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Left-inversion soft-sensor of synchronous generator

Kaifeng Zhang*,†, Xianzhong Dai and Chongxin Huang

Key Laboratory of Measurement and Control of CSE (School of Automation, Southeast University), Ministry of Education, Nanjing 210096, China

SUMMARY

The online measurement of some crucial variables in synchronous generator (such as the power angle, the d-axis and q-axis transient electric and magnetic fields, and the d-axis and q-axis stator circuit currents) is very important in many applications, but they are difficult to be measured by ordinary sensors. In the paper, these variables are estimated (soft-sensed) on the basis of the online directly measurable variables of generator (such as the active and reactive powers and the amplitude of current). First, a new model of synchronous generator is proposed, which fully includes the online measurable variables of synchronous generator and is suitable for designing soft-sensors. Second, aiming at the specialty of the proposed model (a typical kind of nonlinear differential-algebraic subsystem), the original left-inversion soft-sensing algorithm proposed in a previous paper is expanded. Then a left-inversion soft-sensor is designed while the fourth-order transient practical model of synchronous generator is considered, and all of the to-be-estimated variables are soft-sensed. Finally, the simulation is performed on the basis of MATLAB/SimPowerSystems (MathWorks, Natick, MA, USA), in which the sixth-order subtransient practical model of generator is used to simulate the real generator, and the results demonstrate the validity of the proposed method. Copyright © 2011 John Wiley & Sons, Ltd.

KEY WORDS: electrical power systems; measurement; soft-sensor; synchronous generator; power angle

1. INTRODUCTION

In the models of synchronous generator, some variables are very difficult to be online measured using ordinary devices. Choosing the fourth-order practical model as an example, these variables include power angle δ_i, d-axis and q-axis electric and magnetic fields E'_{qi} and E'_{di}, and d-axis and q-axis stator circuit currents I_{di} and I_{qi}. However, the online measurement of these variables is very important in many applications. For example, the online measurement of δ_i is crucial for the real-time monitoring and control of power systems [1,2]; the online measurement of E'_{qi}, E'_{di}, I_{di}, and I_{qi} is necessary for the design of sophisticated controllers, such as various advanced linear and nonlinear controllers [3–5], because these variables should be feedbacked in these controllers.

To realize the online measurement of the aforementioned variables, special devices (such as the optical encoder [6] and the air gap sensor [7]) are proposed. These devices should be installed directly in the generators; then the cost would be very high, and it is not convenient. For this reason, various indirect measurement methods have been discussed. For the indirect methods, the aforementioned to-be-estimated variables are calculated on the basis of the online measurable variables. In most of the literatures, only simple classical generator models are considered. Recently, various estimation techniques are used in this field, such as the curve fitting [8] and the least squares minimization [9]. Presently, the generator models expressed by per unit equations are used in most literatures [8,9], but there is seldom research based on the practical models. In [10], the internal dynamic states of a

*Correspondence to: Kaifeng Zhang, Key Laboratory of Measurement and Control of CSE (School of Automation, Southeast University), Ministry of Education, Nanjing 210096, China.
†E-mail: kaifengzhang@seu.edu.cn

generator are computed through measuring the terminal variables, such as the active and reactive power outputs, the terminal voltage magnitude, and the field current. In [11], a divide-by-difference filter is designed to estimate the power angle of a generator by utilizing the terminal voltage, the active power, and the field voltage. In both [10] and [11], the third-order transient practical model is considered, and the proposed methods could hardly be directly expanded to more complex models, such as the fourth-order nonlinear model of generator.

In this paper, a kind of soft-sensing method (named as the left-inversion soft-sensing method [12]) is used to estimate the crucial variables of synchronous generator. The main idea of the left-inversion soft-sensing method is to construct a soft-sensor on the basis of the model of the object (or the mathematical relation between the to-be-estimated variables and the online measurable variables). The soft-sensor's inputs are the online measurable variables, whereas the outputs are the to-be-estimated variables.

Conceptually, the principle underlying the soft-sensor resembles that of the observer, or soft-sensor is a software-based state estimation [13,14]. However, different from ordinary observers, there are some typical characteristics for soft-sensors. For ordinary observers, new dynamic models should be constructed [9,15,16], and the new model's states will converge to that of the original plant. For soft-sensors, the immeasurable variables are directly calculated on the basis of the measurable variables and the information of the plant, and then there is no converging process. If the effects of noise, filtering, and the computation error are not considered, the estimation results of soft-sensor are accurate in theory. For the left-inversion soft-sensing method proposed in [12], the estimation algorithm is constructive. Thus, compared with other empirical soft-sensing methods [17], the theory of left-inversion soft-sensing method is strict, and there is no need to prove the stability.

However, the left-inversion soft-sensing method proposed in [12] is only suitable for nonlinear ordinary differential equations isolated systems. Yet, the complete model of synchronous generator is a nonlinear differential-algebraic equations (DAEs) subsystem model. Meanwhile, traditional models of synchronous generators (such as Park's model) include few online measurable variables, and then they are not suitable for designing soft-sensors.

For these reasons, a new model of synchronous generator is proposed in the paper first, which is suitable for designing soft-sensors. Second, aiming at the specialty of the proposed model (a typical nonlinear DAE subsystem), the left-inversion soft-sensing algorithm proposed in [12] is expanded, and then a left-inversion soft-sensor is designed when considering the fourth-order transient practical model of synchronous generator. Finally, the simulation is performed on the basis of MATLAB/SimPowerSystems.

2. INTRODUCTION OF ORIGINAL LEFT-INVERSION SOFT-SENSING METHOD

For the left-inversion soft-sensing method proposed in [12], the following general nonlinear system is considered.

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \tag{1}$$

where $\boldsymbol{u} \in \boldsymbol{R}^U$ are the inputs; $\boldsymbol{x} \in \boldsymbol{R}^n$ are the state variables. $\boldsymbol{x} = (\widehat{\boldsymbol{x}}, \breve{\boldsymbol{x}})$. $\widehat{\boldsymbol{x}} \in \boldsymbol{R}^l$ are the to-be-estimated variables; $\breve{\boldsymbol{x}} \in \boldsymbol{R}^{n-l}$ are the online measurable variables.

First, one may assume that there exists an assumed inherent sensor in the nonlinear system of Equation (1) (see Figure 1). For the assumed inherent sensor, the to-be-estimated variables $\widehat{x}$ are the

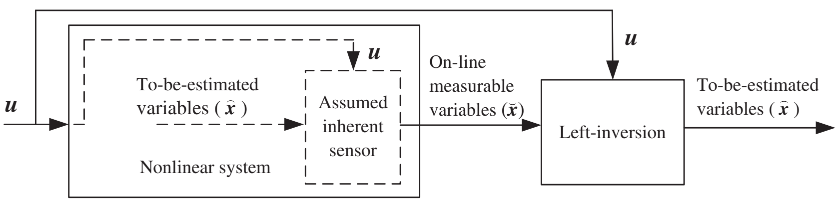

Figure 1. Diagram of original left-inversion soft-sensing method.

