Positioning Based on Signals of Opportunity

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***Abstract*—A novel approach to opportunistic positioning based on timing measurements is presented. The asynchronous radio transmissions from fixed stations and from mobile GPS-equipped nodes are jointly exploited to cooperatively localize a blind node. To this end, a weighted least-squares (WLS) estimator is proposed; its effectiveness is illustrated via simulations in a realistic scenario.**

***Index Terms*—Opportunistic positioning, localization, time difference of arrival (TDOA), asynchronous systems, weighted least-squares (WLS).**

I. Introduction

MANY modern devices are equipped with GPS receiver and therefore are able to identify their position most of the time. However, GPS signals are unavailable in areas without good sky visibility, like indoor or urban canyons. Also, certain systems, e.g., wireless sensor networks, might involve devices without GPS radio due to cost and/or size constraints. This motivates the interest for radio positioning techniques based on terrestrial radio signals. In order to be viable from a cost perspective, such solutions should not require the deployment of new infrastructure but rather reuse *opportunistically* the radio signals transmitted by fixed stations of legacy systems (e.g., broadcasting towers, cellular base stations, WiFi access points). In the ideal scenario where the clocks of all transmitting stations were accurately synchronized, as in satellite-based navigation systems, the standard GPS method could be applied directly, resulting in a sort of "terrestrial GPS-like" system. In practice, transmitters and receivers have independent clocks, hence they are not synchronized. However, some recent papers have started to pioneer a new class of localization techniques for the asynchronous case typically based on time-difference of arrival (TDOA) measurements [1], [2], [3], [4], [5], [6].

Motivated by this framework, we explore the idea of identifying the position of a GPS-less node — hereafter referred to as "blind node" — based on timing measurements by leveraging cooperation with other GPS-enabled mobile terminals (called "helpers"), possibly in addition to signals from fixed transmitters (called "stations") if available. The position scheme we propose can be classified as *asynchronous* and *opportunistic* since it does not require any clock synchronization between nodes and, in addition, performs timing measurements on legacy signals. The idea of opportunistically exploiting terrestrial signals transmitted from fixed stations is not entirely new and has been investigated in [1], [2], [3], [4], [7], [8], [9], often in combination with TDOA or differential-TDOA (DTDOA) methods. One key novelty of our approach lies in the exploitation of "horizontal" cooperation between the mobile nodes (helpers) in addition to "vertical" measurements from the terrestrial infrastructure (stations). Another important element of novelty is that we formulate and solve in closed-form the ranging estimation problem in presence of asynchronous transmissions and uncertainty in the reference node position.

II. Problem Formulation

Reference scenario. We consider three types of nodes. *Stations*: fixed non-cooperative transmit-only nodes whose position is known accurately; *helpers*: mobile cooperative nodes capable of acting as receivers and/or transmitters for timing measurements, whose position is known only approximately, i.e., with an error comparable with that of a commercial GPS module; *blind node*: the node whose position must be determined. We consider a fully centralized scheme, with a central server gathering all data from the nodes (including the blind node) and performing the computation. It is quite straightforward to adapt the basic problem formulation to other (semi-)distributed communication schemes.

The determination of the blind node position involves two distinct stages: pseudo-range[1] estimation and position estimation. In the first stage, timing measurements between various participating nodes, (exact) positions of stations and (non-exact) positions of helpers serve as input for the estimation of the pseudo-ranges between the blind node and a subset of the nodes, located within the reception range of the blind node, that will be referred to as "anchors". In the second stage, the pseudo-range estimates are used as input for determining the position of the blind node. We restrict our attention to planar (2D) localization where the minimum number of anchors required to solve the positioning problem is three.

