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Signal Processing 68 (1998) 307–315

SIGNAL PROCESSING

Identification of power transformer models from frequency response data: A case study

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Received 23 October 1997; received in revised form 16 February 1998

Abstract

A recent frequency-domain, subspace-based algorithm as well as the well-known nonlinear least-squares algorithm are used in the identification of a power transformer whose frequency response has a dynamic range of 1 MHz. When the model complexity is not restricted, both the algorithms produce highly accurate models. Low-complexity models are extracted from the high-order identified ones via the method of balanced truncation. It is observed that this two-step procedure yields more accurate results than an approach of direct identification of a low-order model. The utility of identified models for the purpose of transformer fault detection is also briefly discussed. © 1998 Elsevier Science B.V. All rights reserved.

Zusammenfassung

Ein neuartiger, Unterraum-basierter Algorithmus im Frequenzbereich und der wohlbekannte nichtlineare Kleinste-Quadrate Algorithmus werden zur Identifikation eines Hochspannungstransformators verwendet, dessen Übertragungsfuktion einen Dynamikbereich von 1 MHz besitzt. Bei unbeschränkter Modellkomplexität liefern beide Algorithmen äußerst genaue Modelle. Modelle geringerer Komplexität werden aus denjenigen abgeleitet, die als Modelle höherer Ordnung identifiziert wurden, indem die Methode des ausgewogenen Abschneidens angewandt wird. Es wird beobachtet, daß diese Zweischritt-Prozedur genauere Ergebnisse liefert als ein Ansatz, der auf direkter Identifikation eines Modelles geringerer Ordnung abzielt. Die Tauglichkeit der identifizierten Modelle hinsichtlich der Detektion eines Transformatorausfalls wird ebenfalls kurz diskutiert. © 1998 Elsevier Science B.V. All rights reserved.

Résumé

Un algorithme fréquentiel, basé sur le sous-espace, et récemment introduit, ainsi que l'algorithme aux moindres carrés non linéaires bien connu sont utilisés pour l'identification d'un transformateur de puissance dont la réponse en fréquence a une gamme dynamique de 1 MHz. Quand la complexité du modèle n'est pas restreinte, les deux algorithmes produisent

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[1] This author gratefully acknowledges the support of CIDAC and the Australian Research Council.

0165-1684/98/$19.00
PII: S0165-1684(98)00080-2

des résultats très précis. Les modèles de complexité réduite sont extraits des modèles identifiés d'ordre élevé via la méthode de troncation balancée. Il est observé que cette procédure en deux étapes fournit des résultats plus précis qu'une approche d'identification directe d'une modèle réduit. L'utilité des modèles identifiés à des fins de détection de mauvais fonctionnement du transformateur est également discutée brièvement.

Keywords: Identification; Frequency-response data; Power transformer; Subspace-based algorithm; Nonlinear least-squares

1. Introduction

Frequency response methods are often used in practice to obtain a non-parametric model of a linear system. This identification may be performed without significant a priori knowledge of the plant. Further, it may be accomplished in the presence of significant process and measurement noise, since signal-to-noise ratio may be optimized at each frequency by focusing excitation energy at one frequency at a time and by adjusting excitation amplitude to the plant saturation limit. Also, data obtained from different experiments can easily be combined in the frequency domain.

The problem addressed in this paper is that of fitting a real-rational model to given frequency response data. In the classical prediction error approach [16–19], a system is modeled as a fraction of two real coefficient polynomials and a nonlinear least-squares fit to data is sought. This nonlinear parametric optimization problem is solved by iterative, numerical search. Recently however, non-iterative frequency-domain subspace-based identification algorithms have been developed which deliver state-space models [10,12]. These subspace-based algorithms have been successfully used in the identification of high-order flexible structures [10,12] and power transformers [1].

This paper is a continuation of the work initiated in [1]. There are two main objectives for this paper. Firstly, via case study where the dynamic range of frequency response data is 1 MHz, the properties of frequency-domain subspace-based and nonlinear least-squares identification algorithms are illustrated. The second objective is to develop a recipe for the effective application of both algorithms to the identification of power transformers.

