utility practices have explored maximum utilization level of transformers by applying the $N-1$ approach [7]–[10].

However, the maximized utilization rate of transformers found by the $N-1$ approach has tended to lead to poor reliability of customer service. This is attributed to the interrelation between the failure rate of electricity supply of a transformer and the transformer's CSN. In an $N-1$ system, customer outages occur when two or more pieces of equipment fail simultaneously within one CSN. High utilization rate requires a CSN that is proportionally larger and thus is a greater target for trouble to occur, that is, a large CSN has more exposure to "simultaneously outage." The analysis below exemplifies this fact by estimating the customer service reliability, measured by annual *Energy Not Served* (ENS), in a distribution system loaded to different levels.

Consider a system with 1200 substation transformers, and having the following characteristics:

•) All transformers are of a same type and of a same capacity, and they can be loaded up to 133% in a contingency;
•) All transformers are loaded to a same peak level, i.e., $u_i = u_j$, which is defined as system utilization rate at peak;
•) All transformers have a same failure rate, 0.25%/year.

Ratings of transformers are normalized into per unit (p.u.).

1. In a 66% utilization system, a transformer needs only another unit to support its outage. Therefore, for every transformer in the system its CSN is one unit. Failure to serve the loads under the transformer occurs only if both transformers of this pair fail. That is:

$$\eta_{it} = 0.0025^2 = 6.25 \times 10^{-6};$$
$$\begin{aligned} E_i &= \eta_i \times u_i \times 8760 \\ &= 6.25 \times 10^{-6} \times 0.66 \times 8760 \\ &= 0.0361 \text{ hours/year} \end{aligned} \tag{1}$$
$$\begin{aligned} E &= \sum\nolimits_i E_i \\ &= 0.0361 \times 1200 = 43.362 \text{ hours/year} \end{aligned} \tag{2}$$

2. In an 88.5% utilization system, a transformer's CSN is two other units. Based on the $N-1$ design, failure to serve the load under the transformer occurs only when all three or two out of the three transformers fail, that is, both two or any one transformer fail in the its CSN. Therefore:

$$\begin{aligned} \eta_{it} &= 0.0025^3 + 0.0025 \times 2(0.0025(1-0.0025)) \\ &= 1.25 \times 10^{-5} \\ E_i &= 0.005 \times 0.885 \times 8760 \\ &= 0.0968 \text{ hours/year} \\ E &= 0.0968 \times 1200 = 116.16 \text{ hours/year} \end{aligned}$$

3. In a 100% utilization system, every transformer has a CSN of three other units. Failure to serve the load under the transformer occurs when all the three, any two or any one of the transformers fails in its CSN. That is,

$$\begin{aligned} \eta_{it} &= 0.0025^4 + 0.0025 \times 3(0.0025^2(1-0.0025)) \\ &\quad + 0.0025 \times 3(0.0025(1-0.0025)^2) \\ &= 1.87 \times 10^{-5} \\ E_i &= 0.1638 \text{ hours/year} \\ E &= 196.56 \text{ hours/year} \end{aligned}$$

Despite the fact that this calculation assumes great simplifications of the many details for the example system, real-world variations from these assumptions do nothing to change the basic principle nor dilute the conclusions that are reached: enlarging CSN to achieve a higher utilization rate will, if the $N-1$ criterion assumed, lead to unexpectedly large values of demand not served. Maximum utilization of transformers estimated by the existing studies and utility practices through the $N-1$ approach may lead to poor customer reliability [1], [6], [7]–[9].

IV. A Method to Improve Transformers' Utilization Under Required Customer Reliability

For the reasons mentioned, we propose a method to estimate the maximum utilization of transformers in distribution systems that are reconfigurable under contingencies. The approach guarantees customer service reliability at designated locations and throughout the whole system. It can be applied to estimation of equipment settings in system operation or future capital investment in system planning. The method also identifies the CSN associated with the estimated maximal utilization of transformers. One finding through the proposed method is that the CSN is not naturally defined by the available tie feeders connected to the contingent transformer, but is a function of the transformers utilization and required customer reliability. These results are explained in the next few pages.

