

# A Study on Fuzzy Based Controllers Design for Depth Control of a 3-Joint Carangiform Fish Robot

Khac Anh Hoang, Tuong Quan Vo

**Abstract**— Fish robot can move independently in the water, so controlling of it in six degree of freedoms (6 DOFs) is a difficult problem. To simplify, we can assume the motions of fish robot include two main motions: the movement of fish robot on the horizontal plane and movement of fish robot to the desired depth. This paper proposes a simple method to control a 3-joint Carangiform fish robot swim to the desired depth by using pectoral fins' motions. The role of pectoral fins is similar to the rudder in Z direction that can change the pitch angle of the fish robot. By controlling the rotation angle of the pectoral fins which is called pitch angle, we can control the up and the down motions for fish robot. Fuzzy and Self-tuning PID controllers (Fuzzy-PID) are developed to control the fish robot's pectoral fins in order to make it swim to the desired depth.

## I. INTRODUCTION

Now a days, Autonomous Underwater Vehicles (AUVs) have been widely used for ocean exploration. The current demand requires more advanced AUV devices. Fish robot, a biomimetic robot and also is a kind of AUV which can move independently in the water and has high energy efficiency. Based on [1], they can divided the fish into BCF (Body and/or Caudal Fin) and MPF (Median and/or Paired Fin). The BCF is based on the changing shape of body or caudal fin to create thrust force. The MPF is based on the oscillating of the median and/or paired fins. And, there are four swimming ways of BCF: Anguilliform, Subcarangiform, Carangiform and Thunniform. Therein, Carangiform, which moves the little oscillator body and moves the tail. So, among these types of BCF fish, most of the fish robots are developed based on the Carangiform type because of its flexibility and easiness in control.

When fish robot operates, it will make the decisions and controls itself by sort of pre-defined tasks. Therefore, using the classic controller to control fish robot in operating environment is not effective because it does not know clearly about the behavior of the working environment. So,

Manuscript received June 15, 2013. This research is funded by Viet Nam National University Ho Chi Minh City (VNU-HCM) under Grant number B-2013-20-01.

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we should use the theory of intelligent control or advanced controllers to control the fish robot.

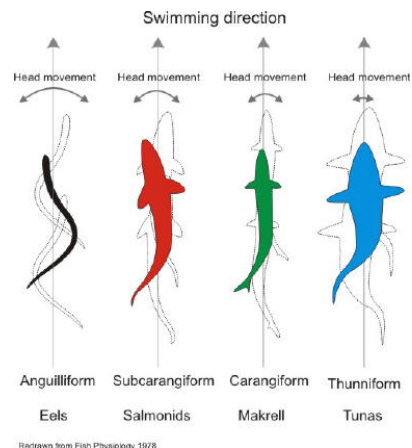


Fig. 1: The four swimming ways of BCF [1]

Gi-Hun Yang et al. [2] describes the dynamics and controlling of fish robot named "Ichthus", it has a 3-DOF serial link-mechanism and is developed by KITECH (Korea Institute of Industrial Technology). The simulation computational dynamics and speed of swimming are completed. By model-based simulations, they have made the actual model and proposed some new control parameters to minimize the power consumption while the fish robot is operating. On the other hand, Le Zhang et al. [3] describes the design fish robot which is capable of 3D locomotion, and presents its depth control method. The dynamics equations and simulation results are not presented in [3], but they make the actual model and introduce their experimental results. Jianxun Wang et al. [4] presents the dynamic model of the robot fish. And, the influence of hydrodynamic forces are evaluated based on estimates Lighthill theory. The contributions of the paper are to introduce the drag and moment coefficients which are required for the movements of underwater robots. They also focus on the operation of the robot fish caused by fluctuations tail, slip, floor number and amplitude of oscillation. Alessio Alessi et al. [5] uses an experimental platform for the validation of bio-inspired control architectures and sensory-motor coordination models. Specifically, they use a moving slider to change the position of the Center of Gravity (COG) of robot to control pitch angle. And, Yonghui Hu et al. [6] presents a quite good structure of a robotic fish with the head module, a pair of pectoral fin modules and two tail modules. The pectoral fins are controlled to change the pitch angle of the robot and therefore the operation depth of the fish robot is changed. Li Wen et al. [7] uses the analytical method which is based on