A power transformer is a critical unit within a power network. A large transformer failure could cause long interruptions and costly repairs. Therefore, it is desirable to detect potential failures as early as possible. Model based diagnosis such as the transfer function method is becoming increasingly popular in transformer condition monitoring. The transfer function method is particularly useful in identifying faults such as winding deformation and displacement, inter-turn and inter-disc faults. As well, in the design of power transformers, high frequency modeling is essential for the study of impulse voltage and switching surge distribution, winding integrity and insulation diagnosis and also for the purposes of condition monitoring, models that are accurate over a bandwidth greater than 1 MHz are required. Most often accurate models in a bandwidth up to 10 MHz are required for condition monitoring purposes. Finally, accurate parameter identification of transformers may lead to economical design of transformer insulation against failure due to ferro-resonance and through fault generated stresses.

The analysis of the frequency response of power transformers was originally proposed by Dick and Even for the detection of winding movement in large power transformers and as a practical maintenance tool [4]. Certain advantages of this approach over the low voltage impulse method [8] were reported in [4]. This method has been mainly limited to interpreting faults by detecting changes in successive frequency response tests. However, this approach does not explain the changes in relation to a suitable mathematical model. In [6,2], transformer frequency response is divided into low, medium, and high frequency ranges and either a second or a third order model fit is sought for each data segment using the nonlinear least-squares method. The models obtained by this procedure poorly fit to the observed data, and in particular are not capable of modeling high frequency dynamics of a transformer.

In [1], the subspace-based algorithm [12] was used in the identification of a three-winding power transformer and accurate high order transfer function models were obtained. It was also demonstrated that at low frequencies, the transfer function of a three-winding power transformer can accurately be modeled by a sixth order model. In this contribution, we demonstrate the same conclusions but with the extensions that via non-linear least-squares identification a fourth order model is shown to accurately describe two-winding transformers.

In the current paper, as in [1] we focus on mathematical models of transformers rather than their equivalent circuits. One reason is that we maintain the idea that once an accurate analytic model of the transformer under consideration is available, it is possible to derive a transformer equivalent circuit by a suitable transformation if necessary. The second reason is that the traditional second or third order transformer equivalent circuits [4,2,11,5,6], do not capture the dynamics of realistic power transformers. Currently we are investigating transformer equivalent circuits that more closely match transfer functions of low order identified models. Apart from the desire to link mathematical and physical models, a mathematical model is of practical interest in its own right for the purposes of studying the time-domain response of transformer and monitoring transformer condition 'in service'.

The paper is organised as follows. In Section 2, we describe the experimental data to which the subspace-based and the nonlinear least-squares identification algorithms will be applied. In Section 3, we present our identification results applied to a particular two-winding transformer which has been in service. Section 4 concludes the paper.

2. Experimental data

In this section, we describe the experimental data set used in the case study. The data set was obtained from the Advanced Technology Center of Pacific Power International, Newcastle, Australia. A two-winding 23/345 kV 390 MVA generator transformer, which has been in service since 1970 at Delta Power/Vales Point Power Station was tested to determine the mechanical integrity of its windings. The transformer was prepared for test by being removed from service and electrically isolated from the transmission system. The transformer had been re-clamped and in service prior to testing. The frequency responses of phases A–C referred to the secondary, whose magnitudes are plotted in Fig. 1, were obtained by injecting a low voltage amplitude into the primary winding of the transformer over a frequency range of 50 Hz to 1 MHz and measuring the output voltage at the secondary winding. We refer the interested reader to [7] for more details on the experimental procedure. The numbers of the nonuniformly spaced frequency points in Fig. 1 are 145, 147 and 161, respectively, for phases A–C. Notice that all the three responses are almost identical. For this reason, we will work only with phase A frequency response.