To maximize utilization of a distribution system, primarily it is accomplished by increasing transformers' utilization, which is the same as to maximize the total load that each substation can accommodate, u_i, as in (3).

$$\text{Max}: \sum\nolimits_i u_i \tag{3}$$

For every substation i, it has a set of transformers $\mathcal{T}_i$, and each transformer n is loaded to u_{in}, and has:

$$u_i = \sum\nolimits_n u_{in}, \quad \forall i, \forall\, n \in \mathcal{T}_i \tag{4}$$

The $N-1$ approach has the following constraints to utilization of the transformers and the system [8]–[10]:

Loading constraint for transformer n at substation i under normal conditions:

$$u_{in} \le R_{in}, \quad \forall i, \forall\, n \in \mathcal{T}_i \tag{5}$$

Loading constraint for demand requirement at substation i under normal conditions:

$$u_i \ge D_i, \ \forall i \tag{6}$$

Loading constraint for contingency support to substation i:

$$u_i - \sum_j p_{ij} \leq A\left(\sum_n R_{in} - R_{in}\right), \quad \forall i,\ \forall j \in \Delta_i,\ \forall n \in \mathcal{T}_i \tag{7}$$

Transfer constraint for transformers $n \in \mathcal{N}_j$ at substations in i's neighborhood, $j \in \Delta_i$:

$$p_{ijn} \leq A \cdot R_{jn} - u_{jn}, \quad \forall i, \forall j \in \Delta_i \quad and\ \forall n \in \mathcal{T}_j \tag{8}$$

Transfer constraint for tie feeder connecting substation i and $j \in \Delta_i$:

$$0 \leq p_{ij} \leq F_{ij}, \quad \forall i,\ \forall j \in \Delta_i \tag{9}$$

$$\text{And}$$

$$p_{ij} = \sum_n p_{ijn}, \quad \forall i,\ \forall j \in \Delta_i,\ \forall n \in \mathcal{T}_j \tag{10}$$

Under nominal (no contingency) conditions, (5) states that every transformer should be loaded at peak less than 100% of its normal rating; and (6) states that transformers at substation i should be loaded to be able to serve the peak demand under i. In a single contingency, the load of any lost transformer at substation i should be picked up by other transformers at the same substation i and/or transferred to other transformers in i's neighboring substation $j \in \Delta_i$, as stated in (7). In addition, the amount of power that can be transferred is further constrained by the transmission limits of tie feeders, F_{ij}, connecting substation i and j, expressed by (9). And the power flow on feeder from substation j to i is the total of the power picked by transformers n in substation j, as stated in (10), which should be less than their contingency operation margin, as stated in (8). Note that the subscript ij is not interchangeable for all the variables here, for i represents the substation of the single contingent transformer.

From (3) to (10), it can be concluded that, in a system of reconfiguration capability, the utilization level of transformers in a substation depends not only on its own operation conditions, which include demand in the region of service and its normal rating, but also on other two factors: the transmission constraint of its substation's tie feeders and the contingency operation margin of transformers in its neighboring substations.

Now further consider the requirements of customer service reliability, which can be specified by

$$e_i \leq E_i, \quad \forall i \tag{11}$$

$$\text{And/or}$$

$$\sum_i e_i \leq E \tag{12}$$

where (11) specifies the maximum value allowed for Energy Not Served (ENS) at a designated substation i; (12) specifies that of the overall system. The choice between (11) and (12) depends on the preference of a power engineer between overall performance on one hand and strict reliability requirements of one or more critical regions on the other. The use of both (11) and (12), however, is not necessarily as a conflict, and an artful assignment of ENS requirements should be able to create good reliability of designated levels throughout the system, which is a very attractive application.

The annual ENS of a substation i is defined by its probability of failure to serve the load times the load in MWh during a year, as shown in (13)

$$e = (\eta_{fi} + \eta_{ti} \cdot (1 - \eta_{fi})) \cdot u_i \times 8760, \quad \forall i \tag{13}$$

where the term $\eta_{fi} + \eta_{ti} \cdot (1 - \eta_{fi})$ defines the failure rate of serving customers under substation i when transformers are set to a utilization level that *marginally* satisfy $N - 1$ criterion. That is the probability that any transformer and/or tie feeder, in addition to a single transformer's failure at substation i, fails in i's CSN.

