

Box 1. Numerical integration algorithm

(1) Specify θ^p and $\hat{\sigma}_{11}^\infty$, and guess an initial bound of opening domain θ^* (e.g., from Taylor's model analysis).

(2) Integrate (3)–(5) over θ and θ_{10} . Note that

$$a_1 = \begin{cases} a_f, & \text{if } \theta \text{ in process domains;} \\ a_0, & \text{otherwise.} \end{cases}$$

(3) Integrate (3)–(5) over θ_{20} for each value of $\theta + \theta_{10}$. Note that

$$a_2 = \begin{cases} a_f, & \text{if } \theta_{20} \text{ in process domains;} \\ a_0, & \text{otherwise.} \end{cases}$$

(4) Integrate Eqs. (4)–(5) from $r = (a_1 + a_2)$ to $\max(20a_1, 20a_2)$.

(5) For a given fracture criterion, find the corresponding threshold stress $\hat{\sigma}_{22}^\infty$ by (26)₁. Calculate the updated opening angle bound θ^* by (26)₂ or (27). If the difference between the initial guess and the updated θ^* -value is not acceptable, then go back to Step (2) and re-iterate until proper convergence is reached.

(6) Go to Step (1) and specify new θ^p and $\hat{\sigma}_{11}^\infty$ values.