3. Experimental identification results

In this section, we will discuss the results obtained from application of two identification algorithms to the transformer data. The first algorithm presented in [12] is a subspace-based identification algorithm. Its theoretical properties and applications to identification of lightly damped flexible structures were reported in [12,13]. See [12] for detailed motivation of this algorithm. This algorithm was first used in power transformer identification in [1]. The other algorithm considered in this paper is the well-known nonlinear least-squares (NLS) identification algorithm [16,18]. The NLS identification algorithm is implemented in the command **invfreqz** in MATLAB's Signal Processing Toolbox [9]. In the current paper, the work initiated in [1] is extended and the performances of the two algorithms are investigated in a case study.

The continuous-time identification problem is converted to an equivalent discrete-time identification problem as in [12] by using the bilinear transformation

$$s = f\frac{z-1}{z+1}.$$

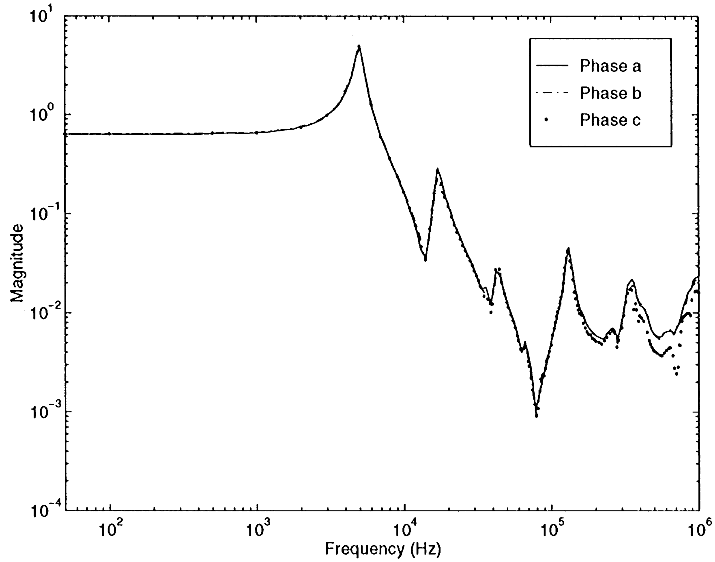

Fig. 1. Phase A–C frequency response magnitudes of the transformer.

Then the estimated continuous-time state-space parameters are obtained by back transformation. We take f twice the maximum of frequencies. For greater flexibility, its value can be adjusted as well.

The quality of estimated models will be assessed by two measures based on the fit between the data and the model: the maximum error

$$\|\hat{G} - G\|_{m,\infty} = \max_{1 \leqslant k \leqslant N} |\hat{G}(j\omega_k) - G_k|, \tag{1}$$

where G_k, $k = 1, \ldots, N$, are given N noise-corrupted samples of the frequency response function G and $\hat{G}$ denotes the identified model, and the root-mean-square error (RMS)

$$\|\hat{G} - G\|_{m,2} = \sqrt{1/N \textstyle\sum_{k=1}^{N} |\hat{G}(j\omega_k) - G_k|^2}. \tag{2}$$

Applying the subspace-based algorithm to the phase A frequency response of the transformer, a sequence of models of order 2–35 are estimated for $q = 40$, which denotes the row dimension of the Hankel matrix in [12] constructed from the sequence $\{G_k\}_{k=1}^{N}$. The results are shown in Fig. 2. From the graph, it is seen that both max and RMS errors first decrease with increasing model order, then remain at the same level for the model orders greater than 10 with few exceptions. The computed ten singular values in the subspace-based algorithm are 4.3569, 1.5975, 0.0990, 0.0398, 0.0180, 0.0162, 0.0079, 0.0063, 0.0041, 0.0015, which suggest that a second order model might adequately describe the dynamics of the transformer. However the max error of the second order model identified directly by the algorithm is 1.0849. In Fig. 3, the measured and the estimated 10th order identified model frequency responses together with the model errors are plotted.

This figure illustrates that the subspace-based method yields an excellent fit in the frequency range [0, 500 kHz]. The 10th order model has a max error 0.2255 in comparison to the smallest error 0.2116 achieved by a 30th order model. The fit to the data in the frequency range [500 kHz, 1 MHz] can be improved if a 30th order model is used. Models delivered by the subspace-based algorithm can be refined further by parametric optimization techniques such as the maximum likelihood or prediction error methods, but improvements by such techniques are usually marginal [14].