In the following, we denote by $\mathcal{S}$, $\mathcal{H}$, and $\mathcal{A} \subseteq \mathcal{S} \cup \mathcal{H}$ the set of integers indexing fixed stations, helpers, and nodes serving as anchors, respectively. Actually, the set of anchors is a selected subset of stations and transmitting helpers located within the reception range of the blind node. The need for this selection idea will be better clarified in Section III. Considering the transmission of a reference signal[2] from

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[1]The term "pseudo-ranges" identifies a set of ranges, i.e., distances, determined up to an additive constant due to an unknown common term.

[2]In order to perform TDOA measurements opportunistically it is sufficient to identify, for each transmitting node, a reference pattern within the transmitted signals that can be conveniently recognized by all receivers. This is straightforward in practice since wireless protocols embed a variety of well-known patterns such as training sequences, preambles, delimiters and alike.

transmitting node i (station or helper) to receiving node k (helper or blind), we introduce the following quantities:

- t_i is the (unknown) transmission time of the reference signal from transmitter i according to the absolute reference clock.
- r_i^k (resp., r_i^*) is the measured time when the reference signal transmitted by node i was received at helper node $k \in \mathcal{H}$ (at the blind node, respectively, denoted by *).

Without loss of generality we take the clock of the blind node as the absolute reference clock. The (unknown) position of the blind node is denoted by $\boldsymbol{p}^* \stackrel{\text{def}}{=} (x^*, y^*)$. For a generic node $i \in \mathcal{S} \cup \mathcal{H}$ we denote by $\boldsymbol{p}_i \stackrel{\text{def}}{=} (x_i, y_i)$ its actual position and by $d_i^* \stackrel{\text{def}}{=} \|\boldsymbol{p}_i - \boldsymbol{p}^*\|$ the distance from the blind node, being $\|\boldsymbol{z}\|$ the Euclidean norm of the vector $\boldsymbol{z} \in \mathbb{R}^{n\times 1}$. For a pair of nodes $i, k \in \mathcal{S} \cup \mathcal{H}$, $d_{ik} \stackrel{\text{def}}{=} \|\boldsymbol{p}_k - \boldsymbol{p}_i\|$ denotes their distance.

Equations. The estimation problem (first stage) involves two types of equations, depending on whether the receiver node is the blind node or an helper. At the design stage, we assume a single-path line-of-sight (LOS) propagation model [10]. Notice that, even in presence of multipath propagation, the single-path model is consistent with the conventional approach to geolocation based on first-arriving components. Considering the blind node as receiver and a generic anchor i (station or helper) as transmitter, we write (Type I equation):

$$r_i^* = t_i + \frac{d_i^*}{c} + \epsilon_i^*, \quad i \in \mathcal{A} \tag{1}$$

where c is the speed of light and ϵ_i^* denotes a Gaussian measurement error with variance $\sigma^2_{\epsilon_i^*}$, i.e., $\epsilon_i^* \sim \mathcal{N}(0, \sigma^2_{\epsilon_i^*})$.

Considering a generic helper k as receiver and a pair of transmitters i, j (stations or other helpers) within its reception range, we can write (Type II equation):

$$r_i^k - r_j^k = t_i - t_j + \frac{d_{ik} - d_{jk}}{c} + \epsilon_i^k - \epsilon_j^k, \quad k \in \mathcal{H},\ i, j \in \mathcal{S} \cup \mathcal{H} \tag{2}$$

where again $\epsilon_i^k \sim \mathcal{N}(0, \sigma^2_{\epsilon_i^k})$ and $\epsilon_j^k \sim \mathcal{N}(0, \sigma^2_{\epsilon_j^k})$ are timing measurement errors that we model as independent random variables (rvs). The variance of the generic measurement error depends on several factors as, for instance, the characteristics of the transmitted signal, namely bandwidth and duration, and the signal-to-noise ratio (SNR) at the receiver. Again, it is useful to select a subset of the triplets (i, j, k), $i, j \in \mathcal{S} \cup \mathcal{H}$ and $k \in \mathcal{H}$, that can give rise to Type II equations (a point better explained in Section III). Finally, notice that the above equations do not include any clock offset term, because in eq. (2) the time difference $r_i^k - r_j^k$ is measured at the common receiving node k, while in eq. (1) the absolute times r_i^* and t_i are referred to the same reference clock of the blind node.

