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Two-stage network structures with undesirable outputs: A DEA based approach



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ARTICLE INFO

Article history:

Received 2 August 2013

Received in revised form 23 September 2013

Accepted 21 October 2013

Available online 9 November 2013

Keywords:

DEA

Weak disposability

Efficiency

Two-stage

Undesirable outputs

ABSTRACT

The non-parametric Data Envelopment Analysis (DEA) literature on network-structured performance analysis normally considers desirable intermediate measures. These measures are the outputs from the first stage and are used as inputs to the second stage. In many real situations, the intermediate measures consist of desirable and undesirable outputs. This subject has recently attracted considerable attention among DEA researchers. The motivation of this study is the application of the weak disposability to modeling network DEA with undesirable intermediate measures. Undesirable products in this paper are studied in two different cases: either as final outputs or as intermediate measures. In both cases, cooperative and non-cooperative game theories are proposed to assess the relative performance of the operational units. A real case on 39 Spanish Airports in 2008 has been illustrated to verify the applicability of the proposed approaches.

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1. Introduction

Since the last decade, there has been a growing interest in use of efficiency and productivity management taking undesirable and pollutant outputs into account. In production theory, parametric and non-parametric techniques have the advantages of imposing of “weak-disposability” assumption, on the functional form of the underlying technology. Data Envelopment Analysis (DEA) initiated by Charnes et al. [1] and extended by Banker et al. [2], has recently made a substantial contribution in analyzing undesirable outputs.

Modeling undesirable factors has received considerable attention not only for measuring efficiency and productivity but also for estimating pollution abatement factor. This approach has been critically debated in Hailu and Veeman [3], Färe and Grosskopf [4], as well as Hailu [5] and Kuosmanen [6]. The traditional approach to modeling weak dis-

posability (reduction of undesirable outputs by decreasing the level of production activity) goes back to Shephard [7] who applied a single abatement factor for all observed activities in the sample. Kuosmanen [6] pointed out that applying a uniform abatement factor is not in line with the usual wisdom of concentrating abatement factors on firms with lower abatement costs.

Podinovski and Kuosmanen [8] developed two further technologies for modeling weak-disposability under relaxed convexity assumption. Recently weak disposability has been applied to network-structured production systems with undesirable outputs and as far as we are aware, there is little DEA-based work considering undesirable variables in network-structured production systems. In real cases, joint production of desirable and undesirable outputs renders difficulties in the measurement of overall performance and two-stage network structures.

In a survey by Cook et al. [9] four categories for efficiency measurement of the two-stage systems were presented: standard DEA approach, efficiency decomposition, network DEA and game-theory approaches. As noted in

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Kao and Hwang [10], in the standard DEA methodology, each process is treated as an independent system. The operation of two processes is represented by conventional envelopment constraints in calculating the system efficiency and the whole efficiency of the system is the product of the efficiencies of the two stages. These two perspectives have been used in various studies as on global companies Zhu [11], on banking Seiford and Zhu [12], on baseball Sexton and Lewis [13] and on IT Wang et al. [14].

Liang et al. [15] have conducted closer examinations for modeling the two-stage network structure using the concept of non-cooperative approach. This perspective is characterized by the leader-follower or Stackelberg game theory.

The above mentioned studies on network DEA either do not consider the existence of undesirable factors in the processes or in the presence of undesirable factors, they did not use the weak disposability assumption in handling undesirable factors. Classical DEA models on two stage production processes normally consider good intermediate measures. In some real occasions, however, the intermediate measures may be undesirable and we should reduce these products. While the network DEA model of Färe and Grosskopf [16] considers different network structures, it cannot provide a performance measurement tool for such production systems. Modeling such a two-stage production systems with undesirable outputs is an important and interesting subject in the context of DEA. The motivation of this study is the application of weak disposability to modeling network DEA with undesirable intermediate measures.

Two different cases are considered: First, the intermediate measures are the inputs to the second stage and in the second one the intermediate measures are the final products. In both cases, cooperative and non-cooperative game theories are proposed to assess the relative performance of the DMUs. The two proposed perspectives (cooperative and non-cooperative game theories) proposed in this paper, are absolutely different and depending on the structure of the production process and the viewpoints of the central decision maker, one of these models is used. So, both models are studied in this paper.

We believe that the contribution of this paper is modeling undesirable in a two stage production system using the weak disposability assumption of Färe and Grosskopf [4]. The non-cooperative and Stackelberg game (leader-follower game theory) are separately studied in our approach.

The remainder of the paper is unfolded as follows. In the next section, a brief description on weak disposability outputs will follow. Then, we proceed to weak disposability in a two-stage structure in Section 3. A real case on Spanish airports is given in Section 4. The conclusion section will summarize the findings and implications of the study.

2. Weakly disposable technology

Modeling undesirable outputs (such as emission of harmful substances in air, energy wasted in power plant) of production activities has attracted considerable attention among researchers. Hailu and Veeman [3] have ex-

tended non-parametric productivity analysis models to include undesirable outputs. They introduced a non-orthodox monotonicity condition on their technology and claimed it is preferable to “weak disposability” concept in DEA. Färe and Grosskopf [4] showed that using monotonicity condition in steal of weak disposability is inconsistent with physical law.

Suppose that there are K DMUs and for DMU_k , data on the vectors of inputs, desirable outputs and undesirable outputs are $x_k = (x_{1k}, \dots, x_{Nk}) \geq 0$, $v_k = (v_{1k}, \dots, v_{Mk}) \geq 0$ and $w_k = (w_{k1}, \dots, w_{Jk}) \geq 0$, respectively.

Further assume $x_k \neq 0$, $v_k \neq 0$ and $w_k \neq 0$. The production technology can be represented by:

$$P(x) = \left\{ (v, w) \mid x \text{ can produce } (v, w), x \in R_+^N \right\}$$

Definition 1. Outputs (desirable and undesirable) are weakly disposable if and only if $(v, w) \in P(x)$ and $0 \leq \theta \leq 1$ imply $(\theta v, \theta w) \in P(x)$, $x \in R_+^N$ (see, Shephard [7]).