The Hankel singular values of the 10th order identified model are computed as 2.5558, 2.2457, 0.1232, 0.1065, 0.0219, 0.0205, 0.0125, 0.0108,

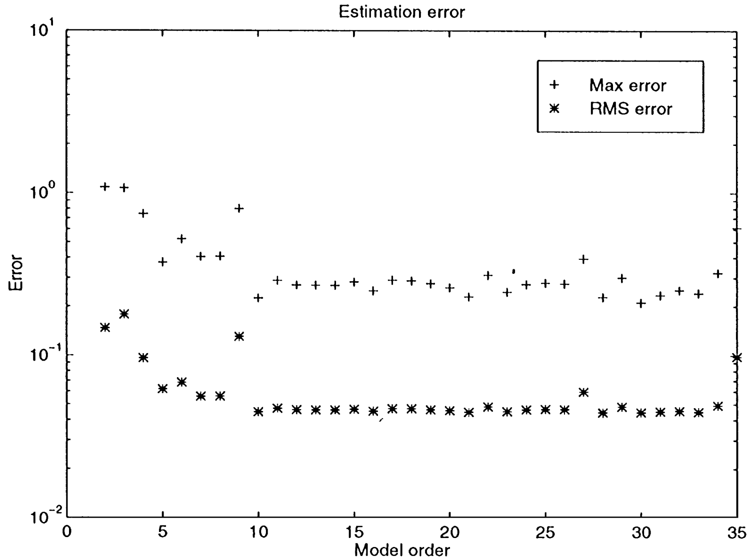

Fig. 2. Model errors (1)–(2) for phase A response of the transformer using subspace-based algorithm.

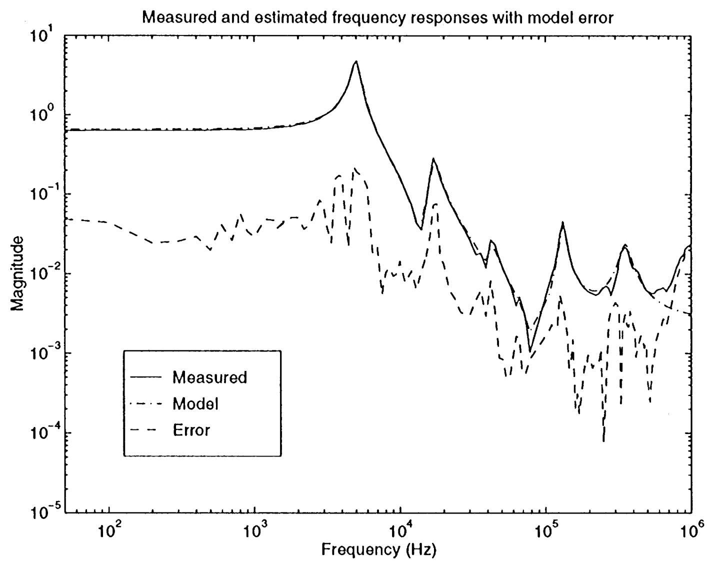

Fig. 3. Measured and estimated 10th order model phase A frequency response magnitudes of the transformer with model error using subspace-based algorithm.

0.0065, 0.0058. If a fourth order model is extracted from the balanced realization of this model, the max error of the reduced model is computed as 0.2248, which slightly improves the max error of the 10th order identified model. However, high frequency dynamics of the transformer is not captured by the fourth order model. Interestingly, the subspace-based algorithm yields a maximum error