Characterization of errors. A major difference between Type I and Type II equations is that the distances d_i^* are unknown variables, while d_{ik} and d_{jk} can be calculated from the coordinates of stations and helpers. However, the position of each helper k is known with some uncertainty (e.g., from an onboard GPS unit) as $\boldsymbol{p}_k + \boldsymbol{v}_k$, where $\boldsymbol{v}_k \sim \mathcal{N}_2(\boldsymbol{0}, \sigma^2_{v_k}\boldsymbol{I}_2)$ is the position error and $\boldsymbol{I}_2$ denotes the two-dimensional identity matrix. In general, the variance $\sigma^2_{v_k}$ depends on k since helpers may be equipped with GPS receivers of different accuracy. The calculated distance between helper k and station i is:

$$\hat{d}_{ik} = \|(\boldsymbol{p}_i - \boldsymbol{p}_k) - \boldsymbol{v}_k\|$$

that is, $\hat{d}_{ik}$ is distributed as the norm of $\mathcal{N}_2(\boldsymbol{p}_i - \boldsymbol{p}_k, \sigma^2_{v_k}\boldsymbol{I}_2)$. Since the distances between helpers and stations are typically large the following approximation holds[3]:

$$\hat{d}_{ik} \approx \|\boldsymbol{p}_i - \boldsymbol{p}_k\| - \boldsymbol{s}_{ik}^T\boldsymbol{v}_k = d_{ik} - \boldsymbol{s}_{ik}^T\boldsymbol{v}_k$$

where $\boldsymbol{s}_{ik} \stackrel{\text{def}}{=} \frac{\boldsymbol{p}_i - \boldsymbol{p}_k}{\|\boldsymbol{p}_i - \boldsymbol{p}_k\|} \in \mathbb{R}^{2\times 1}$ denotes the versor along the line between nodes i and k and ^{T} transpose. Similarly, we suppose that, $\forall i, k \in \mathcal{H}$,

$$\hat{d}_{ik} = \|\boldsymbol{p}_i - \boldsymbol{p}_k + \boldsymbol{v}_i - \boldsymbol{v}_k\| \approx d_{ik} - \boldsymbol{s}_{ik}^T(\boldsymbol{v}_i - \boldsymbol{v}_k)$$

Moreover, Type II equations can be rewritten in terms of the calculated distances $\hat{d}_{ik}$ and $\hat{d}_{jk}$ as

$$r_i^k - r_j^k = t_i - t_j + \frac{\hat{d}_{ik} - \hat{d}_{jk}}{c} + w_{ij}^k, \quad k \in \mathcal{H},\ i, j \in \mathcal{S} \cup \mathcal{H} \tag{3}$$

where w_{ij}^k is a cumulated error term accounting for the relevant timing measurement errors and helper position errors. The rv w_{ij}^k can be approximated by a zero-mean Gaussian rv; it is also not difficult to show that its variance is given by

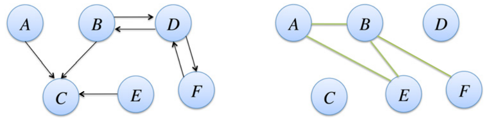
$$(\sigma_{ij}^k)^2 = \begin{cases} \sigma^2_{\epsilon_i^k} + \sigma^2_{\epsilon_j^k} + \frac{\gamma_{i,j}^k \sigma^2_{v_k}}{c^2} & i, j \in \mathcal{S},\ k \in \mathcal{H} \\ \sigma^2_{\epsilon_i^k} + \sigma^2_{\epsilon_j^k} + \frac{\gamma_{i,j}^k \sigma^2_{v_k} + \sigma^2_{v_j}}{c^2} & i \in \mathcal{S},\ j, k \in \mathcal{H} \\ \sigma^2_{\epsilon_i^k} + \sigma^2_{\epsilon_j^k} + \frac{\gamma_{i,j}^k \sigma^2_{v_k} + \sigma^2_{v_j} + \sigma^2_{v_i}}{c^2} & i, j, k \in \mathcal{H} \end{cases}$$