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inputs, the online measurable variables $\breve{x}$ are the outputs, and u are the parameter variables. The general expression of the assumed inherent sensor is

$$\breve{\boldsymbol{x}} = \breve{\boldsymbol{f}}\left(\widehat{\boldsymbol{x}}, \dot{\widehat{\boldsymbol{x}}}, \ddot{\widehat{\boldsymbol{x}}}, \ldots, \boldsymbol{u}\right) \tag{2}$$

On the basis of the algorithm proposed in [12], the assumed inherent sensor as shown in Equation (2) could be constructed. If the constructed assumed inherent sensor is left-invertible in the normal operating area of Equation (1), its left inversion could be derived. For the derived left inversion, $\breve{x}$ are the inputs, $\widehat{x}$ are the outputs, and u are the parameter variables (see Figure 1). The general expression of the left inversion is

$$\widehat{\boldsymbol{x}} = \breve{\boldsymbol{f}}\left(\breve{\boldsymbol{x}}, \dot{\breve{\boldsymbol{x}}}, \ddot{\breve{\boldsymbol{x}}}, \cdots, \boldsymbol{u}\right) \tag{3}$$

Apparently, the left inversion of the assumed inherent sensor is just the soft-sensor of the to-be-estimated variables $\widehat{x}$, and this soft-sensor is named as the "left-inversion soft-sensor."

3. NEW MODEL OF SYNCHRONOUS GENERATOR SUITABLE FOR DESIGNING SOFT-SENSORS

The key of designing soft-sensors is to fully mine and utilize the relations between the online measurable variables and the to-be-estimated variables, and it is very important to consider as more online measurable variables as possible. From this viewpoint, most of the traditional models of synchronous generator (such as Park's model, the practical models) include only few online measurable variables. For example, the fourth-order transient practical model of synchronous generator (connected to an arbitrary external network) is as follows.

The differential equations of the fourth-order transient practical model are (For the explanation of the symbols, please see Section 8.)

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_0 \\ \dot{\omega}_i = (\omega_0/H_i)\left\{P_{mi} - \left[E'_{qi}I_{qi} + E'_{di}I_{di} - \left(x'_{di} - x'_{qi}\right)I_{di}I_{qi}\right] - D_i(\omega_i - \omega_0)/\omega_0\right\} \\ \dot{E}'_{qi} = \left[E_{fi} - E'_{qi} - \left(x_{di} - x'_{di}\right)I_{di}\right]/T'_{di0} \\ \dot{E}'_{di} = \left[-E'_{di} + \left(x_{qi} - x'_{qi}\right)I_{qi}\right]/T'_{qi0} \end{cases} \tag{4}$$

The algebraic equations of the fourth-order transient practical model include the stator voltage equations (Equation (5)) and the *dq–RI* transformation equations (Equation (6)).

$$\begin{cases} U_{di} = E'_{di} - r_{ai}I_{di} + x'_{qi}I_{qi} \\ U_{qi} = E'_{qi} - x'_{di}I_{di} - r_{ai}I_{qi} \end{cases} \tag{5}$$

$$\begin{cases} \begin{bmatrix} U_{Ri} \\ U_{Ii} \end{bmatrix} = \begin{bmatrix} \cos\delta_i & \sin\delta_i \\ \sin\delta_i & -\cos\delta_i \end{bmatrix} \begin{bmatrix} U_{qi} \\ U_{di} \end{bmatrix} \\ \begin{bmatrix} I_{Ri} \\ I_{Ii} \end{bmatrix} = \begin{bmatrix} \cos\delta_i & \sin\delta_i \\ \sin\delta_i & -\cos\delta_i \end{bmatrix} \begin{bmatrix} I_{qi} \\ I_{di} \end{bmatrix} \end{cases} \tag{6}$$

In the model of Equations (4)–(6), many variables, such as δ_i, ω_i, E'_{qi}, E'_{di}, I_{di}, I_{qi}, I_{Ri}, I_{Ii}, U_{Ri}, U_{Ii}, U_{di}, and U_{qi}, cannot be online directly measured using ordinary devices (unless using special devices, such as the optical encoder).

At the same time, in real power engineering, there are some variables that can be easily measured by ordinary devices, such as the current transformers and the voltage transformers. But these online measurable variables are not included in the model of Equations (4)–(6). These online measurable variables include the following:

- The active and reactive powers at the terminal of generator (P_{ti}, Q_{ti}).
- The amplitudes of current and voltage at the terminal of generator (I_{ti}, U_{ti}).

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- The excitation current (I_{fi}).
- The phasor between the terminal voltage and an arbitrary reference (θ_{Ui}). Here, it should be noted that θ_{Ui} is online measured only when the phasor measurement unit is available.

In the following, a new model of synchronous generator will be constructed. The proposed model is *equivalent* to the model of Equations (4)–(6) in nature. Meanwhile, compared with the model of Equations (4)–(6), the proposed model includes more online measurable variables, and then it is more suitable for designing soft-sensors.

In Equations (4)–(6), $(I_{Ri}, I_{Ii}, U_{Ri}, U_{Ii})$ are used to describe the mutual relation between the generator and the rest of power systems. In the new model, a new set of variables, or interface variables $\boldsymbol{v}_i$, is used to describe this relation. Different from $(I_{Ri}, I_{Ii}, U_{Ri}, U_{Ii})$, $\boldsymbol{v}_i$ are all online measurable variables. Meanwhile, $\boldsymbol{v}_i$ should be equivalent to $(I_{Ri}, I_{Ii}, U_{Ri}, U_{Ii})$. For example, $(Q_{ti}, I_{ti}, P_{ti}, \theta_{Ui})$ is an appropriate set (other sets, such as $(U_{ti}, I_{ti}, P_{ti}, \theta_{Ui})$, can also be chosen.) because there is

$$\begin{cases} Q_{ti} = U_{Ii}I_{Ri} - U_{Ri}I_{Ii} \\ I_{ti} = \sqrt{I_{Ri}^2 + I_{Ii}^2} \\ P_{ti} = U_{Ri}I_{Ri} + U_{Ii}I_{Ii} \\ \theta_{Ui} = \arctan(U_{Ri}/U_{Ii}) \end{cases} \tag{7}$$