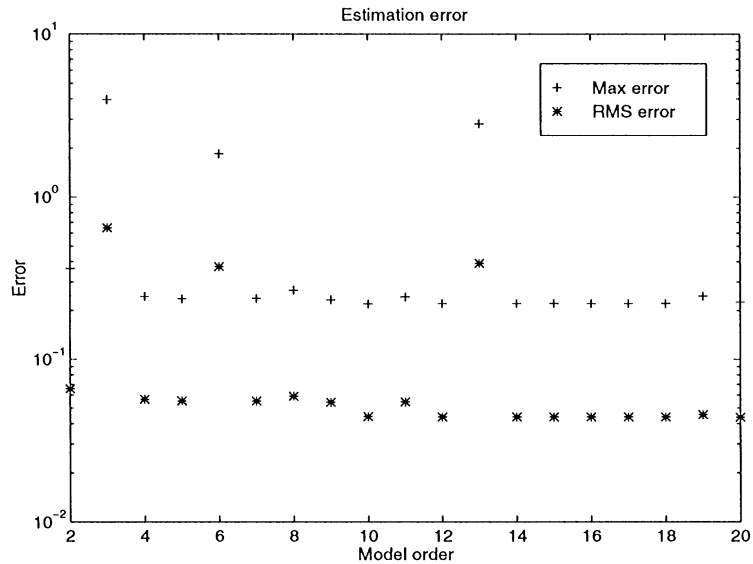

Fig. 4. Model errors (1)–(2) for phase A response of the transformer using nonlinear least-squares algorithm.

0.7446 for a fourth order identified model. The balanced truncation technique yields maximum errors of 0.2651 and 0.2553 for the second and the third order reduced models as opposed to 1.0849 and 1.0710 produced by the direct application of the algorithm.

Next, the NLS algorithm was applied to the phase A frequency response so as to estimate a sequence of models of order 2–20. The results are shown in Fig. 4. Except for the 3th, 6th and 13th order models, the max errors produced by the algorithm are almost equal for all identified models. Fig. 5 shows the excellent fit of the 16th order identified model to the data, in particular at high frequencies. The maximum error of the model is equal to 0.2212. Notice that this identified model misses the third mode.

The first ten Hankel singular values of the 16th order identified model are 2.5740, 2.2665, 0.1267, 0.1095, 0.0231, 0.0216, 0.0138, 0.0125, 0.0087, 0.0085, which agree well with the Hankel singular values of the 10th order identified model delivered by the subspace-based algorithm. The truncations of balanced realization of this model yielded maximum errors of 0.2167, 0.2667 and 0.2641, respectively, for the fourth, the third and the second order reduced models.

In Table 1, the coefficients of the polynomials $\hat{a}(s)$ and $\hat{b}(s)$ in the fourth order model $\hat{G}_{\mathrm{bal}}(s) = \hat{b}(s)/\hat{a}(s)$ obtained by the subspace-based and the NLS algorithms followed by the balanced truncation method are shown. With the exception of $\hat{b}_3$ and $\hat{b}_4$, which contribute to $\hat{G}_{\mathrm{bal}}(s)$ only marginally, the entries for the subspace-based algorithm in the table are little different than those of the NLS algorithm. Indeed, the natural frequencies computed from Table 1 are 5030.6, 17224; 5027.9, 16910 Hz, respectively, for the subspace-based and the NLS algorithms and the corresponding damping coefficients are in turn 0.03229, 0.03740; 0.03182, 0.03662.

We have also tried the continuous-time nonlinear least-squares algorithm as implemented by the **invfreqs** command in MATLAB's Signal Processing Toolbox [9]. This algorithm yielded a similar performance to the NLS algorithm. We reduced high order identified models delivered by this algorithm again by using the balanced truncation method. However, it was not possible to obtain balanced realizations directly from the identified transfer functions, which is related to the fact that if the system order is high, the poles and zeros of the system are sensitive to polynomial factoring. We overcame this problem by first computing the