where $\gamma_{i,j}^k \stackrel{\text{def}}{=} 2(1 - \boldsymbol{s}_{ik}^T\boldsymbol{s}_{jk})$ is a geometrical factor in the interval $[0, 4]$. Since the true helper positions are unknown, the corresponding calculated values are actually used in $\gamma_{i,j}^k$.

III. Resolution Approach

In the previous section, we have derived two types of equations: Type I equations where the parameters to be estimated are the pairs (d_i^*, t_i), $i \in \mathcal{A}$, associated with the anchors, and Type II equations that contain the unknowns t_i and t_j associated with a subset of the triplets (i, j, k) that can be used to compute TDOA measurements. We denote by H_t the number of helpers contributing to Type I and/or Type II equations and by H_a the number of helpers serving as anchors; it follows that $H_t - H_a \geq 0$ is the number of helpers contributing to Type II equations only. Moreover, we denote by S_a the number of stations serving as anchors. Finally, we assume, without loss of generality, that $\mathcal{A} = \{a_1, \ldots, a_{S_a+H_a}\} \subset \{1, \ldots, S_a + H_t\}$ and that the transmitting helpers, not serving as anchors, but contributing to Type II equations, are indexed by the elements of the set $\{a_{S_a+H_a+1}, \ldots, a_{S_a+H_t}\} \subset \{1, \ldots, S_a + H_t\}$ with $a_i \neq a_j$, $\forall i \neq j$. It follows that the parameter vector to be estimated is $\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\tau} \end{bmatrix}$ with $\boldsymbol{\delta} \stackrel{\text{def}}{=} [d_{a_1}^* \cdots d_{a_{S_a+H_a}}^*]^T$ and $\boldsymbol{\tau} \stackrel{\text{def}}{=} [ct_1 \cdots ct_{S_a+H_t}]^T$. In $\boldsymbol{\theta}$ the parameters of interest are just the first $S_a + H_a$ (i.e., $\boldsymbol{\delta}$) while the remaining $S_a + H_t$ (i.e., $\boldsymbol{\tau}$) are nuisance parameters. It is also reasonable to assume

[3]Actually, we suppose that $|v_{xk}| \ll |x_i - x_k|$ and $|v_{yk}| \ll |y_i - y_k|$ with v_{xk} and v_{yk} the Cartesian coordinates of $\boldsymbol{v}_k$, x_i (resp., x_k) and y_i (resp., y_k) the Cartesian coordinates of $\boldsymbol{p}_i$ (resp., $\boldsymbol{p}_k$).



(a) communication graph (b) mutual contact graph

Fig. 1. Example of mutual contact graph derived from communication graph.

that the number of available equations, m say, is greater than $2S_a + H_a + H_t$ and solve the following WLS problem:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left\{ \sum_{i \in \mathcal{A}} \frac{(cr_i^* - d_i^* - ct_i)^2}{\sigma^2_{\epsilon_i^*}} + \right.$$
$$\left. + \sum_{i,j \in \mathcal{S}\cup\mathcal{H}} \sum_{k \in \mathcal{H}} \ell_{ij}^k \frac{\left(cr_i^k - cr_j^k - ct_i + ct_j - \hat{d}_{ik} + \hat{d}_{jk}\right)^2}{(\sigma_{ij}^k)^2} \right\} \quad (4)$$

where $\ell_{ij}^k \in \{0,1\}$ is an indicator function that is equal to one if and only if: a) node k receives and processes signals from nodes i, j; b) the triplet (i,j,k) is used to compute a Type II equation. However, the variables in $\boldsymbol{\theta}$ cannot be univocally identified since they always appear in "pairwise differences" in (4). In fact, if $\boldsymbol{\theta}_0 = \begin{bmatrix} \boldsymbol{\delta}_0 \\ \boldsymbol{\tau}_0 \end{bmatrix}$ is a solution of the WLS problem, it is apparent that

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_0 + \mu[-\mathbf{1}^T_{S_a+H_a} \ \mathbf{1}^T_{S_a+H_t}]^T \quad (5)$$

with $\mu \in \mathbb{R}$ and $\mathbf{1}_n$ a vector of ones, is a solution too.