Here, it should be noted that in the normal operating area of generator, Equation (7) is invertible. Substituting Equations (5) and (6) into Equation (7) and eliminating $I_{Ri}, I_{Ii}, U_{Ri}, U_{Ii}, U_{di}$ and U_{qi}, we have

$$\begin{cases} Q_{ti} = E'_{qi}I_{di} - E'_{di}I_{qi} - x'_{qi}I_{qi}^2 - x'_{di}I_{di}^2 \\ I_{ti} = \left(I_{di}^2 + I_{qi}^2\right)^{0.5} \\ P_{ti} = \left[E'_{qi} + \left(x'_{qi} - x'_{di}\right)I_{di}\right]I_{qi} - r_{ai}\left(I_{di}^2 + I_{qi}^2\right) + E'_{di}I_{di} \\ \theta_{Ui} = \delta_i - \arctan\left[\left(E'_{di} + x'_{qi}I_{qi} - r_{ai}I_{di}\right)/\left(E'_{qi} - x'_{di}I_{di} - r_{ai}I_{qi}\right)\right] \end{cases} \tag{8}$$

Meanwhile, I_{fi} is also online measurable, and there is

$$I_{fi} = \left[E'_{qi} + \left(x_{di} - x'_{di}\right)I_{di}\right]/x_{adi} \tag{9}$$

Combining Equations (4), (8) and (9), one can get the new model of generator (see Figure 2).

The variables in the new model of synchronous generator could be classified as follows:

- $\boldsymbol{x}_i = (\breve{\boldsymbol{x}}_i, \widehat{\boldsymbol{x}}_i)^T$ are the state variables. $\breve{\boldsymbol{x}}_i \in R^{X_i - L_i}$ are the online measurable state variables; $\widehat{\boldsymbol{x}}_i \in R^{L_i}$ are the to-be-estimated state variables.
- $\boldsymbol{w}_i = (\breve{\boldsymbol{w}}_i, \widehat{\boldsymbol{w}}_i)^T$ are the intermediate variables. In theory, these variables can be eliminated. $\breve{\boldsymbol{w}}_i \in R^{W_i - S_i}$ are the online measurable intermediate variables; $\widehat{\boldsymbol{w}}_i \in R^{S_i}$ are the to-be-estimated intermediate variables.
- $\boldsymbol{v}_i \in R^{M_i}$ are the interface variables, all of which are online measurable.
- $\boldsymbol{u}_i \in R^{U_i}$ are the input variables.

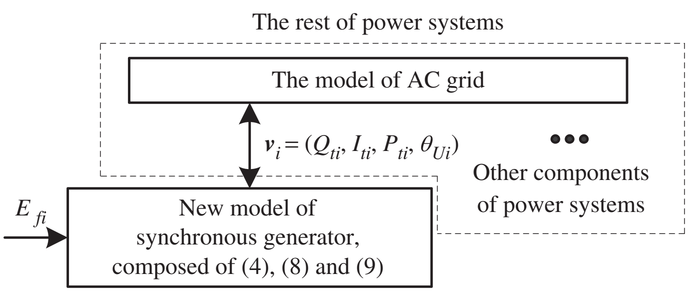

Figure 2. Diagram of the new model of synchronous generator.

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For the new model, there are $\breve{\boldsymbol{x}}_i = \varnothing$ (or there are no online measurable state variables), $\widehat{\boldsymbol{x}}_i = \left(\delta_i, \omega_i, E'_{qi}, E'_{di}\right)$, $\breve{\boldsymbol{w}}_i = \left(I_{fi}\right)$, $\widehat{\boldsymbol{w}}_i = \left(I_{di}, I_{qi}\right)$, $\boldsymbol{v}_i = (Q_{ti}, I_{ti}, P_{ti}, \theta_{Ui})$, and $\boldsymbol{u}_i = (E_{fi})$. Here, it should be noted that ω_i could be directly computed on the basis of δ_i, or there is $\omega_i = \dot{\delta}_i + \omega_0$, and then $\widehat{\boldsymbol{x}}_i$ could also be defined as $\left(\delta_i, E'_{qi}, E'_{di}\right)$.

Thus, the general expression of the new model of synchronous generator as shown in Figure 2 can be expressed as follows.

$$\begin{cases} \dot{\breve{\boldsymbol{x}}}_i = \breve{\boldsymbol{f}}_i^{w}(\breve{\boldsymbol{x}}_i, \widehat{\boldsymbol{x}}_i, \widehat{\boldsymbol{w}}_i, \boldsymbol{u}_i) \\ \dot{\widehat{\boldsymbol{x}}}_i = \widehat{\boldsymbol{f}}_i^{w}(\breve{\boldsymbol{x}}_i, \widehat{\boldsymbol{x}}_i, \widehat{\boldsymbol{w}}_i, \boldsymbol{u}_i) \\ \boldsymbol{v}_i = \boldsymbol{h}_i^{v}(\widehat{\boldsymbol{w}}_i, \breve{\boldsymbol{x}}_i, \widehat{\boldsymbol{x}}_i) \\ \breve{\boldsymbol{w}}_i = \boldsymbol{h}_i^{w}(\widehat{\boldsymbol{w}}_i, \breve{\boldsymbol{x}}_i, \widehat{\boldsymbol{x}}_i) \end{cases} \tag{10}$$

Equation (10) is a typical nonlinear DAE subsystem:

- It is a nonlinear DAE subsystem. There are differential equations and algebraic equations. Via interface variables $\boldsymbol{v}_i$, Equation (10) interact with the rest of power systems (see Figure 2).
- The expressions of the algebraic equations are very typical. Or the online measurable variables $\boldsymbol{v}_i$ and $\breve{\boldsymbol{w}}_i$ could be expressed by $\widehat{\boldsymbol{w}}_i$, $\breve{\boldsymbol{x}}_i$, and $\widehat{\boldsymbol{x}}_i$ explicitly.

It should be noted that when other models of synchronous generator (such the third-order classical or practical models) are considered, the expression of Equation (10) is also with general meanings.

4. THE EXPANSION OF ORIGINAL LEFT-INVERSION SOFT-SENSING ALGORITHM

When constructing soft-sensors, the algorithm proposed in [12] is only suitable for the general nonlinear ordinary differential equation isolated system as shown in Equation (1). For the soft-sensing of synchronous generator, the considered object is a typical nonlinear DAE subsystem as shown in Equation (10), and then original algorithm given in [12] should be expanded (in this section, for the concision of the expressions, the subscript i of all of the variables will be omitted).