Färe and Grosskopf [4] proposed the following technology under variable return to scale satisfying weak-disposability assumption:

$$\begin{aligned} T_{FG} = \left\{ (v, w, x) \mid \right. & \sum_{k=1}^K \theta z^k v_m^k \geq v_m, \quad m = 1, \dots, M, \\ & \sum_{k=1}^K \theta z^k w_j^k = w_j, \quad j = 1, \dots, J, \\ & \sum_{k=1}^K z^k x_n^k \leq x_n, \quad n = 1, \dots, N, \\ & \sum_{k=1}^K z^k = 1, \\ & z^k \geq 0, \quad 0 \leq \theta \leq 1, \quad k = 1, \dots, K \left. \right\}. \end{aligned} \quad (1)$$

The contraction parameter θ in the formulation (1) corresponds to Shephard's definition of weak-disposability. This parameter allows for simultaneous contraction of good and bad outputs.

As Kuosmanen [6] pointed out, this model uses a uniform abatement factor to all firms. To allow for non-uniform abatement factor of the individual firms, he proposed the following production technology:

$$\begin{aligned} T_K = \left\{ (v, w, x) \mid \right. & \sum_{k=1}^K \theta^k z^k v_m^k \geq v_m, \quad m = 1, \dots, M, \\ & \sum_{k=1}^K \theta^k z^k w_j^k = w_j, \quad j = 1, \dots, J, \\ & \sum_{k=1}^K z^k x_n^k \leq x_n, \quad n = 1, \dots, N, \\ & \sum_{k=1}^K z^k = 1, \\ & z^k \geq 0, \quad 0 \leq \theta^k \leq 1, \quad k = 1, \dots, K \left. \right\}. \end{aligned} \quad (2)$$

It should be noted that formulation (1) is a special case of (2) with $\theta^1 = \dots = \theta^K$. Free disposability of the inputs and

good outputs is modeled through the use of inequality constraints regarding v and x . The non-linear technology T_K can be restated in an equivalent linear form by using a simple and effective way. To linearize formulation (2), the intensity weight z^k can be partitioned into two components as $z^k = \lambda^k + \mu^k$. Using this notation, Kuosmanen [6] converted the production technology (2) into the following linear form:

$$T_K^{(L)} = \left\{ (v, w, x) \left| \begin{aligned} \sum_{k=1}^K \lambda^k v_m^k &\geq v_m, \quad m = 1, \dots, M, \\ \sum_{k=1}^K \lambda^k w_j^k &= w_j, \quad j = 1, \dots, J, \\ \sum_{k=1}^K (\lambda^k + \mu^k) x_n^k &\leq x_n, \quad n = 1, \dots, N, \\ \sum_{k=1}^K (\lambda^k + \mu^k) &= 1, \\ \lambda^k, \mu^k &\geq 0, \quad k = 1, \dots, K. \end{aligned} \right. \right\} \quad (3)$$

The above formulation (3) is now a linear form and the right hand sides of the envelopment constraints are faced up with scaling variables. This technology and the foregoing stated linearization procedure are used to modeling undesirable intermediate measures in a two-stage production process.

3. Weak disposability in two-stage decision process

In this section we introduce a two-stage decision process within which the intermediate measures consist of desirable and undesirable outputs. Consider a two-stage production process as shown in Fig. 1.

Suppose again that there are K DMUs and for the first stage of DMU_k the observed data on the vectors of inputs, desirable outputs and undesirable outputs are $x_k = (x_{1k}, \dots, x_{Nk}) \geq 0$, $v_k = (v_{1k}, \dots, v_{Mk}) \geq 0$ and $w_k = (w_{k1}, \dots, w_{jk}) \geq 0$ respectively. The outputs (v_k, w_k) are used as the inputs for the second stage. The second stage is fed up by (v_k, w_k) and an external input vector $z_k = (z_{1k}, \dots, z_{Tk})$. The final product of DMU_k is represented by $y_k = (y_{1k}, \dots, y_{Sk})$. In what follows, two different approaches to this two-stage decision process is introduced. In the first approach a non-cooperative game theory is introduced and the second one considers a centralized model.

3.1. Leader-Follower game theory

In this section, the leader-follower approach is developed to analyze this extended two stage structure. In non-cooperative game (Stackelberg or leader-follower game), there is a preference on the leader and follower. In this case, the leader is more preferable than the follower.

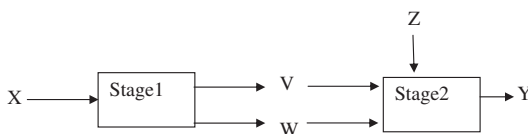


Fig. 1. Two-stage process of DMU_k .

So, the leader determines the most-efficient statuses and then the follower identifies its optimal statuses based on the information from the leader. In our approach to two-stage decision process, the first stage is the leader and the second stage is the follower.