the treating of the undulating body and flapping tail independently to model the dynamics of the fish robot. Besides, they find that the harmonic control of the Strouhal number and caudal fin angle are the principal mechanism through which the robotic fish can obtain high thrust efficiency while swimming. They realized that robotic fish can always swim near an “universal” Strouhal number that approximates to the swimming of live fish [7]. Zhao Wei Ma et al. [8] proposes the adaptive neural method to control robotic fish by using a pair of coupled neural oscillators and have good simulation results. The main ideas of them is to make the fish robot has a better operation with the changing of the working environment. Moreover, Y. Hu et al. [9] introduces the cooperative method among three autonomous fish robot. The purpose of this cooperation is to push the box from the start position to the goal position. The experimental results show that their propose methods of the cooperating among many fish robot to carry out a specific task is feasible. With the ideas is quite similar to [9], Stefano Marras and Maurizio Porfiri [10] research about the collective behaviors in a fish school when they analyze a robotic fish and real fish swimming together in a water tunnel at different flow velocities. The results indicate that the robot fish can integrate with the real fish.

Besides, about the Carangiform fish robot, we also have some prior researches about the optimization problem for fish robot velocity and fish robot gait as introduced in [11] [12]. And, we also focus on the turning motion control for a 3-joint Carangiform fish robot by using Sliding Mode and Fuzzy Sliding Mode controllers as discussed in [13].

In this paper, we study on the design of the intelligent controllers as Fuzzy and Fuzzy-PID to control a 3-joint Carangiform fish robot type to swim to the desired depth.

Modeling and simulating are important key in controlling the robot fish, these will be a helpful factor to prepare for the experiments of the real robot.

In addition to the researches about Carangiform fish robot in this paper, we also present a new method for modeling the dynamics of this fish robot. In this dynamics model, the fish robot is considered which is similar to an underwater vehicle moving in 3 dimensions of working environment. Then, the design of the depth controllers will be discussed. These controllers help the fish robot swim to the desired depth and keep this depth while swimming. For example, the conventional PID controller is mostly used for clear system or linear system. If we develop this controller for a non-linear system, the performance of the system are not good as expected. In this paper, we propose a Fuzzy based controllers which are called Fuzzy and Fuzzy-PID controllers to control the depth motion of our fish robot. Then, some simulation results are introduced to prove the effectiveness and the feasibility of our proposed methods.

## II. FISH ROBOT MODELING

### A. Mechanical Design

With the simple design, the fish robot has 3-Joint and pectoral fins to control the up-down motion. The structure of

our fish robot is described in Fig 2.

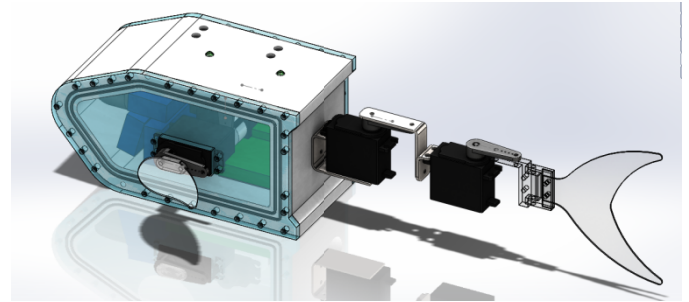


Fig. 2: Structure of 3-Joint Carangiform fish robot.

The control parameters for fish robot are introduced in table 1.

DOF	Motions & rotation	Linear & Angular Velocities	Position & Euler Angles
1	surge	u	x
2	sway	v	y
3	heave	w	z
4	roll	p	$\phi$
5	pitch	q	$\theta$
6	yaw	r	$\psi$

### B. Dynamics Equations

The idea of controlling our fish robot swim to the desired depth is the combination between the thrust force and the changing angle of the fish robot by using pectoral fins. Firstly, the fish robot moves in horizontal plane by its caudal part oscillations. This caudal part's oscillation will generate the propulsion force that make fish robot swim forward. Secondly, the pectoral fins are operated and resulting in changing the pitch angle for fish robot. Then, the combination between the propulsion force generated by the caudal part and the change direction of pitch angles, the fish robot will swim to the desired depth. Based on this assumption, the mathematical model of fish robot is developed.

Based on [14], the motion of the vehicle in the 3D environment is described by Eq. (1):

$$M \cdot \dot{v} + [C(v) + D(v)] \cdot v + g(q) = T \quad (1)$$

Where:

M: Inertia matrix.

C(v): Coriolis and Centripetal matrix.

D(v): Damping matrix.

