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# Event driven tracking control algorithm for marine vessel based on backstepping method

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ABSTRACT

In this paper, an event driven tracking control algorithm based backstepping method is proposed for marine vessel. A tracking controller is designed by using backstepping method at the beginning to guarantee global asymptotic tracking of the desired position or trajectory. The proposed event driven tracking control algorithm is achieved by extending the designed continuous time tracking controller. The event driven conditions are developed to determine the updated time instants of the event driven controller. The proposed event driven tracking controller could ensure that the error of tracking is uniform ultimately bounded and no Zeno behavior. Compared to the existing continuous or discrete controller, the communication quantity will be reduced by introducing the event driven conditions, and meanwhile it will lead to little executions of actuator. The performance of this algorithm is finally demonstrated through the simulation results.

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## 1. Introduction

This paper investigates the problem of tracking control for marine vessel. There are many research results about control method of dynamic position, trajectory tracking or path following for marine vessel [1,2]. The backstepping method is often used to research the marine control in view of the fact that it is a better way to deal with nonlinearity problem and it is convenience for the further design of adaptive control schemes. Related results can be referred to the literatures as [3,4]. In the existing research results, the designed controllers broadly fall into two categories: continuous and discrete control. In particular, discrete control has been widely used in practical applications in view of the lower installation cost of microprocessors. The relative studies on discrete control for marine vessel could refer to the literatures [5,6]. These discrete control approaches depend on periodic sampling of the sensors and execution of control law, which requires to communicate the state (which includes the actuator state) information at every sampling instant. This will lead to unnecessary energy consumption and the wear of actuator due to the frequent changes of actuator status.

Some other control mechanisms have recently attracted significant attention, such as data driven control [7,8] and event driven control [9]. The data driven is widely used in some areas [10,11]. The event driven control could be considered as sampled-data control which may be aperiodic sampling. Compared with the periodic sampling of traditional discrete-time control, the sampling of calculation or execution of the event driven control is activated only when certain events occur or meet certain conditions. For the practical applications of the marine vessel, it is no doubt that the event driven tracking controller can make better use of the communication resources and extend the service life of propellers or thruster. Thus the event driven control will be more suitable for some applications of marine control.

Relevant studies on the event driven control are widely discussed recently. The state-feedback event driven control scheme [12–14,17] and model-based event driven control strategy [15] are proposed for the linear systems, respectively. The theoretical studies on event driven control are carried out for nonlinear systems [16]. Specifically, event driven control method is developed for input–output linearizable nonlinear systems which have internal dynamics, and applied to practical reactor temperature [18]. A robust event driven approach is proposed for nonlinear uncertain systems in the strict-feedback form based small-gain method, the designed controller is robust to the external disturbances [19]. The decentralized asynchronous event driven control methods are proposed for linear system and nonlinear system, respectively. The architecture in which is a system with full state feedback and with distributed sensors [20]. The small-gain approach to event driven control is also developed for nonlinear systems with state feedback and output feedback [21]. A framework and a universal

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formula are proposed for event driven stabilization of nonlinear systems [22,23]. The above theoretical results of event driven control for nonlinear system is useful for the practical applications. The issues of event driven tracking control for nonlinear systems are studied in [24,25], the proposed event driven control scheme is achieved by extending the continuous-time tracking control law, and assuming a reference system with exogenous input, the solution of which is desired trajectory. Recently, the applications of the event driven tracking control are investigated for unicycle mobile robots and fully actuated surface vessel in [26] and [27], respectively. The event driven control has great potential applications in marine control in view of the limited communication resources and the requirement of service life. However, existing results of event driven control are extremely rare.

The main contribution of this paper is that an event driven tracking control algorithm is proposed for marine vessel to ensure that the error of tracking is uniform ultimately bounded and no Zeno behavior. The continuous-time nonlinear tracking controller is firstly designed using backstepping method. The proposed algorithm is achieved by constructing the event driven conditions and extending the designed continuous-time tracking controller. Compared with the traditional tracking controller for the marine vessel, the designed control input is updated only when meet certain event conditions. The proposed event driven control method could significantly reduce the controller implementation frequency and the communication quantity. Thus the efficient energy consumption could be guaranteed and the service time of vessel could be improved, meanwhile it can ensure that the tracking performance is similar to that of the conventional method, if not better.