The expanded soft-sensing algorithm:

Step 1. We calculate the rank of the following Jacobi matrix (the online measurable variables $\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}$ to the to-be-estimated variables $\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{w}}$):

$$\boldsymbol{J}_1 = \frac{\partial\left(\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}\right)^T}{\partial(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{w}})} \tag{11}$$

If $rank(\boldsymbol{J}_1) = L + S$, the desired assumed inherent sensor could be constructed by $L + S$ independent variables of $\left(\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}\right)$. Define these $L + S$ variables as Z_1, and then there should exist $rank(\partial Z_1 / \partial(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{w}})) = L + S$. Thus, according to the Implicit Function Theorem, there is

$$(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{w}}) = \boldsymbol{f}_1^{-1}(Z_1) \tag{12}$$

This is just the expression of the left-inversion soft-sensor.

Otherwise, if $rank(\boldsymbol{J}_1) < L + S$, this means that the derivatives of some variables of $\left(\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}\right)$ are required to construct the assumed inherent sensor. The algorithm goes to step 2.

Step 2. The first-order derivative of one interface variables (assuming it is v_1) is considered. Then from Equation (10), it can be seen that some other directly immeasurable variables ($\dot{\widehat{\boldsymbol{w}}}_i$) may appear in $\dot{v}_1$. Assuming the number of $\dot{\widehat{\boldsymbol{w}}}_i$ in $\dot{v}_1$ is q_2, we calculate the rank of the following Jacobi matrix:

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$$\boldsymbol{J}_2 = \frac{\partial\left(\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}, \dot{v}_1\right)^T}{\partial\left(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{w}}, \dot{\widehat{w}}_1, \dot{\widehat{w}}_2, \ldots, \dot{\widehat{w}}_{q_2}\right)} \tag{13}$$

If $rank(\boldsymbol{J}_2) = L + S + q_2$, the desired assumed inherent sensor could be constructed by $L + S + q_2$ independent variables of $\left(\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}, \dot{v}_1\right)$. Define them as Z_2, and then the following left-inversion soft-sensor could be constructed.

$$\left(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{w}}, \dot{\widehat{w}}_1, \dot{\widehat{w}}_2, \ldots, \dot{\widehat{w}}_{q_2}\right) = \boldsymbol{f}_2^{-1}(Z_2) \tag{14}$$

Otherwise, if $rank(\boldsymbol{J}_2) < L + S + q_2$, the algorithm goes to the next step.

Step $M+1$. The first-order derivative of v_M is considered. Assuming the number of $\dot{\widehat{\boldsymbol{w}}}_i$ in $\dot{\boldsymbol{v}}$ (or $\dot{v}_1, \ldots, \dot{v}_M$) is q_{M+1}, we calculate the rank of the following Jacobi matrix:

$$\boldsymbol{J}_{M+1} = \frac{\partial\left(\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}, \dot{\boldsymbol{v}}\right)^T}{\partial\left(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{w}}, \dot{\widehat{w}}_1, \dot{\widehat{w}}_2, \ldots, \dot{\widehat{w}}_{q_{M+1}}\right)} \tag{15}$$

If $rank(\boldsymbol{J}_{M+1}) = L + S + q_{M+1}$, the soft-sensor could be constructed successfully. Otherwise, the algorithm goes to the next step.

Step $M+2$. The first-order derivative of one online measurable intermediate variables (assuming it is $\breve{w}_1$) is considered. Assuming the number of $\dot{\widehat{\boldsymbol{w}}}_i$ in $\left(\dot{\boldsymbol{v}}, \dot{\breve{w}}_1\right)$ (or $\left(\dot{v}_1, \ldots, \dot{v}_{2m}, \dot{\breve{w}}_1\right)$) is q_{M+2}, we calculate the rank of the following Jacobi matrix:

$$\boldsymbol{J}_{M+2} = \frac{\partial\left(\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}, \dot{\boldsymbol{v}}, \dot{\breve{w}}_1\right)^T}{\partial\left(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{w}}, \dot{\widehat{w}}_1, \dot{\widehat{w}}_2, \cdots, \dot{\widehat{w}}_{q_{M+2}}\right)} \tag{16}$$

If $rank(\boldsymbol{J}_{M+2}) = L + S + q_{M+2}$, the soft-sensor could be constructed successfully. Otherwise, the algorithm goes to the next step.

Step $M+1+(W\text{-}S)$. The first-order derivative of $\breve{w}_{W-S}$ is considered. Assuming the number of $\dot{\widehat{\boldsymbol{w}}}_i$ in $\left(\dot{\boldsymbol{v}}, \dot{\breve{\boldsymbol{w}}}\right)$ is $q_{M+1+(W-S)}$, we calculate the rank of the following Jacobi matrix:

$$\boldsymbol{J}_{M+1+(W-S)} = \frac{\partial\left(\boldsymbol{v}, \breve{\boldsymbol{w}}, \dot{\breve{\boldsymbol{x}}}, \dot{\boldsymbol{v}}, \dot{\breve{\boldsymbol{w}}}\right)^T}{\partial\left(\widehat{\mathrm{x}}, \widehat{\boldsymbol{w}}, \dot{\widehat{w}}_1, \dot{\widehat{w}}_2, \ldots, \dot{\widehat{w}}_{q_{M+1+(W-S)}}\right)} \tag{17}$$

If $rank$ $(\boldsymbol{J}_{M+1+(W-S)}) = L + S + q_{M+1+(W-S)}$, the soft-sensor could be constructed successfully. Otherwise, the algorithm goes to the next step.

Similarly, Step $M + (W - S) + 2$ to $M + 1 + (W - S) + (X - L)$ can be taken to consider the second-order derivative of the online measurable state variables (Here, the detailed procedures are omitted). After Step $M + 1 + (W - S) + (X - L)$, if the soft-sensor still cannot be constructed successfully, the algorithm should stop. In this case, it means that the higher-order derivatives, such as $\ddot{\boldsymbol{v}}, \ddot{\breve{\boldsymbol{w}}}, \dddot{\breve{\boldsymbol{x}}}, \ldots$, should be used to construct the soft-sensor. Yet, as is well known, the high-order differentiators of variables are very difficult to acquire precisely in real engineering. Thus, above soft-sensing method is not useful for this object.

Finally, it should be noted that for the algorithm proposed in [12], in every step when judging the invertibility of corresponding Jacobi matrix, the Jacobi matrix should be full rank in all of the operating areas of the object. However, in this paper, we think that this requirement is too strict. Actually, if the Jacobi matrix is full rank in most of the operating areas of the object apart from certain very small areas, it is still possible to construct the soft-sensor successfully. In Section 6, the simulation results will demonstrate the reasonability of this idea.