An algebraic representation of the production technology of the first stage is given as follows:

$$T_1 = \left\{ (v, w) \left| \begin{aligned} \sum_{k=1}^K (\rho^k + \mu^k) x_n^k &\leq x_n \quad n = 1, \dots, N \\ \sum_{k=1}^K \rho^k v_m^k &\geq v_m \quad m = 1, \dots, M \\ \sum_{k=1}^K \rho^k w_j^k &= w_j \quad j = 1, \dots, J \\ \sum_{k=1}^K (\rho^k + \mu^k) &= 1 \\ \rho^k, \mu^k &\geq 0 \end{aligned} \right. \right\} \quad (4)$$

The above linear technology is in terms of unknown variables μ and ρ . In applying the model described herein, attention is paid to the radial measure. In our proposed model, we want to measure the efficiency of DMU_o , in terms of the abatement potential in undesirable outputs. This is obtained as the optimal value of the following model:

$$\begin{aligned} e_o^{(1)*} &= \text{Min} \theta_o \\ \text{s.t.} & \sum_{k=1}^K (\rho^k + \mu^k) x_n^k \leq x_n^o, \quad n = 1, \dots, N, \\ & \sum_{k=1}^K \rho^k v_m^k \geq v_m^o, \quad m = 1, \dots, M, \\ & \sum_{k=1}^K \rho^k w_j^k = \theta_o w_j^o, \quad j = 1, \dots, J, \\ & \sum_{k=1}^K (\rho^k + \mu^k) = 1, \\ & \rho^k, \mu^k \geq 0, \quad k = 1, \dots, K. \end{aligned} \quad (5)$$

The objective function minimizes the equal-proportional reduction factor for all undesirable outputs from preserving the current level of inputs and desirable outputs. Clearly, model (5) is a linear programming problem and it is always feasible and bounded. For an inefficient leader in DMU_o (stage 1), we have

$$\begin{aligned} \sum_{k=1}^K (\rho^k + \mu^k) x_n^k &= x_n^o - S_n^{(x)} \\ \sum_{k=1}^K \rho^k v_m^k &= v_m^o + S_m^{(v)} \\ \sum_{k=1}^K \rho^k w_j^k &= \theta_o w_j^o. \end{aligned} \quad (6)$$

in which $S_n^{(x)}$ and $S_m^{(v)}$ are the slack variables of the first and second constraints in (5), respectively. Stage 1 can be improved by deleting inputs as well as undesirable output excesses and augmenting the output shortfalls. It is easy to show that the improved leader is now efficient. Having obtained the efficiency of the first stage, we evaluate stage 2, preserving the efficiency statuses of the first stage. Following Kuosmanen [6], the minimal weakly disposable technology can be formulated in terms of non-uniform scalar factor θ across all DMUs. Under these assumptions, the empirical production set $P_2(z, y)$ can be written as:

$$T_2 = \left\{ (v, w, y) \mid \sum_{k=1}^K \theta^k \lambda^k v_m^k \leq v_m \quad m = 1, \dots, M \right. \\ \sum_{k=1}^K \theta^k \lambda^k w_j^k = w_j \quad j = 1, \dots, J \\ \sum_{k=1}^K \lambda^k y_r^k \geq y_r \quad r = 1, \dots, S \\ \sum_{k=1}^K \lambda^k z_t^k \leq z_t \quad t = 1, \dots, T \\ \sum_{k=1}^K \lambda^k = 1 \\ \lambda^k \geq 0, \quad \theta^k \geq 1, \quad k = 1, \dots, K \left. \right\} \quad (7)$$

As for treatment of weak disposability, formulation (7) uses the abatement factors θ^k that scales down both good and bad outputs by the same fraction, consistent with Färe and Grosskopf [17] definition. Now, we come to transform the non-linear technology (7) into a linear format using the same manner of Kuosmanen [6]. Let $\theta^k \lambda^k = \beta^k$ and $\alpha^k = (1 - \theta^k) \lambda^k$. Then we must have $\beta^k + \alpha^k = \lambda^k$. Rearranging the terms in (7), we obtain an equivalent representation for production technology (7) as follows:

$$\widehat{T}_2 = \left\{ (v, w) \mid \sum_{k=1}^K \beta^k v_m^k \leq v_m, \quad m = 1, \dots, M \right. \\ \sum_{k=1}^K \beta^k w_j^k = w_j, \quad j = 1, \dots, J \\ \sum_{k=1}^K (\beta^k + \alpha^k) y_r^k \geq y_r, \quad r = 1, \dots, S \\ \sum_{k=1}^K (\beta^k + \alpha^k) z_t^k \leq z_t, \quad t = 1, \dots, T \\ \sum_{k=1}^K (\beta^k + \alpha^k) = 1, \\ \beta^k, \quad \alpha^k \geq 0, \quad k = 1, \dots, K \left. \right\} \quad (8)$$

This technology is linear in terms of unknown variables β and α . Based upon Stackelberg game theory for two-stage process, the second stage only considers optimal solutions that maintain the first stage's efficiency statuses. To this ends, the second stage treats the triple (v, w, z) subject to the restriction that efficiency score of the first stage re-

mains at optimality. To evaluate the second stage, we solve the following linear programming problem:

$$e_o^{(2)*} = \text{Min} \quad \phi_o \\ \text{s.t.} \\ \sum_{k=1}^K \beta^k v_m^k = \sum_{k=1}^K \rho^{k*} v_m^k, \quad m = 1, \dots, M \\ \sum_{k=1}^K \beta^k w_j^k = \theta_j^* w_j^0, \quad j = 1, \dots, J \\ \sum_{k=1}^K (\beta^k + \alpha^k) y_r^k \geq y_r^0, \quad r = 1, \dots, S \\ \sum_{k=1}^K (\beta^k + \alpha^k) z_t^k \leq \phi_t z_t^0, \quad t = 1, \dots, T \\ \sum_{k=1}^K (\beta^k + \alpha^k) = 1, \\ \beta^k, \quad \alpha^k \geq 0, \quad k = 1, \dots, K. \quad (9)$$

In this model, the second stage treats the m th desirable output and j th undesirable output as constants $\sum_{k=1}^K \rho^{k*} v_m^k$ and $\theta_j^* w_j^0$, respectively. These constants are the optimal output values of the first stage of DMU_o . So, the right hand side in the first two constraints in (9) is maintaining the efficiency of the stage 1. It should be pointed out that a system is efficient if and only if the two component processes are efficient.