Problem (4) can be restated in more succinct form[4] as

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\boldsymbol{A}\boldsymbol{\theta} - \boldsymbol{b}\|^2 \quad (6)$$

It is well-known that the solution to problem (6) is unique only if the matrix $\boldsymbol{A}$ is a full-column rank one. It turns out that, for the problem at hand, the matrix $\boldsymbol{A}$ cannot be full-column rank. However, it is possible to prove that, by proper selection of Type I and II equations, the rank of $\boldsymbol{A}$ is equal to $2S_a + H_a + H_t - 1$, i.e., $\boldsymbol{A}$ is rank deficient by just one, and hence that the solution is unique up to a known vector multiplied by an unknown constant, namely that (5) describes the overall set of solutions of the problem at hand. The unknown μ can be removed in the position estimation stage, as described later. An alternative approach is to differentiate equations pairwise, if possible, so as to get rid upfront of the nuisance parameter $\boldsymbol{\tau}$. However, such a DTDOA-based approach produces aggregation of the error terms.

The condition that ensures that the rank of $\boldsymbol{A}$ is equal to $2S_a + H_a + H_t - 1$ can be represented more conveniently in terms of graph properties. To illustrate, consider the sample communication graph in Fig. 1(a), where each vertex represents a station or helper node (the blind node is not included in this graph) and a directed edge from one node to another (for instance, from A to C) means that the signal transmitted by the former is received by the latter. From the communication graph of Fig. 1(a) we derive the so called mutual contact graph (MCG) depicted in Fig. 1(b), wherein two generic transmitting nodes (A and B for example) are tied by an (undirected) edge if and only if they have at least one common receiver (C in this case). It is possible to show that, selecting Type I and Type II equations corresponding to nodes that belong to a connected component of the MCG, the matrix $\boldsymbol{A}$ is rank deficient by one or, otherwise stated, that the following Proposition holds.

Proposition 1: Suppose that the selected nodes of the MCG are such that $\forall j \in \{2,\ldots,S_a+H_t\}$ *there exist* $i \in \{1,\ldots,S_a+H_t-1\}$, $i<j$, *tied to* j. *Then, it is possible to construct a matrix* $\boldsymbol{A}$ *for problem (6) whose rank is* $2S_a + H_a + H_t - 1$.

Proof Define the following vectors $i = 1,\ldots,S_a+H_a$

$$\boldsymbol{e}_i^{(1)} \stackrel{\text{def}}{=} \Big[\underbrace{0\cdots0}_{i-1} \ \frac{1}{\sigma_{\epsilon^*_{a_i}}} \ \underbrace{0\cdots0}_{S_a+H_a-i}\Big] \in \mathbb{R}^{1\times(S_a+H_a)}$$
$$\boldsymbol{e}_i^{(2)} \stackrel{\text{def}}{=} \Big[\underbrace{0\cdots0}_{a_i-1} \ \frac{1}{\sigma_{\epsilon^*_{a_i}}} \ \underbrace{0\cdots0}_{S_a+H_t-a_i}\Big] \in \mathbb{R}^{1\times(S_a+H_t)}$$

and $\forall j \in \{2,\ldots,S_a+H_t\}$

$$\boldsymbol{e}_j \stackrel{\text{def}}{=} \Big[\underbrace{0\cdots0}_{i-1} \ \frac{1}{\sigma_{ij}^k} \ \underbrace{0\cdots0}_{j-i-1} - \frac{1}{\sigma_{ij}^k} \ \underbrace{0\cdots0}_{S_a+H_t-j}\Big] \in \mathbb{R}^{1\times(S_a+H_t)}$$