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5. CONSTRUCTION OF THE LEFT-INVERSION SOFT-SENSOR OF SYNCHRONOUS GENERATOR

On the basis of the expanded algorithm proposed in Section 4 and aiming at the model of synchronous generator as shown in Figure 2, the left-inversion soft-sensor of synchronous generator will be constructed in this section.

Step 1. We calculate the rank of the following Jacobi matrix:

$$\boldsymbol{J}_1 = \frac{\partial\left(P_{ti}, I_{ti}, I_{fi}, Q_{ti}, \theta_{Ui}\right)^T}{\partial\left(\delta_i, E'_{qi}, E'_{di}, I_{di}, I_{qi}\right)} \tag{18}$$

It could be calculated that $\det(\boldsymbol{J}_1) = I_{ti}\left[E'_{di} - \left(x_{di} - x'_{qi}\right)I_{qi}\right]/x_{adi}$. In the next section, the simulation results will show that in many normal operating points there is $\det(\boldsymbol{J}_1) = 0$. Then there is $rank(\boldsymbol{J}_1) < 5$.

Step 2. The first-order derivative of P_{ti} is considered. From the third equation in Equation (8), or $P_{ti} = \left[E'_{qi} + \left(x'_{qi} - x'_{di}\right)I_{di}\right]I_{qi} - r_{ai}\left(I_{di}^2 + I_{qi}^2\right) + E'_{di}I_{di}$, it could be seen that two additional immeasurable variables (or $\dot{I}_{di}$ and $\dot{I}_{qi}$) will appear. Then we calculate the rank of the following Jacobi matrix:

$$\boldsymbol{J}_2 = \frac{\partial\left(P_{ti}, I_{ti}, I_{fi}, Q_{ti}, \theta_{Ui}, \dot{P}_{ti}\right)^T}{\partial\left(\delta_i, E'_{qi}, E'_{di}, I_{di}, I_{qi}, \dot{I}_{di}, \dot{I}_{qi}\right)} \tag{19}$$

Apparently, in all of the operating areas of power systems, there is $rank(\boldsymbol{J}_2) < 7$.

Step 3. The first-order derivative of I_{ti} is considered, and then we calculate the rank of the following Jacobi matrix:

$$\boldsymbol{J}_3 = \frac{\partial\left(P_{ti}, I_{ti}, I_{fi}, Q_{ti}, \theta_{Ui}, \dot{P}_{ti}, \dot{I}_{ti}\right)^T}{\partial\left(\delta_i, E'_{qi}, E'_{di}, I_{di}, I_{qi}, \dot{I}_{di}, \dot{I}_{qi}\right)} \tag{20}$$

It could be calculated that $\det(\boldsymbol{J}_3) = \left[E'_{di} - \left(x_{di} - x'_{qi}\right)I_{qi}\right] \cdot \left[\left(x'_{qi} - x'_{di}\right)\left(I_{qi}^2 - I_{di}^2\right) + E'_{di}I_{qi} - E'_{qi}I_{di}\right]/x_{adi}$. In the next section, the simulation results will show that in many normal operating points there is $\det(\boldsymbol{J}_3) = 0$. Then there is $rank(\boldsymbol{J}_3) < 7$.

Step 4. The first-order derivative of I_{fi} is considered, and then we calculate the rank of the following Jacobi matrix:

$$\boldsymbol{J}_4 = \frac{\partial\left(P_{ti}, I_{ti}, I_{fi}, Q_{ti}, \theta_{Ui}, \dot{P}_{ti}, \dot{I}_{ti}, \dot{I}_{fi}\right)^T}{\partial\left(\delta_i, E'_{qi}, E'_{di}, I_{di}, I_{qi}, \dot{I}_{di}, \dot{I}_{qi}\right)} \tag{21}$$

In most of the operating areas of power systems, there is $rank(\boldsymbol{J}_4) = 7$. In fact, in the next section, the simulation results will show that in most of the operating areas (apart from a very small transient area), there is

$$rank\left(\boldsymbol{J}'_4\right) = rank\left(\frac{\partial\left(P_{ti}, I_{ti}, I_{fi}, \theta_{Ui}, \dot{P}_{ti}, \dot{I}_{ti}, \dot{I}_{fi}\right)^T}{\partial\left(\delta_i, E'_{qi}, E'_{di}, I_{di}, I_{qi}, \dot{I}_{di}, \dot{I}_{qi}\right)}\right) = 7 \tag{22}$$

Thus, the assumed inherent sensor is constructed successfully, and then the left-inversion soft-sensor could be derived. First, the expressions of $\dot{I}_{di}$ and I_{di} could be derived (For the concision of the expressions, Q_{ti} is held here).

$$\dot{I}_{di} = \left[x_{adi}\dot{I}_{fi} - \left(E_{fi} - x_{adi}I_{fi}\right)/T'_{di0}\right]/\left(x_{di} - x'_{di}\right) \tag{23}$$

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$$I_{di} = \left(acp_2 - a^2p_0 - bcp_3\right)/\left(b^2p_3 + a^2p_1 - acp_3 - abp_2\right) \tag{24}$$

where

$$\begin{aligned}
a &= \left(P_{ti} + r_{ai}I_{ti}{}^2\right)^2 + \left(Q_{ti} + x_{di}I_{ti}{}^2\right)^2 \\
b &= -2x_{adi}I_{fi}I_{ti}{}^2\left(Q_{ti} + x_{di}I_{ti}{}^2\right) \\
c &= \left(x_{adi}I_{fi}\right)^2I_{ti}{}^4 - I_{ti}{}^2\left(P_{ti} + r_{ai}I_{ti}{}^2\right)^2 \\
p_0 &= \left[\left(E_{fi} - x_{adi}I_{fi}\right)/T'_{di0} + \left(x'_{qi} - x'_{di}\right)\dot{I}_{di} - \left(\dot{P}_{ti} + 2r_{ai}\dot{I}_{ti}I_{ti}\right)x_{adi}I_{fi}/\left(P_{ti} + r_{ai}I_{ti}{}^2\right)\right]I_{ti}{}^2 \\
&\quad + x_{adi}I_{fi}\dot{I}_{ti}I_{ti} - \left(Q_{ti} + x'_{qi}I_{ti}{}^2\right)\dot{I}_{di} \\
p_1 &= \left(Q_{ti} + x_{di}I_{ti}{}^2\right)/T'_{qi0} - \left(x_{di} - x'_{qi}\right)\dot{I}_{ti}I_{ti} + \left(Q_{ti} + x_{di}I_{ti}{}^2\right)\left(\dot{P}_{ti} + 2r_{ai}\dot{I}_{ti}I_{ti}\right)/\left(P_{ti} + r_{ai}I_{ti}{}^2\right) \\
p_2 &= \left(x'_{di} - x'_{qi}\right)\dot{I}_{di} - x_{adi}I_{fi}/T'_{qi0} - \left(E_{fi} - x_{adi}I_{fi}\right)/T'_{di0} \\
p_3 &= \left(x_{di} - x_{qi}\right)/T'_{qi0}
\end{aligned}$$