3.2. Centralized model

In real world situations, there are many cases that sub-DMUs cooperate together to achieve the overall performance of the whole system. For example, marketing and production departments work together to maximize the company's profit. This section addresses the centralized approach to evaluate the relative performance of the two-stage structures. In Our approach, the two-stage process is viewed as one stage where the two stages jointly determine one optimal plan to maximize the total efficiency of the whole system. The intermediate measure v is a bad output and it should be abated in both stages. However, the intermediate measure v is a good output of the first stage that is used as input to the second stage. At a rational sight, it might appear that v should be increased in the first stage and against it should be decreased in the second stage. Regarding the good outputs v , two rational treatments exist: (i) one can leave it unchanged, because it must increase from one side and simultaneously; must be reduced from the other side, (ii) the intermediate measure v is a good product of the system that can be used by the system itself. So, it seems to be rational to the whole system to increase the desirable output v . In our approach to two-stage procedure evaluation, we have used the second approach. If we want to measure the efficiency of DMU_o in terms of abatement potential in undesirable outputs and the reduction potential in inputs, we can solve the following linear programming problem:

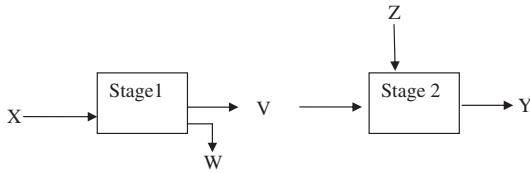


Fig. 2. Two-stage process for DMU_k.

$$Min e_o = \frac{1}{2} \left[\frac{1}{N+J} \left[\sum_{n=1}^N \beta_n + \sum_{j=1}^J \theta_j \right] + \frac{1}{J+T} \left[\sum_{j=1}^J \theta_j + \sum_{t=1}^T \phi_t \right] \right]$$

s.t.

Stage 1 constraints:

$$\sum_{k=1}^K (\rho^k + \mu^k) x_n^k \leq \beta_n x_n^o, \quad n = 1, \dots, N,$$

$$\sum_{k=1}^K \rho^k v_m^k \geq v_m^o, \quad m = 1, \dots, M,$$

$$\sum_{k=1}^K \rho^k w_j^k = \theta_j w_j^o, \quad j = 1, \dots, J,$$

Stage 2 constraints:

$$\sum_{k=1}^K \rho^k v_m^k \geq v_m^o, \quad m = 1, \dots, M,$$

$$\sum_{k=1}^K \rho^k w_j^k = \theta_j w_j^o, \quad j = 1, \dots, J,$$

$$\sum_{k=1}^K (\rho^k + \mu^k) z_t^k \leq \phi_t z_t^o, \quad t = 1, \dots, T,$$

$$\sum_{k=1}^K (\rho^k + \mu^k) y_r^k \geq y_r^o, \quad r = 1, \dots, S,$$

generic constraints

$$\sum_{k=1}^K (\rho^k + \mu^k) = 1,$$

$$\rho^k, \mu^k \geq 0, \quad k = 1, \dots, K,$$

$$0 \leq \theta_j, \beta_n, \phi_t \leq 1, \quad \text{for all } j, n, t.$$

In the formulation above, the constraints $0 \leq \theta_j, \beta_n, \phi_t \leq 1$ are the requirements for dominance. The objective function can be decomposed into two terms: the first term

Table 1
Data set for Spanish airports.

Airport	x ₁	x ₂	x ₃	v ₁	w ₁	w ₂	z ₁	z ₂	y ₁	y ₂
A Coruna	87,300	5	4	17.719	1218	23783.4	10	3	1174.970	283.571
Albacete	162,000	2	2	2.113	58	1376.5	4	1	19.254	8.924
Alicante	135,000	31	16	81.097	7642	142445.8	42	9	9678.304	5982.313
Almeria	144,000	15	5	18.280	1114	20149.1	17	4	1024.303	21.322
Asturias	99,000	7	9	18.371	1310	23893.5	11	3	1530.245	139.465
Badajoz	171,000	1	2	4.033	137	2365.4	4	1	81.010	0
Barcelona	475,000	121	65	321.693	33,036	645924.6	143	19	30272.084	103996.489
Bilbao	207,000	21	12	61.682	4592	80848.2	36	7	4172.903	3178.758
Cordoba	62,100	23	1	9.604	14	254.4	1	0	22.230	0
El Hierro	37,500	3	2	4.775	27	641.6	5	1	195.425	171.717
Fuerteventura	153,000	34	10	44.552	3920	72179.7	34	8	4492.003	2722.661
Girona-Costa Brava	108,000	17	7	49.927	4992	100305.6	18	3	5510.970	184.127
Gran Canaria	139,500	55	38	116.252	7463	136380.7	86	19	10212.123	33695.248
Granada-Jaen	134,550	11	3	19.279	951	17868.8	12	3	1422.014	66.889
Ibiza	126,000	25	12	57.233	6193	152840.1	48	8	4647.360	3928.387
Jerez	103,500	9	5	50.551	1174	19292.2	13	3	1303.817	90.428
La Gomera	45,000	3	2	3.393	17	420.7	5	1	41.890	7.863
La Palma	99,000	5	5	20.109	423	8286.0	13	2	1151.357	1277.264
Lanzarote	108,000	24	16	53.375	5104	101685.6	49	8	5438.178	5429.589
Leon	94,500	5	2	5.705	442	7191.5	3	1	123.183	15.979
Madrid Barajas	927,000	263	230	469.746	52,526	908360.0	484	53	50846.494	329186.631
Malaga	144,000	43	30	119.821	15,548	277663.8	85	16	12813.472	4800.271
Melilla	64,260	5	2	10.959	218	2979.6	4	1	314.643	386.340
Murcia	138,000	5	5	19.339	1344	24103.1	18	4	1876.255	2.730
Palma de Mallorca	295,650	86	68	193.379	26,038	501486.0	204	16	22832.857	21395.791
Pamplona	99,315	7	2	12.971	666	11691.8	4	1	434.477	52.942
Reus	110,475	5	5	26.676	943	18240.8	8	3	1278.074	119.848
Salamanca	150,000	6	2	12.450	427	6626.1	4	2	60.103	0
San Sebastian	78,930	6	3	12.282	713	11184.0	6	2	403.191	63.791
Santander	104,400	8	5	19.198	1004	17842.0	8	2	856.606	37.482
Santiago	14,400	16	12	21.945	2007	34322.3	19	5	1917.466	2418.798
Saragossa	302,310	12	3	14.584	1095	19547.6	6	2	594.952	21438.894
Seville	151,200	23	10	65.067	2567	51084.9	42	6	4392.148	6102.264
Tenerife North	153,000	16	16	67.800	1783	32637.0	37	5	4236.615	20781.674
Tenerife South	144,000	44	22	60.779	5254	110818.9	87	14	8251.989	8567.093
Valencia	144,000	35	18	96.795	4998	102719.2	42	8	5779.343	13325.799
Valladolid	180,000	7	5	13.002	843	14760.6	8	2	479.689	34.650
Vigo	108,000	8	6	17.934	1535	25593.6	12	3	1278.762	1481.939
Vitoria	157,500	18	3	12.225	669	11585.8	7	2	67.818	34989.727