The remainder sections are organized as follows. Section 2 presents the mathematical model of marine vessel. Section 3 discusses the proposed event-triggered trajectory tracking control algorithm. In Section 4, simulations are carried out to validate the developed tracking algorithm. Finally, Section 5 provides the conclusions.

## 2. Mathematical model of marine vessel

In this paper, the tracking operations for marine vessel is focused on low-speed applications, so 3 degrees of freedom (DOF) model for marine vessel is considered. For marine vessel, $\eta = [N,\ E,\ \psi]^T$ represents north, east locations and heading in the frame of earth reference. $v = [u,\ v,\ r]^T$ denotes the velocities of surge, sway and yaw in the frame of body reference. The mathematical model for marine vessel is as follows [1]:

$$\begin{aligned} &\dot{\eta} = R(\psi)v \\ &M\dot{v} + C(v)v + D(v)v = \tau \end{aligned} \tag{1}$$

$R(\psi)$ is a matrix that used to transform from body to earth reference frame, the form of which is:

$$R(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

It satisfies $RR^T = R^TR = I_3$. $M \in \mathbb{R}^{3\times3}$, $C(v) \in \mathbb{R}^{3\times3}$ and $D(v) \in \mathbb{R}^{3\times3}$ are system inertia, Coriolis-centripetal and damping matrices, respectively. $\tau$ is the control input of forces and torques to vessel.

## 3. Event driven tracking algorithm

In order to reduce costs, the new designed tracking control algorithm is better to expand and deepen the existing controller, due to nonlinear kinematic and nonlinear dynamics that are included in the maneuvering model of marine vessel. In the practical applications, the controller are often designed using the backstepping method. Therefore, in this section, the tracking controller is designed based on backstepping method firstly, and then the event driven tracking controller is achieved by extending the designed control law.

### 3.1. Tracking controller design based on backstepping method

Define the position tracking error is:

$$\tilde{\eta} = \eta - \eta_d \tag{3}$$

Differentiating the above equation, then we have:

$$\dot{\tilde{\eta}} = \dot{\eta} - \dot{\eta}_d \tag{4}$$

Assume that a reference velocity of marine vessel is $v_r$, define the velocity error as:

$$\tilde{v} = v - v_r \tag{5}$$

Define a Lyapunov function as:

$$V_1 = \frac{1}{2}\tilde{\eta}^T\tilde{\eta} \tag{6}$$

Differentiating the above equation yields:

$$\dot{V}_1 = \tilde{\eta}^T R(\psi)\tilde{v} + \tilde{\eta}^T R(\psi)v_r - \tilde{\eta}^T\dot{\eta}_d \tag{7}$$

The reference velocity is chosen as:

$$v_r = R^T(\psi)(\dot{\eta}_d - \Lambda\tilde{\eta}) \tag{8}$$

where $\Lambda$ is a parameter matrix which is positive definite.

Define a Lyapunov function as:

$$V_2 = V_1 + \frac{1}{2}\tilde{v}^T M\tilde{v} \tag{9}$$

The derivative of above equation is as:

$$\dot{V}_2 = \tilde{\eta}^T R(\psi)\tilde{v} - \tilde{\eta}^T\Lambda\tilde{\eta} + \tilde{v}^T[\tau - C(v)v - D(v)v - M\dot{v}_r] \tag{10}$$

The control input can be chosen as:

$$\tau = C(v)v + D(v)v + M\dot{v}_r - R^T(\psi)\tilde{\eta} - \Gamma\tilde{v} \tag{11}$$

where $\Gamma$ is the parameter matrix which is positive definite.

Then Eq. (10) can change to be:

$$\dot{V}_2 = -\tilde{\eta}^T\Lambda\tilde{\eta} - \tilde{v}^T\Gamma\tilde{v} \le 0 \tag{12}$$

The above control input can guarantee the tracking operation is performed.