where i is the node with $i<j$ tied to j while k denotes the node computing the corresponding TDOA. The $(2S_a + H_a + H_t - 1) \times (2S_a + H_a + H_t)$ matrix

$$\boldsymbol{A}' = \begin{bmatrix} \boldsymbol{e}_1^{(1)} & \boldsymbol{e}_1^{(2)} \\ \vdots & \vdots \\ \boldsymbol{e}_{S_a+H_a}^{(1)} & \boldsymbol{e}_{S_a+H_a}^{(2)} \\ & \boldsymbol{e}_2 \\ \mathbf{0}_{(S_a+H_t-1)\times(S_a+H_a)} & \vdots \\ & \boldsymbol{e}_{S_a+H_t} \end{bmatrix}$$

with $\mathbf{0}_{(S_a+H_t-1)\times(S_a+H_a)} \in \mathbb{R}^{(S_a+H_t-1)\times(S_a+H_a)}$ a zero entries matrix, has rank $2S_a + H_a + H_t - 1$ (it is easy to check that its rows are linearly independent vectors). This concludes the proof since $\boldsymbol{A}'$ is a minor of $\boldsymbol{A}$. ***Q.E.D.***

In the example of Fig. 1(b) the connected component is formed by A, B, E, and F (which correspond to indexes 1, 2, 3, and 4 in the application of the Proposition). Anchors are not explicitly indicated in the MCG; however, the Proposition states that the construction of $\boldsymbol{A}'$ relies on the selection of the Type I equations associated with the anchors belonging to the set $\{A, B, E, F\}$ and, in addition, on three, judiciously chosen, Type II equations (for instance, one for each of the pairs (A,B), (B,E), and (B,F)). We have to discard, instead, possible Type I equations corresponding to nodes C and D.

Position estimation. From eq. (5) we obtain the vector of pseudo-ranges as

$$\hat{\boldsymbol{\delta}} = \boldsymbol{\delta}_0 - \mu\mathbf{1}_{S_a+H_a} \quad (7)$$

therefore the estimate $\hat{\boldsymbol{p}}^*$ of the blind node position can be obtained by directly applying the standard algorithm used in GPS, that is an iterative least squares procedure including a common bias term [11]. In other words, the cooperative approach allows to compensate for the lack of synchronization between the anchors, reducing the problem to a range-based positioning formally identical to the (synchronous) GPS case.

[4]The actual expressions for $\boldsymbol{b} \in \mathbb{R}^{m\times1}$ and $\boldsymbol{A} \in \mathbb{R}^{m\times(2S_a+H_a+H_t)}$ depend on the specific Type I and II equations that has to be considered.

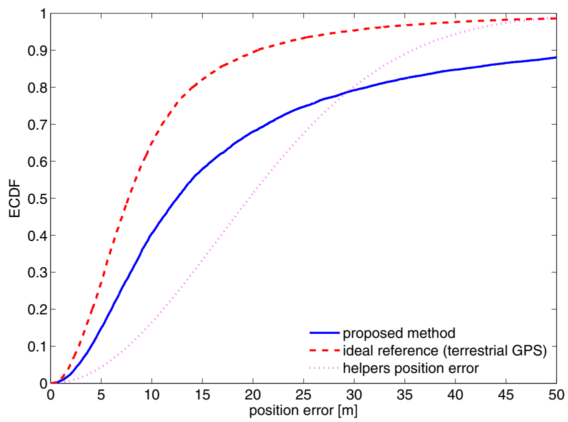


Fig. 2. Simulation results for a scenario assuming LOS and NLOS links.