Feeder changeover failure rate η_{fi}, i.e., the failure rate of switching the normally open points, and transformer failure rate η_{ti} in substation i's CSN are given by:

$$\eta_{fi} = \mathrm{H}_f \cdot \sum_{j=1}^{\tilde{F}_i} \left[\binom{j}{\tilde{F}_i} \cdot (\mathrm{H}_f)^j \cdot (1 - \mathrm{H}_f)^{\tilde{F}_i - j}\right], \quad \forall i \tag{14}$$

$$\eta_{ti} = \mathrm{H}_t \cdot \sum_{n=1}^{\tilde{T}_i} \left[\binom{n}{\tilde{T}_i} \cdot (\mathrm{H}_t)^n \cdot (1 - \mathrm{H}_t)^{\tilde{T}_i - n}\right], \quad \forall i \tag{15}$$

where H_f and H_t are failure rates for each feeder switching and transformer unit respectively. In practical cases, each unit can be assigned with different failure rates.

In (14) and (15) $\tilde{T}_i$ and $\tilde{F}_i$ are the numbers of transformer and feeder units in substation i's CSN. They are found by two binary variables k_{ij} and $q_{ijn} \in \{0, 1\}$, which indicate the membership of substation j and its transformer n to substation i's CSN. Their relation can be described by (16)

$$q_{ijn} \leq k_{ij} \leq \sum_j q_{ijn}, \quad \forall i,\ \forall j \in \Delta_i,\ \forall n \in \mathcal{T}_j \tag{16}$$

which states that a substation j is in the substation i's CSN, and so does the tie feeder connecting j to i, if and only if j has at least one transformer unit that supports i's single transformer contingency.

The two indicators are deployed to specify the membership of substation i's CSN by constraining contingency power transfer, as in (17) and in (18), which replaces (9):

$$0 \leq p_{ijn} \leq q_{ijn} \cdot M, \quad \forall i,\ \forall j \in \Delta_i,\ \forall n \in \mathcal{T}_j \tag{17}$$

where M is a very big positive number.

$$0 \leq p_{ij} \leq k_{ij} \cdot F_{ij}, \quad \forall i,\ \forall j \in \Delta_i \tag{18}$$

Therefore,

$$\tilde{T}_i = \sum_j \sum_n q_{ijn} + T_i - 1,\ \forall i \tag{19}$$

$$\tilde{F}_i = \sum_j k_{ij}, \quad \forall i \tag{20}$$

Equation (13) to (20) indicate that the number of transformer and feeder units, $\tilde{T}_i$ and $\tilde{F}_i$, in substation i's CSN is limited by their combined failure rate and consequently the customer reliability requirements in the region of service.

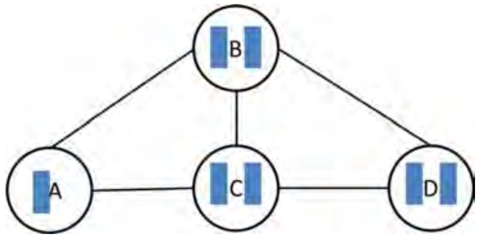


Fig. 1. Testing system topology. The system is planned in a meshed topology, and operated radially with all tie feeders open.

Following the discussions above, readers should distinguish the concept of neighboring units from that of CSN for a certain substation. In other words, CSN of substation i is not necessarily the full set of transformers in neighboring substations j to which i is physically connected through a tie feeder. For example, in an interconnected neighborhood of five substations and each equipped with two transformers, for a given transformer, the former concept refers to all the nine transformers that have operation margin to pick up the load of one lost transformer; while its CSN may have only five transformers, which together has a failure rate under its reliability requirements.

The CSN found under the proposed approach, however, is never limited to a probabilistic concept. The identified CSN can be a reference for power system planners in system reinforcement or for system operators in contingency resource dispatch, and result in satisfactory customer reliability and high system utilization rate.

It is also interesting to note that the constraints which specify feeder capacity of power transfer and neighboring transformers' operation margin, from (5) to (11), determines the amount of how much power each transformer in the CSN can provide to support a single contingency; constraints that specify reliability requirements, from (13) to (20), decides which transformer units should be in the CSN. This structure infers a potential two-stage algorithm of solving the mixed-binary-integer problem: 1) determining which units are in the CSN, or in other words, which substations in neighborhood to be connected to for contingency support; and 2) determining how much contingency support each transformer in the connected substations can provide. An analogy of the algorithm can be found in the problem of electricity markets' unit commitment and economic dispatch [12].