T: Input vector.

and:  $v = [u \quad v \quad w \quad p \quad q \quad r]^{-1}$

We only focus on the motion of fish robot in Oxz plane. So the Eq. (1) becomes Eq. (2):

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{\theta} \\ \dot{u} \\ \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} u \\ u \tan \theta \\ q \\ (F \cos \theta + F_{fin} \sin \alpha - F_{dragx}) / m \\ (mX_g \dot{q} - F_{fin} \cos \alpha + F \sin \theta + B - W - F_{dragz}) / m \\ (mX_g w + M_H + M_{fin}) / I_{yy} \end{bmatrix} \quad (2)$$

Where:

$F$ : Thrust force of tail.

$F_{fin}$ : Fin lift.

$\alpha$ : Angle of attack.

$F_{dragx}$ : Axial Ox drag force.

$F_{dragz}$ : Axial Oz drag force .

$m$ : Mass of robot

$W$ : Gravity force

$B$ : Buoyancy force

$I_{yy}$ : Inertial tensor.

Then, by analyzing Eq. (2), the depth equation of fish robot is calculated by Eq. (3)

$$M_H = (Z_g W - Z_b B) \sin \theta + (X_g W - X_b B) \cos \theta \quad (3)$$

Where:

$\begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix}$  position of gravity in body-fixed frame

$\begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix}$  position of buoyancy in body-fixed frame

The calculation method for depth motion of fish robot is introduced in Fig. 3.  $W$  is the direction of the depth control for fish robot.

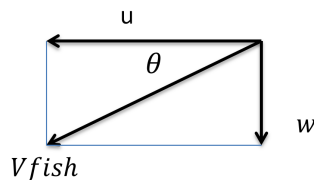


Fig. 3: The depth calculation method of fish robot

### III. CONTROLLERS DESIGN

#### A. The Fuzzy Controller

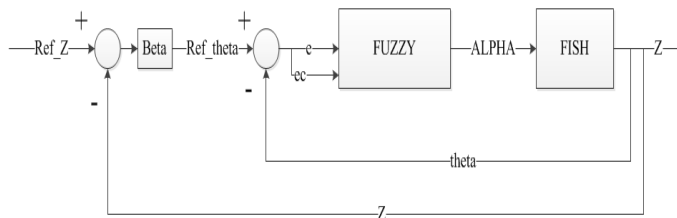


Fig. 4: The Fuzzy Controller.

In reality, Fuzzy logic controller is used to control many systems including linear and non-linear systems. In our

problem, the Fuzzy controller is introduced in Fig. 4 above. The Fuzzy controller has two inputs (the error ( $e$ ) and the rate of the change of error ( $ec$ )) and one outputs ( $\alpha$ ). The triangle membership function is used for input and output variables. The defuzzification method chosen is COG.

We define the fuzzy variables and the rule for Fuzzy controller as:

- $e$  (the error) = {NB (Negative Big), NM (Negative Medium), NS (Negative Small), ZE (Zero), PS (Positive Small), PM (Positive Medium), PB (Positive Big)}.
- $ec$  (the change of error) = NB (Negative Big), NM (Negative Medium), NS (Negative Small), ZE (Zero), PS (Positive Small), PM (Positive Medium), PB (Positive Big)}.
- $\alpha$  = {NB (Negative Big), NM (Negative Medium), NS (Negative Small), ZE (Zero), PS (Positive Small), PM (Positive Medium), PB (Positive Big)}.

The Fuzzy rules are introduced in Table 2.

TABLE 2. FUZZY RULES FOR  $\alpha$

$\alpha$	$ec$							
		NB	NM	NS	ZE	PS	PM	PB
e	NB	PB	PB	PM	PS	PS	ZE	ZE
	NM	PB	PB	PM	PS	PS	ZE	NS
	NS	PM	PM	PM	PS	ZE	NS	NS
	ZE	PM	PM	PS	NS	NS	NM	NM
	PS	PS	PS	ZE	NS	NS	NM	NM
	PM	PS	ZE	NS	NM	NM	NM	NB
	PB	PB	PB	PB	NM	NM	NB	NB

#### B. The Fuzzy-PID Controller

In our study, the Fuzzy logic controller is combined with conventional PID controller to make the Fuzzy-PID controller. With the unclear influences of the working environment, Fuzzy logic controller will tune the gains of PID controller. The  $K_p$ ,  $K_i$ ,  $K_d$  gains are changed by Fuzzy to match with the operating conditions of the fish robot. The proposed Fuzzy-PID controller is introduced in Fig. 5 below:

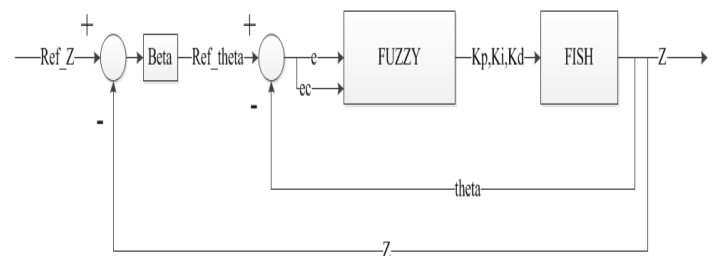


Fig. 5: The Fuzzy-PID Controller.