Considerations. The model presented in this work is extremely versatile and applies to a wide range of real-world scenarios, exploiting (opportunistically) different types and scales of "infrastructure" and with different degrees of "cooperation". As limit case, it can capture an "infrastructure-driven" scenario where only fixed stations serve as anchors ($H_a = 0$) and a small number of helpers (in the limit case only one) serves exclusively to compensate for the asynchronous nature of transmissions. On the other hand, it can model a fully "cooperation-driven" scenario with no station ($S_a = 0$) where the entire procedure relies on several cooperating helpers.

IV. Simulation Results

We present Monte Carlo simulation results in a realistic scenario in order to validate the proposed approach. We consider a squared area of 3×3 km^2 with 10 stations and 20 helpers. At each of the 10000 trials, the positions of the blind node and all stations and helpers are extracted randomly. The propagation effects over all links are simulated by considering a path loss exponent $\alpha = 4$ and a Rayleigh or Rice fast fading according to the non-line-of-sight (NLOS) or LOS condition of the link. Links are randomly assigned to the NLOS class with probability $1/3$. In addition to this (fast) fading model, shadowing effects are also taken into account through a lognormal factor with standard deviation 6 dB. More precisely, the SNR at the receiver is modeled as $SNR = \frac{E_s/\mathcal{N}_0}{d^\alpha} S A^2$ where the energy contrast $E_s/\mathcal{N}_0$ is equal to 30 dB at 1 km from the transmitter, d is the length of the link (in km), S is a lognormal rv, $A = \sqrt{X^2 + Y^2}$ with $Z = X + jY \sim \mathcal{CN}(\rho\delta_{i1}, 2\sigma_i^2)$ and, in turn, δ_{i1} is the Kronecker symbol. As to i, $i = 0$ (resp., $i = 1$) implies a Rayleigh (resp., Rice) rv: in both cases the mean square value of A is equal to 1; moreover, for the Ricean fading we set ρ such that $10\log_{10}\frac{\rho^2}{2\sigma_1^2} = 6$ dB. Links with an SNR below 5 dB are considered obstructed, i.e., they are discarded; on the other side, the SNR is upper limited to 60 dB (for nodes in spatial proximity). All NLOS links experience also an additional error in the measured ranges, due to the longer propagation path, which is modeled as a random positive bias uniformly distributed up to 1% of the true distance. The modeling setup described above determines in general a communication graph that is not fully meshed. On average the number of stations (resp., helpers) effectively acting as anchors is approximatively $S_a = 7$ (resp., $H_a = 13$). Similarly, about one fourth of all helper-station and helper-helper links are obstructed due to destructive fading effects, i.e., their SNR is below the sensitivity threshold of 5 dB. The standard deviations of the GPS position errors are set to $\sigma_{v_i} = 50/3$ m while the variances of timing errors have been set according to the Cramer-Rao lower bound [10]. To this end, we assume root-raised-cosine-rolloff pulses with a normalized mean square bandwidth $\beta^2 \approx 0.1$, a symbol duration T with $1/T = 2$ MHz, and an estimation interval of 64 symbols. On average, this results in a timing standard deviation of the order of 10^{-7} s. Since the exact position of the helpers is unknown, the localization algorithm must use estimates of $\sigma_{\epsilon_i^k}$ based on the noisy helpers positions. As to the blind node, whose position is unknown to the algorithm, the centroid of the helper positions is taken as a guess for the evaluation of the SNR.

Fig. 2 reports the empirical cumulative distribution function (ECDF) of the module of the positioning error of the blind node. The dashed curve indicates the reference ECDF in the ideal case that all anchors (selected stations and helpers, i.e., $S_a + H_a$ transmitting nodes) were perfectly synchronized (a sort of "terrestrial GPS-like" system). The CDF of helper position errors is plotted as a dotted curve: interestingly, for a sufficiently large number of helpers, the position of the blind node can be determined with a median error that is smaller than the uncertainty on the helper positions. Remarkably, in the majority of cases the localization accuracy of the proposed method is only a few meters worse than the ideal reference.

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