Substituting Equations (23) and (24) into the following equations, one can have the expressions of I_{qi}, E'_{qi}, E'_{di}, and δ_i.

$$\begin{cases}
I_{qi} = \left[x_{adi}I_{fi}I_{ti}{}^2 - (Q_{ti} + x_{di}I_{ti}{}^2)I_{di}\right]/(P_{ti} + r_{ai}I_{ti}{}^2) \\
E'_{qi} = x_{adi}I_{fi} - \left(x_{di} - x'_{di}\right)I_{di} \\
E'_{di} = \left\{\left[(P_{ti} + r_{ai}I_{ti}{}^2) - E'_{qi}I_{qi} + \left(x'_{qi} - x'_{di}\right)I_{di}I_{qi}\right]^2 \right. \\
\qquad \left. + \left(E'_{qi}I_{di} - Q_{ti} - x'_{qi}I_{qi}{}^2 - x'_{di}I_{di}{}^2\right)^2\right\}^{0.5}/I_{ti} \\
\delta_i = \theta_{Ui} + \arctan\left[\left(E'_{di} + x'_{qi}I_{qi} - r_{ai}I_{di}\right)/\left(E'_{qi} - x'_{di}I_{di} - r_{ai}I_{qi}\right)\right]
\end{cases} \tag{25}$$

The expressions of Equations (23)–(25) are the final left-inversion soft-sensor of synchronous generator (see Figure 3). Here, for $\dot{I}_{di}$ and $\dot{I}_{qi}$ are not the to-be-estimated variables, there is no need to calculate them in Figure 3.

6. SIMULATIONS

6.1. Introduction of the simulations

To study the performance of the proposed left-inversion soft-sensor, simulations are performed on the basis of MATLAB/SimPowerSystems. The simulation system is as shown in Figure 4. In the simulations, a three-phase fault occurs at 0.1 s and lasts 0.1 s.

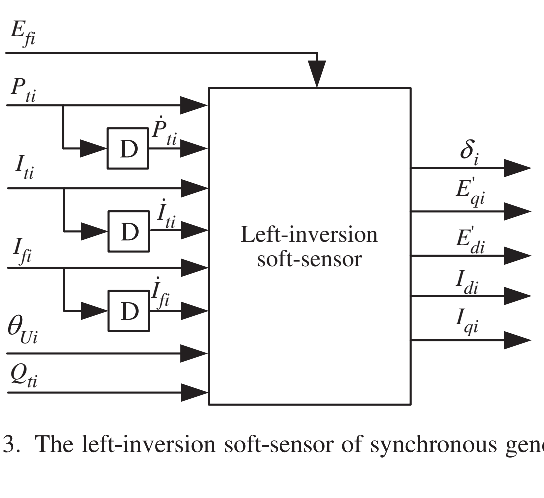

Figure 3. The left-inversion soft-sensor of synchronous generator.

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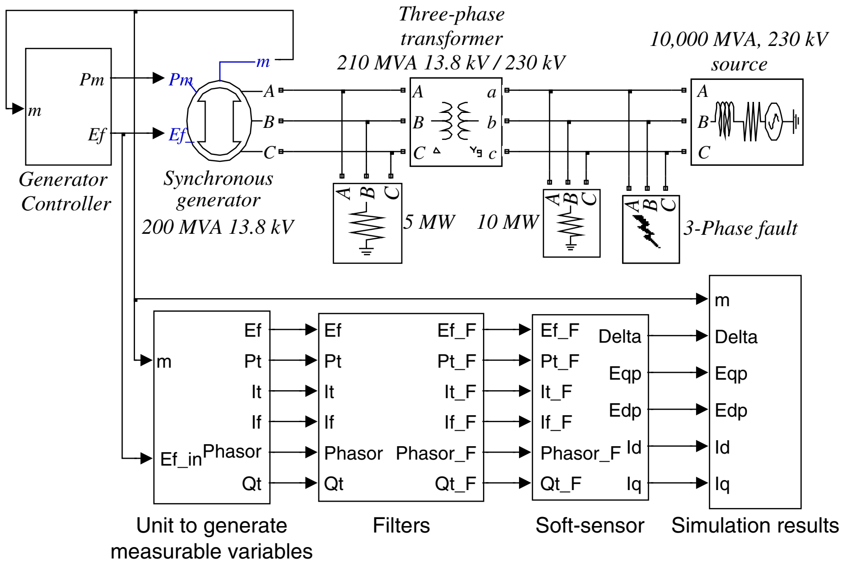

Figure 4. The simulation system.

In Figure 4, an output of synchronous generator, or m, is a vector containing 22 signals [18]. Some of the input variables of the soft-sensor are included in m, and others could be simply calculated from m. Thus, the inputs of the soft-sensor could be generated first. Then these variables should be filtered, and the left-inversion soft-sensor as shown in Equations (23)–(25) could be designed. Here, the to-be-estimated variables can also be directly exported from m (see Figure 4), and they could be seen as the "real" results. Thus, in the simulation, the soft-sensor results could be compared with the "real" results to justify the validity of the proposed method.

Here, it should be noted that the model of synchronous generator used in the simulation is a more complex model, or the sixth-order practical model (subtransient model). Thus, when deriving the soft-sensor of Equations (23)–(25), the synchronous generator used in the simulation could be seen as a "*black box*." Meanwhile, the d-axis saturation characteristic of synchronous generator is also considered in the simulation. For the detailed explanation about the sixth-order practical model and the saturation curve of the synchronous generator, please refer to [18].

6.2. Design of filters and differentiators

Because a complex generator model and a large disturbance are considered in the simulation, there would be some harmonics in signals, especially in the transient period (The simulation result of I_{ti} is given in Figure 5). Thus, the inputs of the soft-sensor should be filtered first.

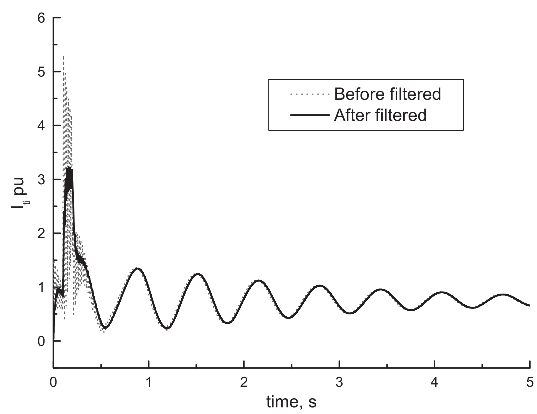

Figure 5. The simulation results of I_{ti} before and after filtered.