The structure of the Fuzzy-PID controller includes four main elements:

- 1- The inputs.
- 2- The Fuzzy controller.
- 3- PID controller
- 4- Output  $u$ .

The Fuzzy controller has two inputs (the error ( $e$ ) and the rate of the change of error ( $ec$ )) and three outputs ( $K_p$ ,  $K_i$ ,  $K_d$ ). The triangle membership function is used for input and

output variables. Also, the defuzzification method chosen is COG.

We define the fuzzy variables and the rule for Fuzzy-PID controller as:

- $e$  (the error) = {NB (Negative Big), NM (Negative Medium), NS (Negative Small), ZE (Zero), PS (Positive Small), PM (Positive Medium), PB (Positive Big)}.
- $ec$  (the change of error) = {NB (Negative Big), NM (Negative Medium), NS (Negative Small), ZE (Zero), PS (Positive Small), PM (Positive Medium), PB (Positive Big)}.
- $(K_p, K_i, K_d)$  = {NB (Negative Big), NM (Negative Medium), NS (Negative Small), ZE (Zero), PS (Positive Small), PM (Positive Medium), PB (Positive Big)}.

The Fuzzy rules are presented in Table 3, 4 and 5.

TABLE 3. FUZZY RULES FOR K<sub>p</sub>.

K <sub>p</sub>	ec							
	NB	NM	NS	ZE	PS	PM	PB	
e	NB	PB	PB	PM	PS	PS	ZE	ZE
	NM	PB	PB	PM	PS	PS	ZE	NS
	NS	PM	PM	PM	PS	ZE	NS	NS
	ZE	PM	PM	PS	NS	NS	NM	NM
	PS	PS	PS	ZE	NS	NS	NM	NM
	PM	PS	ZE	NS	NM	NM	NM	NB
	PB	PB	PB	PB	NM	NM	NB	NB

TABLE 4. FUZZY RULES FOR K<sub>i</sub>.

K <sub>i</sub>	ec							
	NB	NM	NS	ZE	PS	PM	PB	
e	NB	NB	NB	NM	NM	NS	ZE	ZE
	NM	NB	NB	NM	NS	NS	ZE	ZE
	NS	NB	NB	NM	NS	NS	NM	NM
	ZE	ZE	ZE	ZE	ZE	ZE	ZE	NS
	PS	NM	NS	ZE	PS	ZE	ZE	PS
	PM	ZE	ZE	PS	PS	PS	PB	PB
	PB	ZE	NB	PS	PM	PM	PB	PB

TABLE 5. FUZZY RULES FOR K<sub>d</sub>.

K <sub>d</sub>	ec							
	NB	NM	NS	ZE	PS	PM	PB	
e	NB	NB	NS	NB	NB	NB	NM	PS
	NM	PS	NS	NB	NM	NM	NS	PS
	NS	PB	PB	PM	PB	PM	PM	PB
	ZE	PS	PM	PS	PB	PB	PB	PS
	PS	ZE	NS	NS	NS	NS	ZE	ZE
	PM	PB	PS	PS	PS	PM	PS	PB
	PB	NB	ZE	NS	PM	PS	PS	PB

IV. SIMULATION RESULTS

The simulations are carried out which is based on the depth equation of fish robot as introduced in section 2 above. And, we also apply the two different controllers as presented in section 3 above called Fuzzy and Fuzzy-PID to operate the depth control problems for our fish robot.

Besides, the chosen desired depth to check our controllers is 1 meter. In this depth, the fish robot will be controlled to swim from the water surface to the desired depth which is called the down motion and it also control the fish robot swim from the desired depth to the water surface which is

called the up motion. The simulation results will introduce the depth respond of fish robot, the changing by time of fish robot's pitch angle.