$\frac{1}{N+J} \left[\sum_{n=1}^N \beta_n + \sum_{j=1}^J \theta_j \right]$ is the Russell-input/bad output measure of efficiency of the first stage and the second one, $\frac{1}{J+T} \left[\sum_{j=1}^J \theta_j + \sum_{t=1}^T \phi_t \right]$ is the measure for the second stage.

3.3. Undesirable final outputs

This section addresses the centralized approach to evaluate the performance of the two stages when undesirable outputs are characterized as final outputs. As Fig. 2 shows the desirable intermediate measure v of the first stage is consumed for the second stage. The undesirable intermediate measure w leaves the system in this stage. Hence, in this approach one wants improving the efficiency of the first stage through increasing outputs, will affect the efficiency of the second stage. In other words, we left the undesirable intermediate measure as final output and did not consider them as the input for the second stage.

This would be the case in situations such as manufacturer-retailer relationship in a supply chain management and modeling airport operations. Using DEA for modeling this situation not only reduces the inputs and increase

the desirable outputs but also reduces the undesirable outputs. Therefore, in the discussion to follow, a version of centralized network DEA model under variable return to scale and weak disposability property for final outputs of the first stage can be written as follows:

$$\text{Min } e_o = \frac{1}{2} \left[\frac{1}{N+J} \left[\sum_{n=1}^N \beta_n + \sum_{j=1}^J \theta_j \right] + \frac{1}{J} \left[\sum_{t=1}^T \phi_t \right] \right]$$

s.t.

Stage 1 constraints:

$$\begin{aligned} \sum_{k=1}^K (\rho^k + \mu^k) x_n^k &\leq \beta_n x_n^o, & n = 1, \dots, N \\ \sum_{k=1}^K \rho^k v_m^k &\geq v_m^o, & m = 1, \dots, M \\ \sum_{k=1}^K \rho^k w_j^k &= \theta_j w_j^o, & j = 1, \dots, J \end{aligned} \tag{11}$$

Table 2 Efficiency score and projection for Stage 1.

Airport	e_o^{1*}	v_1^*	w_1^*	w_2^*	x_1^*	x_2^*	x_3^*
A Coruna	0.3251	17.7190	395.9391	7731.3445	78375.8286	5.000	4.000
Albacete	0.2014	2.1130	11.6833	277.2762	112666.8227	2.000	1.9939
Alicante	0.5564	81.0970	4242.3424	79263.3603	135000.000	25.7824	16.000
Almeria	0.2688	18.2800	299.4928	5416.9756	80251.9378	15.000	2.9294
Asturias	0.2728	18.3710	357.4269	6519.2202	76282.5283	7.000	3.6089
Badajoz	1	4.0330	137.0000	2365.4000	171000.000	1.000	2.000
Barcelona	1	321.6930	33036.000	645924.6000	475020.0000	121.000	65.0000
Bilbao	0.3730	61.6820	1713.0203	30159.9749	133926.9648	13.8523	11.9293
Cordoba	1	9.6040	14.0000	254.4000	62100.000	23.000	1.000
El Hierro	1	4.07750	27.0000	641.6000	37500.000	3.000	2.000
Fuerteventura	0.2782	44.5520	1090.7215	20083.6616	116641.8814	18.8145	10.000
Girona-Costa Brava	0.3481	49.9270	1737.7146	34916.3675	108000.000	12.6857	6.6387
Gran Canaria	1	116.2520	7463.0000	136380.7000	139500.000	55.000	38.000
Granada-Jaen	0.4368	19.2790	415.3811	7804.7963	71154.5099	11.000	3.000
Ibiza	1	57.2330	6193.000	152840.1000	126000.000	25.000	12.000
Jerez	1	50.5510	1174.0000	19292.2000	103500.000	9.000	5.000
La Gomera	1	3.3930	17.0000	420.7000	45000.000	3.000	2.000
La Palma	1	20.1090	423.0000	8286.0000	99000.000	5.000	5.000
Lanzarote	0.3736	53.3750	1906.9521	37991.6874	108000.000	11.9547	6.3978
Leon	0.1344	5.7050	59.4138	966.6842	66542.6079	5.000	1.9039
Madrid Barajas	1	469.7460	52526.0000	903860.0000	927000.000	263.000	230.000
Malaga	1	119.8210	15548.0000	277663.8000	144000.000	43.000	30.000
Melilla	1	10.9590	218.000	2979.6000	64260.000	5.000	2.000
Murcia	0.2945	19.3390	395.7726	7097.7288	73638.7112	5.000	3.7105
Palma de Mallorca	0.6552	193.3790	17060.9591	328590.2194	267737.2523	78.9153	47.0421
Pamplona	1	12.9710	666.0000	11691.8000	99315.000	7.000	2.000
Reus	1	26.6760	943.0000	18240.8000	110475.000	5.000	5.000
Salamanca	1	12.4500	427.0000	6626.1000	150000.000	6.000	2.000
San Sebastian	0.3076	12.2820	219.3020	3439.9342	57029.6777	6.000	2.2220
Santander	0.3691	19.1980	370.6255	6586.3551	73730.1553	8.000	3.3718
Santiago	0.1918	21.9450	384.9760	6583.5885	79197.0276	16.000	2.7566
Saragossa	0.2023	14.5840	221.5452	3954.9568	65860.7582	12	2.4826
Seville	0.9956	65.0670	2555.6539	50859.1052	120646.3647	17.6125	10
Tenerife North	1	67.8000	1783.0000	32637.0000	153000.000	16.000	16.000
Tenerife South	0.4486	60.7790	2357.0339	49715.2472	140286.7337	18.1309	13.9779
Valencia	1	96.7950	4998.0000	102719.200	144000.000	35.000	18.000
Valladolid	0.2536	13.0020	213.8123	3743.7702	54999.4493	7.000	2.4913
Vigo	0.2189	17.9340	336.0528	5603.1270	60209.1849	8.000	2.6252
Vitoria	0.1751	12.2250	117.1179	2028.2575	62601.9937	18.000	1.6155