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Choosing I_{ti} as an example, a filter with the transfer function of $1/(0.02s+1)$ is designed. For the designed filter, the amplitude decay is 0.98 and the phase shift is 1.83°. The simulation result of I_{ti} after filtered is also given in Figure 5. Meanwhile, real differentiators with the transfer function of $s/(0.02s+1)$are designed to replace the ideal differentiators (D) in Figure 3

6.3. Simulation results and discussions

For the aforementioned simulation system and the corresponding disturbance, the determinants of Jacobi Matrixes $\boldsymbol{J}_1$ and $\boldsymbol{J}_3$ (see Equations (18) and (20)) are given in Figure 6, and that of $\boldsymbol{J}_4^{'}$ (see Equation (22)) are given in Figure 7. It can be seen that in many normal operating points, there are $\det(\boldsymbol{J}_1)=0$ and $\det(\boldsymbol{J}_3)=0$. However, in most operating areas, there is $\det\left(\boldsymbol{J}_4^{'}\right) \neq 0$. Only in a small transient area of 0.5–0.7 s, there exist the points of $\det\left(\boldsymbol{J}_4^{'}\right) = 0$.

The soft-sensing results are given in Figures 8–12. In these figures, the solid lines are the soft-sensing results, and the dash lines are the "real" results. It can be seen that in most operating areas, the soft-sensing results could match the "real" results well, or the performance of the proposed soft-sensor is quite well.

Meanwhile, one can also see that in two small areas, the performance is not as well as that in other areas. First, in 0.1 s when the disturbance occurs, the value of some signals (such as the current) may change significantly, and then the derivative of these signals will be very large. Thus, some pulses may exist in the results. Second, in the area of 0.5–0.7 s, for there are some points of $\det\left(\boldsymbol{J}_4^{'}\right) = 0$ (see Figure 7), some singular points may exist, and then the denominator of some expressions (see Equations (23)–(25)) may equal to zero. Thus, the values of the soft-sensed signals would be very large. Because these areas are very small, by using appropriate filters, these pulses could be filtered. Overall, the soft-sensing results are satisfactory and useful.

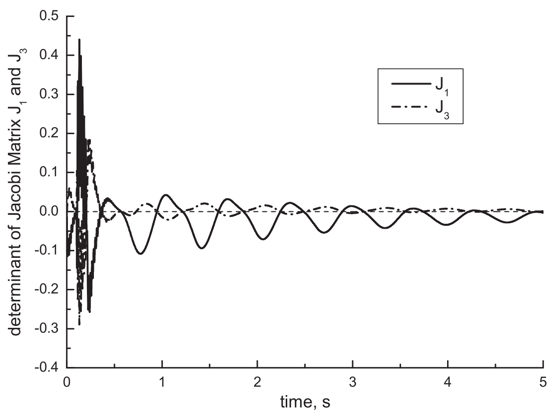

Figure 6. The determinants of Jacobi matrixes J_1 and J_3.

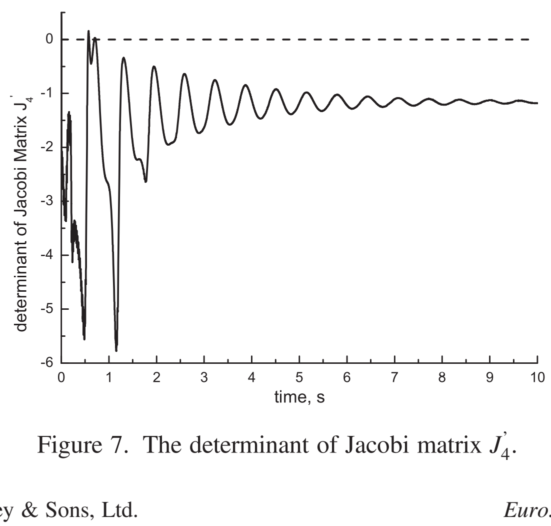

Figure 7. The determinant of Jacobi matrix $J_4^{'}$.

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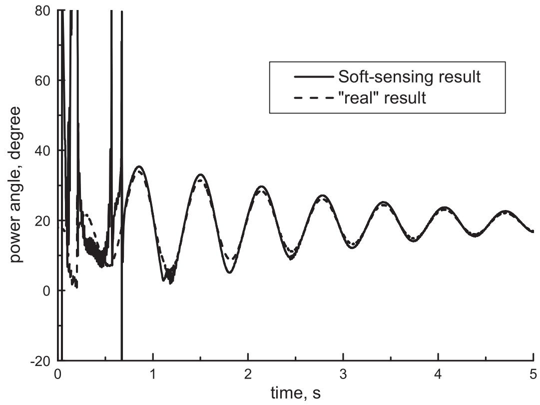

Figure 8. The soft-sensing results of δ_i.

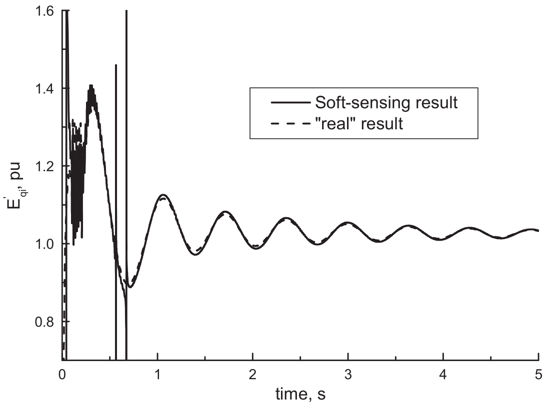

Figure 9. The soft-sensing results of $E^{'}_{qi}$.

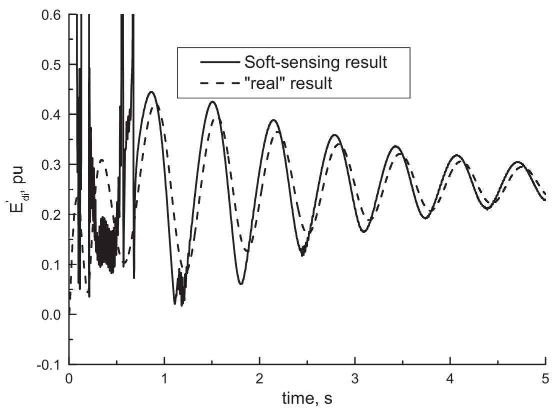

Figure 10. The soft-sensing results of $E^{'}_{di}$.