In our proposed methods, about the pitch angle of fish robot, the less oscillation of fish robot's pitch angle is the better and also the fast stability of pitch angle is the better. And, about the time of fish robot swim to the desire depth called 'depth reaching time', the shortest time is the better. However, when choosing the suitable controller for our real fish robot, we also consider two elements that are the less pitch angle oscillation and the fast 'depth reaching time'.

Based on the simulation results, we can check our fish robot dynamic equations. And, we just pay much attention to the depth equation of our fish robot in this paper. Besides, we also check the responds and the performances of our fish robot when we apply the Fuzzy and Fuzzy-PID controllers.

In the simulation graphs will be discussed below, the continuous line is the desired depth, the continuous curve is the respond of fish robot's depth and the dash line is the oscillation of fish robot's pitch angle.

A. The Down Motion

The desired depth is 1 meter and the water surface is defined is 0 meter. The fish robot will be controlled to swim from the water surface to the desired depth.

• The Fuzzy Controller

The respond of fish robot is introduced in Fig. 6. Base on Fig. 6, the Fuzzy controller takes about 47 to 50 seconds to control fish robot swim to the desired depth. And, the fish robot's pitch angle oscillates quite strong at the beginning time but it will gradually decrease and come to stable at about 50 seconds. The time that pitch angle come to stable is nearly equal to the time that fish robot swim to the desired depth. When fish robot swim to the desired depth, the fish robot's pitch angle still has small oscillations. The reason of this phenomenon is that when fish robot already reach to the desired depth, it whole body still have some light oscillations which are similar to the real fish.

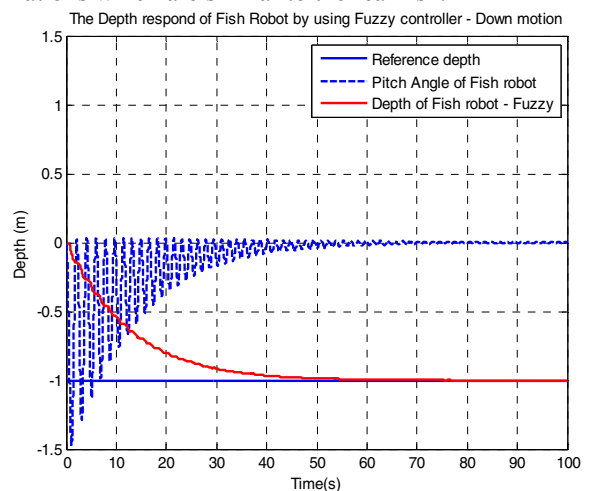


Fig. 6: The response of fish robot down motion (Fuzzy controller)

- *The Fuzzy-PID Controller*

The respond of fish robot when applying Fuzzy-PID controller to control fish robot swim from the desired depth to the water surface is introduced in Fig. 7.

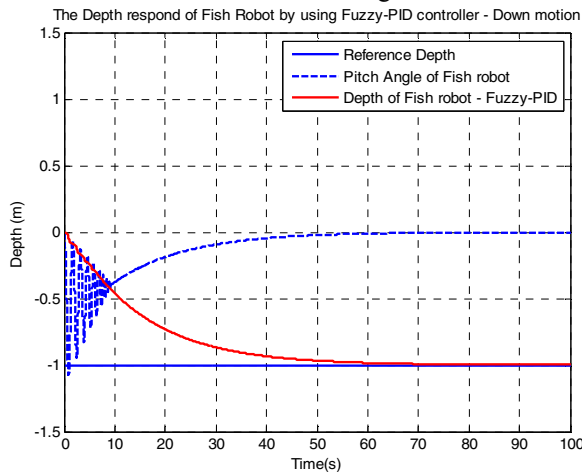


Fig. 7: The response of fish robot down motion (Fuzzy-PID controller)

In Fig. 7, fish robot will take about 60 seconds to swim to the desired depth. This time is longer than when we apply the Fuzzy controller as in Fig. 6 above. However, the performance of fish robot in this case is better because of the ‘depth reaching time’ is very short. And, the fish robot’s pitch angle does not oscillate as much as the Fuzzy controller.

*B. The Up Motion*

Similarity to the down motion, fish robot will also be controlled by applying Fuzzy and Fuzzy-PID controllers to make it swim from the desired depth to the water surface. This is the opposite problem of the down motion above.

In Fig. 8 and Fig. 9, the current position of fish robot is supposed to be at the desired depth and this desired depth is defined as 0 meter. And, the water surface is defined as 1 meter. The fish robot will be control to swim from the current position (0 meter) to the water surface (1 meter).