V. Numerical Examples

This section demonstrates the proposed method through a simple numerical case, shown in Fig. 1. The picture shows a testing distribution system that consists of four substations $A, B, C,$ and D. Substation A has one transformer and all the other three substations have two. All the tie feeders between substations are normally open and can be connected under a contingency to any transformer. Parameters of the testing system are shown in Table II. All numerical values in this section are normalized values.

The proposed method, as formulated in the equations from (3) to (8) and (10) to (20), is applied on the testing system to find out transformers' maximum utilization. The following assumptions are made:

TABLE II
Some Parameters for the System of Fig. 1

	A	B	C	D
R_{in}	(5)	(5, 5)	(10, 10)	(5, 5)
D_i	3	9	18	7

R_{in} shows normal rating for transformers in each substation; D_i shows peak load under each substation.

- Changeover failure of feeders is 0. Tie feeders are reliable and guaranteed to be switched successfully in a contingency and of a failure rate of 0;
- Failure rate of every transformer is the same and is 0.25%/year;
- Contingency overloading rate of every transformer is the same and is 130%;

To demonstrate the trade-off between customer reliability requirements and transformers' utilization, in this particular example, (12) is written into a penalty term to be included at the objective function; whereas in real applications, (12) can be used as a hard constraint if needed. Hence, (3) is rewritten as:

$$\text{Max} \sum_i u_i - w \cdot \sum_i e_i \tag{21}$$

where w is the weight of the system reliability penalty and is of unit 1/hours. The choice of w depends on the intended emphasis on the system-wide reliability versus utilization. The modified formulae find the maximum utilization of transformers and optimize system reliability, defined by (21), and keeps local reliability under control, as specified by (12).

The system is tested under the following four scenarios:

1. No reliability requirement for all regions in testing system; no transmission limits for tie feeders that connect the substation of the contingent transformer to its neighboring substations; low weight of reliability penalty in objective function $(w = 0.5)$;
2. No reliability requirement for all regions; a weak link between substation B and C $(F_{BC} = 2)$, and transmission capacities for all other tie feeders $(F_{ij} = 5)$; and $w = 1$;
3. Reliability requirements of two categories: a) strict requirement for critical loads under substation D ($E_D = 1$ hour/year); b) requirement in proportion to peak demand for all other regions ($E_A = 1$ hour/year, $E_B = 2$ hour/year, $E_C = 4$ hour/year); transmission limits of the tie feeders and weight of reliability penalty the same as Scenario 2;
4. Reliability requirements and transmission limits of tie feeders the same as Scenario 3; high weight of reliability penalty $(w = 1.5)$.

Results of the four scenarios are presented in Table III. In Scenario 1, all transformers' utilization rates are pushed to their normal rating. Without reliability requirement for each substation and transmission limits of tie feeders, the load that can be transferred from the contingent transformers in substation i is constrained only by the contingent operation margin of its neighboring substation j, shown in columns p_A to p_D. Note that the CSN of a transformer unit is less than the natural neighboring transformers that are defined by tie feeder connection. For example, in a contingency of transformer outage, substation A can be connected to substation B and C, which have four