Stage 2 constraints:

$$\sum_{k=1}^K \rho^k v_m^k \geq v_m^o, \quad m = 1, \dots, M$$

$$\sum_{k=1}^K (\rho^k + \mu^k) z_t^k \leq \phi_t z_t^o, \quad t = 1, \dots, T$$

$$\sum_{k=1}^K (\rho^k + \mu^k) y_r^k \geq y_r^o, \quad r = 1, \dots, S$$

generic constraints

$$\sum_{k=1}^K (\rho^k + \mu^k) = 1,$$

$$\rho^k, \mu^k \geq 0, \quad k = 1, \dots, K$$

$$0 \leq \theta_j, \beta_n, \phi_t \leq 1, \quad \text{for all } j, n, t.$$

In the formulation above, the constraints $0 \leq \theta_j, \beta_n, \phi_t \leq 1$ are the requirements for dominance. The objective function can be decomposed into average of

two terms: the first term $\frac{1}{N+J} [\sum_{n=1}^N \beta_n + \sum_{j=1}^J \theta_j]$, is the Russell-input /undesirable output measure of efficiency of the first stage and the second one, $\frac{1}{T} [\sum_{t=1}^T \phi_t]$, is the Russell input measure for the second stage.

To highlight the practical implication of the proposed approaches, we apply the procedures to a real case consisting of 39 Spanish airports in 2008 taken from Lozano et al. [18].

4. An illustrative application

After formulating our methodological framework, it has been illustrated through empirical analysis. In efficiency analysis using parametric and nonparametric techniques, the airports efficiency has long been a primary interest of research due to its socio-economic significance. (See Zhu [19], Gillen and Lall [20] and Yu et al. [21]). The existing DEA studies on airport benchmarking consider an airport as a single process. A slack-based network DEA approach has been proposed by Yu et al. [21] but it does not consider undesirable outputs. As Lozano et al. [18] stated considering undesirable outputs not only increase the relation of

Table 3
Efficiency scores and projections for Stage 2.

Airport	$e_o^{(2)}$	z_1^*	z_2^*	y_1^*	y_2^*
A Coruna	0.4607	4.6072	0.6493	1174.97	283.571
Albacete	0.2504	1.0015	0.0005	22.2416	8.924
Alicante	0.9174	38.5329	5.6282	9578.304	17236.65
Almeria	0.2611	4.4384	0.6436	1024.303	35.593
Asturias	0.5167	5.6841	0.8312	1530.245	139.465
Badajoz	0.2955	1.1821	0.0321	81.01	1.9719
Barcelona	1	143	19	30272.08	103996.5
Bilbao	0.5672	20.4197	3.6713	4172.903	4486.831
Cordoba	1	1	0	22.23	0
El Hierro	0.3123	1.5614	0.1077	195.425	171.717
Fuerteventura	0.4631	15.745	2.7926	4492.003	2722.661
Girona-Costa Brava	1	18	3	5510.97	184.127
Gran Canaria	0.5619	48.324	7.1008	10212.12	33695.25
Granada-Jaen	0.4449	5.3385	0.7667	1422.014	66.889
Ibiza	0.4165	19.9924	3.3321	4647.36	4992.297
Jerez	1	13	3	1303.817	90.428
La Gomera	0.2124	1.062	0.0113	41.89	7.863
La Palma	0.406	5.2782	0.812	1151.357	1277.264
Lanzarote	0.4055	19.8717	3.2444	5438.178	5429.589
Leon	0.4382	1.3146	0.0562	123.183	15.979
Madrid Barajas	1	484	53	50846.49	329186.6
Malaga	0.6455	54.8648	7.7187	12813.47	30800.28
Melilla	0.4906	1.9624	0.1895	314.643	386.34
Murcia	0.3746	6.7424	1.0134	1876.255	62.1957
Palma de Mallorca	0.8871	105.445	14.193	22832.86	72807.11
Pamplona	0.5921	2.3685	0.2529	434.477	52.942
Reus	0.8622	6.8972	1.2636	1278.074	119.848
Salamanca	0.4585	1.8341	0.2085	111.306	6.2852
San Sebastian	0.3646	2.1876	0.2122	403.191	63.791
Santander	0.5584	4.4672	0.709	856.606	37.482
Santiago	0.3803	7.2248	1.2214	1917.466	2418.798
Saragossa	1	6	2	594.952	21438.89
Seville	0.5843	24.5426	3.5061	4711.957	15021.02
Tenerife North	0.7397	27.3681	3.6984	5474.498	20781.67
Tenerife South	0.3659	31.8373	4.7712	8251.989	11.676
Valencia	0.8374	35.1719	5.7288	6244.433	17811.89
Valladolid	0.3036	2.4291	0.2542	479.689	34.65
Vigo	0.4257	5.1086	0.8002	1278.762	1481.939
Vitoria	1	7	2	67.818	34989.73

Table 4

Total efficiency along with stages scores and dominance factors.