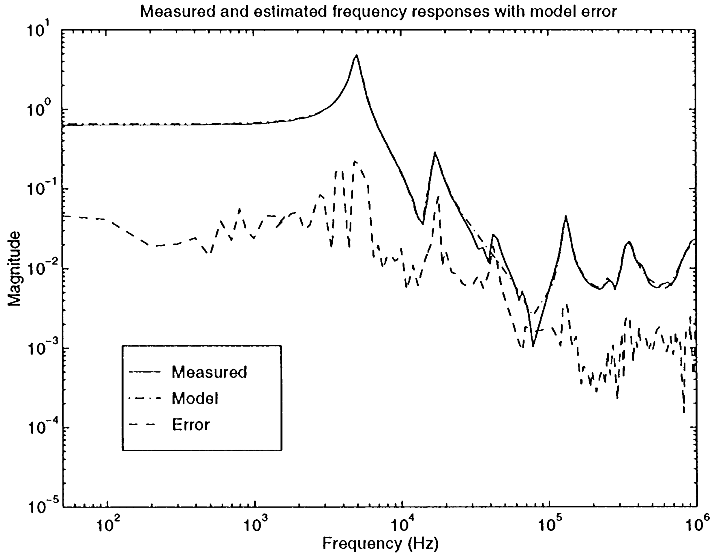

Fig. 5. Measured and estimated 16th order model phase A frequency response magnitudes of the transformer with model error using nonlinear least-squares algorithm.

Table 1
Comparison of the subspace-based and the NLS parameter estimates. The fourth order reduced models were obtained by truncating balanced realizations of the identified models by the subspace-based and the nonlinear least-squares algorithms. In the table, $\hat{G}_{\mathrm{bal}}(s) = \hat{b}(s)/\hat{a}(s)$, $\hat{a}(s) = s^4 + \sum_{k=0}^{3} \hat{a}_k s^k$ and $\hat{b}(s) = \sum_{k=0}^{4} \hat{b}_k s^k$

	Subspace	NLS
$\hat{a}_0$ $(\times 10^{19})$	1.1816	1.1373
$\hat{a}_1$ $(\times 10^{13})$	6.4329	6.1236
$\hat{a}_2$ $(\times 10^{10})$	1.2847	1.2415
$\hat{a}_3$ $(\times 10^{4})$	2.0275	1.9585
$\hat{b}_0$ $(\times 10^{18})$	7.7543	7.3389
$\hat{b}_1$ $(\times 10^{13})$	2.0651	2.1110
$\hat{b}_2$ $(\times 10^{9})$	1.0762	0.9716
$\hat{b}_3$ $(\times 10^{3})$	0.8832	1.5087
$\hat{b}_4$ $(\times 10^{-3})$	2.6445	− 4.3861

frequency response of the high order identified model at an arbitrarily chosen set of frequencies and then using the subspace-based algorithm with the computed frequency response samples at the chosen set of frequencies to obtain a state-space realization of the identified transfer function. This method exactly retrieves an nth order transfer function when the frequency response measurements are noise-free and the number of measurements is at least $n + 2$ and, moreover, the returned state-space realization is nearly balanced [12]. Finally a balancing transformation is performed on this particular realization. We used the same set of frequencies and $q = 40$. This procedure and the NLS algorithm followed by the balanced truncation yielded almost the same reduced order models.

In Figs. 6 and 7, the measured and the 4th order reduced model phase A frequency response magnitudes of the transformer with errors are plotted for the two identification algorithms followed by the balanced model reduction. Not surprisingly, the final models are almost identical since the identified models by the two algorithms are almost the same in the bandwidth of the reduced models.

Figs. 6 and 7 also indicate that the dynamics of a two-winding power transformer operating under normal conditions can be captured by a 4th order model in the frequency range [0, 40 kHz]. This accurate model was obtained in two steps. In the first step, an *over-parameterized* model structure was chosen and either the subspace-based or the nonlinear least-squares identification algorithm was used. In the second step, a low complexity