TABLE III
MAXIMUM UTILIZATION OF TRANSFORMERS IN SYSTEM OF FIG. 1

Case #	Subs i	p_A	p_B	p_C	p_D	T_i	$u_i = \sum_n u_{in}$	e_i
1	A	×	3	6	×	2	(5)	0.547
	B	1.5	×	6	3	3	(5,5)	1.638
	C	1.5	3	×	3	6	(10,10)	6.529
	D	×	3	6	×	3	(5,5)	1.638
	$\sum_i u_i$		45		$\sum_i e_i$		10.352	
2	A	×	3	5	×	2	(5)	0.547
	B	1.5	×	2	3	3	(5,5)	1.638
	C	1.5	2	×	3	6	(9.5,10)	6.366
	D	×	3	5	×	2	(5,5)	1.094
	$\sum_i u_i$		44.5		$\sum_i e_i$		9.645	
3	A	×	3	5	×	2	5	0.547
	B	1.5	×	2	3.87	3	(5,5)	1.638
	C	1.5	2	×	3.87	4	(9.17,9.17)	3.999
	D	×	3	5	×	2	(4.57,4.57)	0.999
	$\sum_i u_i$		42.46		$\sum_i e_i$		7.183	
4	A	×	3	5	×	1	(5)	0.274
	B	1.5	×	2	3.87	3	(5,5)	1.638
	C	1.5	2	×	3.87	4	(8,10)	3.927
	D	×	3	5	×	2	(4.57,4.57)	0.999
	$\sum_i u_i$		42.13		$\sum_i e_i$		6.838	

u_i shows utilization rate of each substation. p_j shows the load that can be transferred to substation j in case of a single transformer contingency in substation i. T_i shows the CSN size in transformer units under the obtained utilization rate.

transformers in total for its contingency support, but its CSN size is limited to two to minimize the system reliability penalty and result in the system's annual Energy Not Served (ENS) less than 0.6.

In Scenario 2, load that can be transferred to a neighboring substation is constrained by transmission limits which is less than 2 between substation B and C and less than 5 between all other substations. Because of these constraints, the utilization of one transformer in substation C is reduced to 9.5 from 10, which in turn increases its contingent operation margin from 3 to 3.5. The relaxed contingent operation margin enables more load transfer from substation D to C from less transformer units, and substation D's CSN is reduced from 3 to 2. Consequently its annual ENS is reduced from 1.638 to 1.094.

In Scenario 3, regional reliability requirements are specified for each substation. Substation A and B has satisfied their requirements ($E_A = 1$ hour/year and $E_B = 2$ hour/year) and thus need no more changes on the current settings. Substation C improves its annual ENS by reducing its CSN from six transformer units to four, which in turn gives a less utilization of 18.33; while substation D keeps its size of CSN but reduces its utilization from 10 to 9.13 in total. The adjusted utilization rate of substation C and D leads to annual ENS within the required range ($E_C = 4$ hour/year and $E_D = 1$ hour/year).

In Scenario 4, the weight of system reliability penalty is increased from 0.5 to 1.5. As a result, substation C's utilization is decreased from 18.33 to 18, creating a larger contingent operation margin of 5 for one of its two transformers. This increased margin itself is sufficient to support the transformer loss in substation A. Hence, substation A's CSN is reduced to 1. While the total utilization rate of the system is reduced from 42.46 (in Scenario 3) to 42.13, system's total annual ENS is improved from 7.183 to 6.838. This scenario, together with Scenario 3, shows that system can be set to operate in a mode of more efficient or more reliable. The trade-off between the two targets and its reflection on penalty weight in the objective function can be subtle and lead to another research topic.