Airport	$e_o^{(total)}$	$e_o^{(1)}$	$e_o^{(2)}$	β_1	β_2	β_3	θ_1	θ_2	ϕ_2	ϕ_1
A Coruna	0.7654	0.8383	0.6925	1	1	1	0.6	0.59	0.82	0.76
Albacete	0.7698	0.7858	0.7537	0.91	1	0.99	0.65	0.38	1	0.99
Alicante	1	1	1	1	1	1	1	1	1	1
Almeria	0.4154	0.4573	0.3735	0.53	0.47	0.62	0.36	0.3	0.43	0.4
Asturias	0.5718	0.5764	0.5671	0.82	1	0.48	0.3	0.29	1	0.68
Badajoz	1	1	1	1	1	1	1	1	1	1
Barcelona	1	1	1	1	1	1	1	1	1	1
Bilbao	0.6734	0.7017	0.6451	0.65	0.8	1	0.52	0.54	0.81	0.71
Cordoba	0.875	1	0.75	1	1	1	1	1	1	0
El Hierro	1	1	1	1	1	1	1	1	1	1
Fuerteventura	0.5945	0.6513	0.5376	0.84	0.56	1	0.44	0.42	0.72	0.58
Girona-Costa Brava	1	1	1	1	1	1	1	1	1	1
Gran Canaria	1	1	1	1	1	1	1	1	1	1
Granada-Jaen	1	1	1	1	1	1	1	1	1	1
Ibiza	0.5885	0.68	0.4971	1	0.69	0.92	0.45	0.33	0.57	0.63
Jerez	1	1	1	1	1	1	1	1	1	1
La Gomera	1	1	1	1	1	1	1	1	1	1
La Palma	1	1	1	1	1	1	1	1	1	1
Lanzarote	0.7394	0.7903	0.6884	1	0.8	0.68	0.76	0.71	0.57	0.71
Leon	1	1	1	1	1	1	1	1	1	1
Madrid Barajas	1	1	1	1	1	1	1	1	1	1
Malaga	1	1	1	1	1	1	1	1	1	1
Melilla	1	1	1	1	1	1	1	1	1	1
Murcia	1	1	1	1	1	1	1	1	1	1
Palma de Mallorca	1	1	1	1	1	1	1	1	1	1
Pamplona	1	1	1	1	1	1	1	1	1	1
Reus	1	1	1	1	1	1	1	1	1	1
Salamanca	1	1	1	1	1	1	1	1	1	1
San Sebastian	0.5552	0.6206	0.4897	0.84	0.88	0.72	0.35	0.32	0.74	0.55
Santander	0.5968	0.5866	0.607	0.73	0.83	0.6	0.42	0.36	0.86	0.79
Santiago	0.4022	0.4279	0.3764	0.66	0.64	0.36	0.25	0.23	0.57	0.45
Saragossa	1	1	1	1	1	1	1	1	1	1
Seville	1	1	1	1	1	1	1	1	1	1
Tenerife North	1	1	1	1	1	1	1	1	1	1
Tenerife South	0.7181	0.7946	0.6415	1	0.75	1	0.65	0.57	0.6	0.74
Valencia	1	1	1	1	1	1	1	1	1	1
Valladolid	0.439	0.4384	0.4396	0.37	0.78	0.46	0.32	0.26	0.6	0.58
Vigo	0.5056	0.5652	0.446	0.75	1	0.58	0.26	0.24	0.69	0.6
Vitoria	1	1	1	1	1	1	1	1	1	1

the analysis but also it leads to a fairer performance assessments. The motivation of this study is the application of weak disposability in modeling network DEA with undesirable intermediate measures. To gain further insight, we apply the proposed approaches on a real case consisting of 39 Spanish airports taken from Lozano et al. [18]. Table 1 reports the data set. Similar to the previous works on airports performance evaluations, the processes of airport are divided into two stages: aircraft movement process and aircraft loading process. There are three inputs to the first stage characterized by total runway-area (x_1), apron capacity (x_2) and number of boarding gates (x_3). The two outputs of the second stage are reported by annual passenger movement (y_1) and cargo landed (y_2). The two additional inputs for the second stage are characterized as number of baggage belts (z_1) and number of check-in counters (z_2). The desirable intermediate measure is aircraft traffic movement (v_1), and finally, the two other undesirable intermediate measures are recorded as the number of delayed flights (w_1) and accumulated flight delays (w_2). As Lozano et al. [18] argued, these intermediate measures (w_1) and (w_2) are considered as first stage's final outputs. The important point of this study and the main differences

of our model with those that proposed previously, is that we believe that the number of delayed flights and accumulated flight delays affect directly on the second stage (Aircraft loading process), we have used these undesirable outputs as inputs to the second stage. To see how weak disposability assumption influences on the two-stage network structure, both non-cooperative and cooperative game approaches are considered. We first applied the leader-follower game approach proposed in Section 3.2 to this data set and we assumed that the aircraft movement process is leader. Table 2 shows the optimal scores of the first stage obtained from our proposed non-cooperative (leader-follower) approach together with the optimal values for the target points. The same results for the second stage are reported in Table 3.

As Table 2 shows, when the aircraft movement process is leader, 17 airports are efficient in the first stage. Keeping these efficient airports in the second stage, seven airports are efficient in the second stage. Looking at the second columns of Tables 2 and 3, one can find that only four airports are efficient in aggregate sense (Airports Barcelona, Cordoba, Jerez and Madrid). Referring to the columns four and five in Table 2, we see that the mean reductions of

Table 5
Projections points in centralized model.