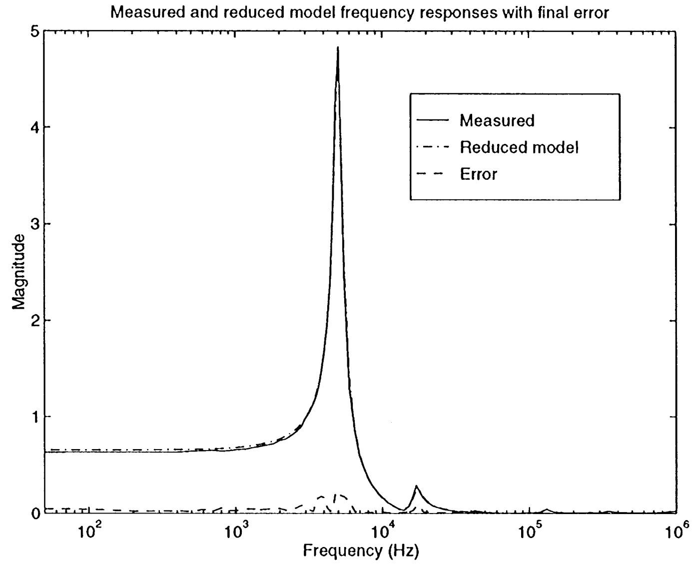

Fig. 6. Measured and estimated 4th order model phase A frequency response magnitude of the transformer with model error. The 4th order model is obtained by balanced truncation from the 10th order identified model using subspace-based algorithm.

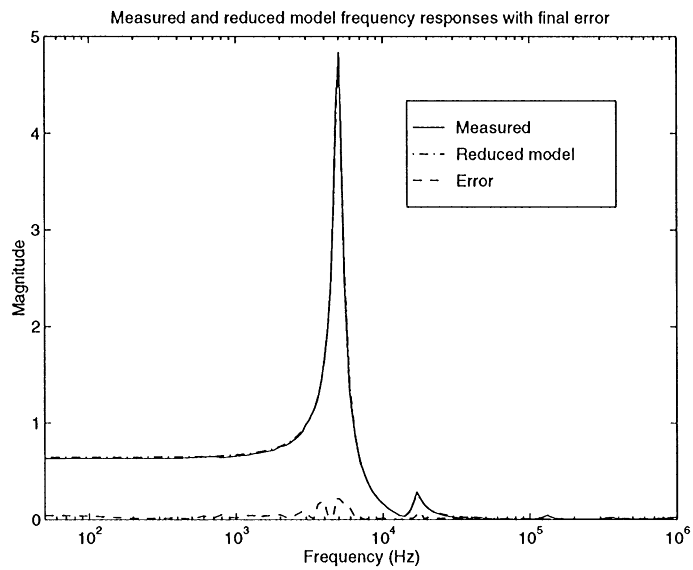

Fig. 7. Measured and estimated 4th order model phase A frequency response magnitude of the transformer with model error. The 4th order model is obtained by balanced truncation from the 16th order identified model using nonlinear least-squares algorithm.

model was extracted from the high order model. The approach that integrates subspace-based identification algorithm with model reduction and parameter estimation algorithm was proposed by Jacgues et al. [15] in the identification of lightly damped flexible structures. The other approach which integrates nonlinear least-squares identification with model reduction was proposed by Bayard [3] again in the context of flexible structure identification.

Table 2
Correlation of model parameter changes to transformer fault type

Parameter	Type of fault
Inductances (Primary & Secondary)	Disc deformation Local breakdown Winding short
Capacitances (Primary & Secondary)	Disc movement Buckling due to large mechanical forces and moisture ingress
Resistances (Primary & Secondary)	Shortened or broken disc Partial discharge
Capacitance between Primary & Secondary	Aging of insulation

The next step is to link the coefficients $\{\hat{a}_k\}_{k=0}^{3}$ and $\{\hat{b}_k\}_{k=0}^{2}$ to the primary, secondary and the core impedance in a transformer equivalent circuit. Then it will be possible to correlate the parameters of the equivalent circuit to a fault type as shown in Table 2. This approach is currently under investigation.

4. Conclusions

In a case study, we applied a recently developed subspace-based and the nonlinear least-squares identification algorithms to obtain parametric transfer function of a two-winding power transformer from frequency response data. The two algorithms yielded high order accurate models of the transfer function of the power transformer studied. Next we reduced the identified models to obtain 4th order models. This integrated approach for model development resulted in higher accuracy than a direct identification approach.

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