For there is noises in actual practice, the effect of noise on the proposed method should be evaluated. Here, we generate a white noise signal whose noise power is 5×10^{-6} (SNR = 21.3 dB) by using the band-limited white noise block of MATLAB and add it to the measured variables (terminal currentI_{ti}) when doing the tests. The simulation results of I_{ti} and $E^{'}_{qi}$ are given in Figures 13 and 14. From Figure 13, it can be seen that the noise power is considerable. However, the soft-sensing results of $E^{'}_{qi}$ are still satisfactory. The absolute error of $E^{'}_{qi}$ is no more than 0.04 pu after 1 s.

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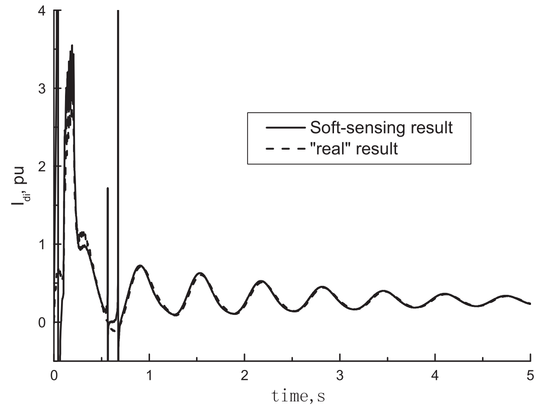

Figure 11. The soft-sensing results of I_{di}.

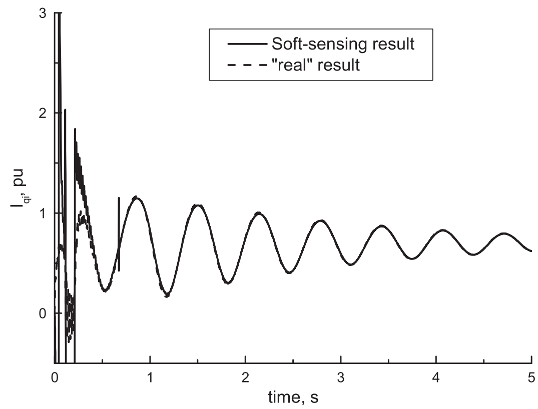

Figure 12. The soft-sensing results of I_{qi}.

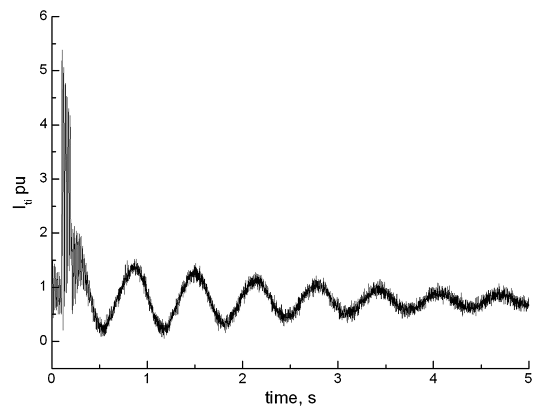

Figure 13. The simulation results of I_{ti} with noise.

7. CONCLUSIONS

A left-inversion soft-sensor is designed in the paper to estimate the crucial variables in synchronous generator, including δ_i, $E^{'}_{qi}$, $E^{'}_{di}$, I_{di}, and I_{qi}. Compared with the indirect method proposed in [10], the soft-sensing method proposed in this paper is more general. The proposed method of constructing the new model of synchronous generator and the proposed expanded algorithm are also valuable and applicable for other cases. Thus, even other models of synchronous generator (such the third-order classical or practical models) are considered, the proposed soft-sensing method can also be used.

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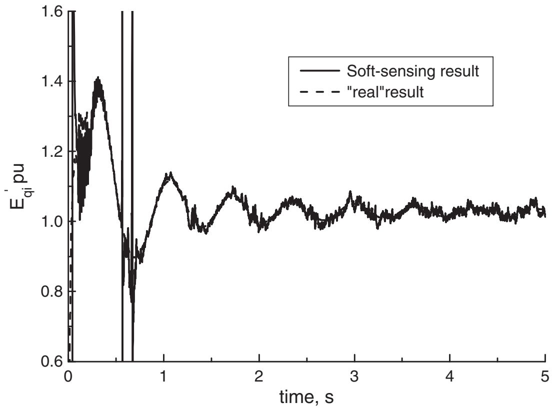

Figure 14. The soft-sensing results of E'_{qi} under noise.

8. LIST OF SYMBOLS

δ_i	Power angle of synchronous generator (degree)
ω_i	Rotor speed of synchronous generator (rad/s)
ω_0	Synchronous speed (rad/s)
H_i	Inertia constant of synchronous generator (s)
D_i	Damping constant of synchronous generator (pu)
P_{mi}	Mechanic power of synchronous generator (pu)
E'_{qi}, E'_{di}	*q*-axis and *d*-axis transient electric and magnetic fields of synchronous generator (pu)
x_{di}, x_{qi}	*d*-axis and *q*-axis synchronous reactances of synchronous generator (pu)
x'_{di}, x'_{qi}	*d*-axis and *q*-axis transient reactances of synchronous generator (pu)
E_{fi}	Excitation control input of synchronous generator (pu)
r_{ai}	Resistance of the armature of synchronous generator (pu)
x_{qdi}	*d*-axis armature reaction reactance of synchronous generator (pu)
T'_{di0}, T'_{qi0}	*d*-axis and *q*-axis transient open circuit time constants of synchronous generator (s)
U_{di}, U_{qi}	*d*-axis and *q*-axis components of the terminal voltage (pu)
I_{di}, I_{qi}	*d*-axis and *q*-axis stator circuit currents (pu)
U_{Ri}, U_{Ii}	*R*-axis and *I*-axis (or called as x-axis and y-axis, respectively) components of the terminal voltage (pu)
I_{Ri}, I_{Ii}	*R*-axis and *I*-axis components of the terminal current (pu)
P_{ti}, Q_{ti}	Active and reactive powers at the terminal of generator (pu)
I_{ti}, U_{ti}	Amplitudes of current and voltage at the terminal of generator (pu)
I_{fi}	Excitation current of synchronous generator (pu)
θ_{Ui}	Phasor between the terminal voltage and an arbitrary reference (degree)
x_i	State variables
$\breve{x}_i$	Online measurable state variables
$\widehat{x}_i$	To-be-estimated state variables
w_i	Intermediate variables
$\breve{w}_i$	Online measurable intermediate variables
$\widehat{w}_i$	To-be-estimated intermediate variables
v_i	Interface variables
u_i	Input variables
J_i	Jacobi matrix

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