- *The Fuzzy Controller*

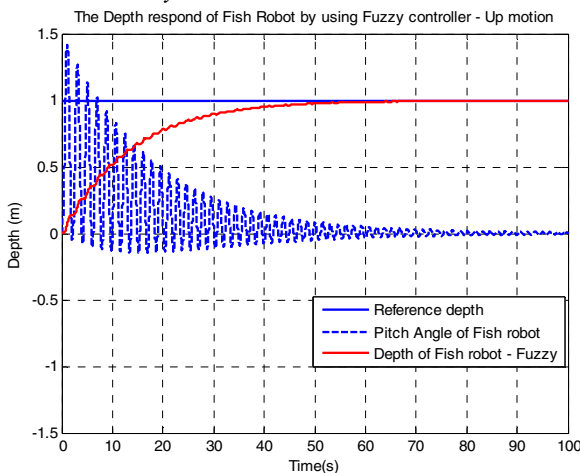


Fig. 8: The response of fish robot up motion (Fuzzy controller)

The respond of fish robot when apply Fuzzy controller to control it swim from the desired depth to the water surface is presented in Fig. 8 above.

In Fig.8, fish robot takes about 55 to 60 seconds to reach from the desired depth to the water surface. And, the oscillation of pitch angle is also quite big.

- *The Fuzzy-PID Controller*

The respond of fish robot when we apply Fuzzy-PID controller to control fish robot swim from the desired depth to the water surface called the up motion is introduced in Fig. 9.

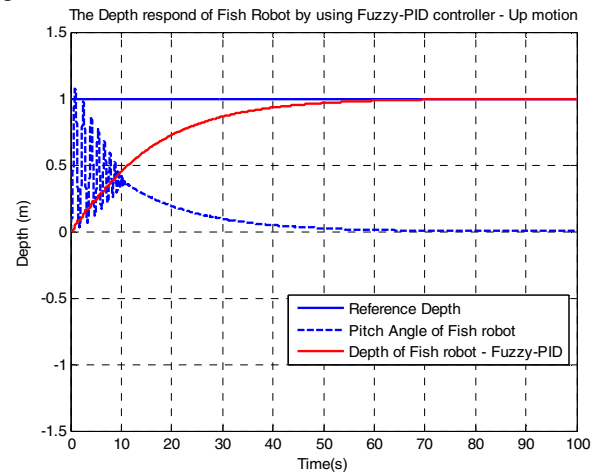


Fig. 9: The response of fish robot up motion (Fuzzy-PID controller)

In Fig. 9, the fish robot will also take about nearly 60 seconds to swim from the desired depth to the water surface. However, similarity to the down motion, when we apply the Fuzzy-PID controller, the stability of the fish robot’s pitch angle takes quite short time. This is the better point of Fuzzy-PID controller in comparison to the Fuzzy controller in the depth control problem for our fish robot.

Besides, if we compare about the respond times between the up and down motions with the same desired depth, the time responds in up motion will take longer than the time respond in down motion. In the up motion, It will take longer time to control fish robot swim from the desired depth to the water surface (Fig. 8 and Fig. 9).

Therefore, based on the simulation results, we can get some conclusions for our fish robot in the depth control problem. The oscillation of fish robot’s pitch angle when using Fuzzy-PID controller is smaller than the Fuzzy controller. And, the Fuzzy-PID controller takes shorter time than the Fuzzy controller to make the fish robot’s pitch angle come to stable state. Besides, in the same depth, the up motion will take longer time than the down motion when we apply both Fuzzy and Fuzzy-PID for our fish robot.

V. CONCLUSION

In this paper, the depth dynamic equation of a 3-joint Carangiform fish robot is introduced. Then, based on the depth dynamic equation of the fish robot, we develop two different intelligent controllers as Fuzzy and Fuzzy-PID

controllers to control for fish robot depth problem. The simulation results when applying Fuzzy, Fuzzy-PID controllers to control fish robot swim from the water surface to the desired depth which is called the down motion and to control fish robot swim from the desired depth to the water surface which is called the up motion are quite good and quite stable.

In the next step, we are now making our real fish robot and we will continue to do some experiments to check the agreements between the simulation results and the experimental results for the depth control problem of our fish robot. Besides, some other controllers will be considered to control the depth problem for our fish robot such as optimal control and robust control.

#### ACKNOWLEDGMENT

This research is funded by Viet Nam National University Ho Chi Minh City (VNU-HCM) under Grant number B-2013-20-01.

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