Airport	x_1^c	x_2^c	x_3^c	v_1^c	w_1^c	w_2^c	z_1^c	z_2^c	y_1^c	y_2^c
A Coruna	87,300	5	4	17.72	727.78	14122.09	8.23	2.27	1174.97	283.57
Albacete	147138.4	2	1.99	2.11	37.51	523.12	4	0.99	123.38	70.59
Alicante	135,000	31	16	81.1	7624	142445.8	42	9	9578.3	5982.31
Almeria	76773.82	7	3.12	18.28	399.26	6143.86	7.25	1.62	1024.3	304.26
Asturias	80844.01	7	4.3	18.37	390.88	6914.48	11	2.04	1530.24	472.33
Badajoz	171,000	1	2	4.03	137	2365.4	4	1	81.01	0
Barcelona	475,020	121	65	321.69	33,036	645924.6	143	19	30272.08	103996.5
Bilbao	133543.2	16.77	12	61.68	2410.8	43649.86	29.04	4.96	4172.9	10836.53
Cordoba	62,100	23	1	9.6	14	254.4	1	0	22.23	0
El Hierro	37,500	3	2	4.78	27	641.6	5	1	195.43	171.72
Fuerteventura	129137.3	19.07	10	44.55	1706.24	30046.04	24.53	4.62	4492	6765.66
Girona-Costa Brava	108,000	17	7	49.93	4992	100305.6	18	3	5510.97	184.13
Gran Canaria	139,500	55	38	116.25	7463	136380.7	86	19	10212.12	33695.25
Granada-Jaen	134,550	11	3	19.28	951	17868.8	12	3	1422.01	66.89
Ibiza	126,000	17.3	11.09	57.23	2788.87	50977.44	27.58	5.04	4647.36	7518.65
Jerez	103,500	9	5	50.55	1174	19292.2	13	3	1303.82	90.43
La Gomera	45,000	3	2	3.39	17	420.7	5	1	41.89	7.86
La Palma	99,000	5	5	20.11	423	8286	13	2	1151.36	1277.26
Lanzarote	108,000	19.12	10.94	53.38	3885.92	72199.4	28.05	5.68	5438.18	5429.59
Leon	94,500	5	2	5.7	442	7191.5	3	1	123.18	15.98
Madrid Barajas	927,000	263	230	469.75	52,526	908360	484	53	50846.49	329186.6
Malaga	144,000	43	30	119.82	15,548	277663.8	85	16	12813.47	4800.27
Melilla	64,260	5	2	10.96	218	2979.6	4	1	314.64	386.34
Murcia	138,000	5	5	19.34	1344	24103.1	18	4	1876.26	2.73
Palma de Mallorca	295,650	86	68	193.38	26,038	501486	204	16	22832.86	21395.79
Pamplona	99,315	7	2	12.97	666	11691.8	4	1	434.48	52.94
Reus	110,475	5	5	26.68	943	18240.8	8	3	1278.07	119.85
Salamanca	150,000	6	2	12.45	427	6626.1	4	2	60.1	0
San Sebastian	66125.17	5.27	2.16	12.28	250.42	3540.28	4.47	1.09	403.19	373.57
Santander	75,780.8	6.64	2.98	19.2	419.81	6468.58	6.89	1.57	856.61	307.3
Santiago	94,974.94	10.18	4.36	21.94	497.53	7975.6	10.84	2.27	1917.47	2418.8
Saragossa	302,310	12	3	14.58	1095	19547.6	6	2	594.95	21438.89
Seville	151,200	23	10	65.07	2567	51084.9	42	6	4392.15	6102.26
Tenerife North	153,000	16	16	67.8	1783	32,637	37	5	4236.62	20781.67
Tenerife South	144,000	33.21	22	60.78	3419.17	62889.89	52.49	10.42	8251.99	18311.92
Valencia	144,000	35	18	96.8	4998	102719.2	42	8	5779.34	13325.8
Valladolid	67,422.79	5.48	2.28	13	268.3	3853.36	4.81	1.16	479.69	365.15
Vigo	81,426.74	8	3.46	17.93	392.26	6140.41	8.29	1.79	1278.76	1481.94
Vitoria	157,500	18	3	12.23	669	11585.8	7	2	67.82	34989.73

the first and second undesirable outputs are respectively 973.05 and 18441.61, respectively.

We have also applied the centralized model (10) to the airport data with the results reported in Tables 4 and 5. The first three columns of Table 4 report the total efficiency score along with the stages' scores. The last seven columns in this table show the dominance factors. As the table shows, of 39 airports, 24 airports are fully efficient. Testing the columns six and seven in Table 1, four and five in Table 2 and six and seven in Table 5, we find out that the mean reductions of the first and second undesirable outputs in non-cooperative approach is substantially greater than the mean reductions in cooperative approach. This means that in this example the non-cooperative approach will substantially reduce the bad outputs.

5. Conclusions

Performance analysis in two-stage network structures has recently attracted considerable attention among DEA researchers. The existing studies on network DEA either do not consider the existence of undesirable products in the processes or they did not use the weak disposability

assumption in handling undesirable factors. When the intermediate measures in two-stage processes consist of desirable and undesirable, the existing approaches in network DEA cannot provide a good estimation of the efficiency. This paper introduces a two-stage DEA approach to analyze the performance of these processes with undesirable intermediate measures. Two different two-stage structures have been considered and in each structure, two different cases (one with bad outputs as final outputs and another with bad outputs as intermediate measures) are considered. The contribution of this paper is to apply the weak disposability assumption in a two-stage process in the presence of desirable and undesirable outputs. The approach was illustrated with a real data set on 39 Spanish Airports in 2008.

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