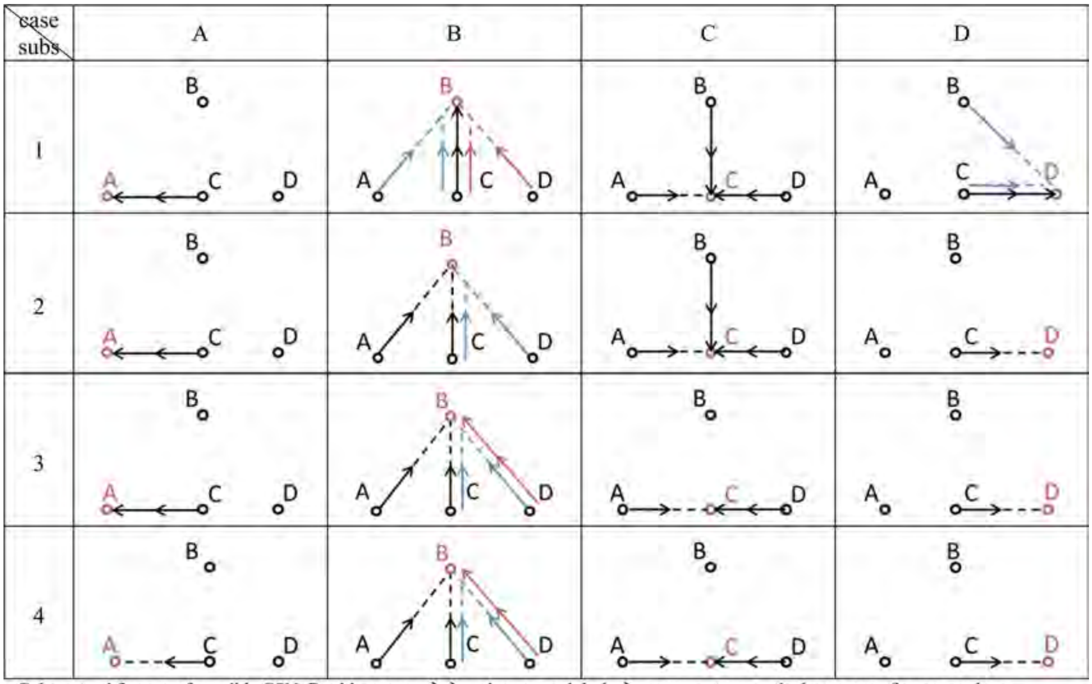
Despite that the utilization of the testing system is maximized by the proposed method, readers may notice that the solution is not unique in all scenarios discussed. In the mixed-binary-integer problem, the decision variables are utilization rate of every transformer, u_{in}, their CSN, $\tilde{\mathcal{F}}_i$ and $\tilde{\mathcal{T}}_i$, and loads transferred to their CSN in a contingency, p_{ijn}. Table III lists all possible CSN of the results in Table II. Furthermore, the utilization of transformers in a substation has infinite combinations of a maximal total. For example, in Scenario 3, any combination of utilization rates of two transformers in substation C is an optimal solution if it makes up a total of 18.33. The underdetermined property of the problem can be attributed to (19) and (20). By involving more conditions, for instance contingent transfer cost as a function of transferring distance and feeders' thermal dissipation rate, or specifying equipment's physical characteristics, such as failure rate of transformers as a function of its utilization rate, $\mathrm{H}_{t,in}(u_{in})$ (as the function presented in [11]), the optimal solution can be narrowed to a unique one.

VI. CONCLUSION

High utilization level of transformers at distribution levels of power systems could lead to deteriorated customer service reliability. This can be attributed to the incompatibility between the supply-side reliability planning approach (the $N-1$ approach) and customer-side reliability requirements, which tends to be violated by enlarged *Contingency Support Neighborhood* (CSN) in highly-utilized systems.

For this reason, a method beyond the $N-1$ methodology is sought for to maximize utilization of transformers while as-

TABLE IV
ILLUSTRATION OF ALL POSSIBLE CONTINGENCY SUPPORT NEIGHBORHOOD (CSN) IN TABLE III



Colors stand for sets of possible CSN. Double arrows →→ and arrow and dash →-- represent respectively two transformers and one transformer, in their originated substation, serving as its pointed substation's contingency support units.

suring good customer service quality at the same time. This paper proposes a methodology to estimate the maximal transformer's utilization under given customer reliability requirements. The method is formulated as a mixed-binary-integer optimization problem. The formula enables the specification of customer reliability requirements at a system level as well as at designated localities. The major conclusions of this study are: 1) The utilization rate of a distribution system is constrained by its tie feeder strength and reliability requirement of customer service; 2) The CSN are not regions that are partitioned at a system level but should be defined for every transformer of concern under its maximum utilization; 3) The CSN of a transformer is not necessary the full set of units which are accessible through tie feeders.

The proposed method can be applied to find the maximum utilization of transformers, and extensively that of other power delivery equipment, in distribution systems. Moreover, it can be applied in the planning stage to identify the CSN associated with the maximum utilization of concerned transformers for reinforcement of their contingency support capability.

Future work mainly includes two parts. One is to include more information of physical characteristics of distribution systems into the mixed-binary-integer formula of the proposed method. This can contribute to the uniqueness of the solution, which is likely to give a more optimal and accurate maximum utilization in practice. The other part is to develop an algorithm for solving the mixed-binary-integer problem for large systems of many units. A potential direction is to solve the problem with two steps: for the transformer of concern, first determining which units are in its CSN, that is, which substations to be connected to for contingency support; secondly deciding how much contingency support the units in the CSN can offer.

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