### 3.2. Event driven tracking controller design

Define

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

to facilitate the following expressions. Derivative of the reference velocity as Eq. (8) is:

$$\dot{v}_r = -rSv_r - R^T(\psi)\Lambda R(\psi)v - R^T(\psi)\Lambda\dot{\eta}_d + R^T(\psi)\ddot{\eta}_d \tag{13}$$

Then the control input is rewritten to be:

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$$\tau = C(v)v + D(v)v + MR^T(\psi)\ddot{\eta}_d - MR^T(\psi)\Lambda\dot{\eta}_d - MR^T(\psi)\Lambda R(\psi)v - MrSR^T(\psi)(\dot{\eta}_d - \Lambda\tilde{\eta}) - R^T(\psi)\tilde{\eta} - \Gamma\tilde{v} \tag{14}$$

It is obvious that the following equation is true:

$$\psi = \eta(3) = (\tilde{\eta} + \eta_d)(3) \tag{15}$$

where $\cdot(3)$ is denoted the third element of the corresponding vector:

$$v = \tilde{v} + v_r = \tilde{v} + R^T(\psi)(\dot{\eta}_d - \Lambda\tilde{\eta}) \tag{16}$$

In order to obtain the event driven tracking controller, define $\zeta = [\tilde{\eta}^T,\ \tilde{v}^T,\ \eta_d^T,\ \dot{\eta}_d^T,\ \ddot{\eta}_d^T]^T$. Thus the control input (14) is denoted $\tau = \tau(\zeta)$.

If we define $t_i\{i = 0, 1, 2, \ldots\}$ as the time series that control input updated, the event driven tracking controller can be denoted as:

$$\tau(t) = \tau(\zeta(t_i)), \quad t \in [t_i, t_{i+1}) \tag{17}$$

The time instants $t_i$ of event driven tracking controller are determined dynamically by an event driven condition. The event driven condition is developed in the following contents to guarantee the event driven tracking controller satisfies the following requirements:

- The tracking error of the marine vessel is uniformly ultimately hounded.
- No Zeno behaviors occur in the execution times.

A measurement error of the state and the exogenous input vector $\zeta$ are existed between the event driven tracking controller and the continuous tracking controller. In order to facilitate further analysis, for $t \in [t_i, t_i + 1)$, the measurement error is represented as:

$$e(t) = \zeta(t_i) - \zeta(t) = \begin{bmatrix} \tilde{\eta}(t_i) - \tilde{\eta}(t) \\ \tilde{v}(t_i) - \tilde{v}(t) \\ \eta_d(t_i) - \eta_d(t) \\ \dot{\eta}_d(t_i) - \dot{\eta}_d(t) \\ \ddot{\eta}_d(t_i) - \ddot{\eta}_d(t) \end{bmatrix} \tag{18}$$

Note that for each $i$, $e(t_i) = 0$. It is evident that $e$ is discontinuous when $t = t_i$.

According to the definition of measurement error, the event driven tracking controller is changed to:

$$\tau(t) = \tau(\zeta(t) + e(t)), \quad \text{for } t \in [t_i, t_i + 1) \tag{19}$$

In order to use directly the stability analysis of the continuous tracking control input based on the backstepping method, the event driven tracking controller can be changed to:

$$\tau(\zeta(t_i)) = \tau(\zeta(t)) + \tau(\zeta(t) + e(t)) - \tau(\zeta(t)) \tag{20}$$

If the tracking task is performed using the event driven tracking controller, then the derivative of $V_2$ is represented as:

$$\dot{V}_2 = -\tilde{\eta}^T\Lambda\tilde{\eta} - \tilde{v}^T\Gamma\tilde{v} + \tilde{v}^T[\tau(\zeta(t) + e(t)) - \tau(\zeta(t))] \tag{21}$$

In the practical tracking operations, the desired reference signals satisfy the following condition: $\| [\eta_d^T,\ \dot{\eta}_d^T,\ \ddot{\eta}_d^T]^T \| \le a$, and the initial position and the velocity satisfy the following condition $\| [\tilde{\eta}(0)^T,\ \tilde{v}(0)^T]^T \| \le b$, where $a > 0$, $b > 0$.

For the actual marine vessel, it is obvious the continuous control input as Eq. (14) is Lipschitz on compact sets. Here the compact sets can be defined as:

$$S = \{\zeta\colon \| [\tilde{\eta}_p^T, \tilde{v}^T]^T \| \le \mu,\ \| [\eta_d^T, \dot{\eta}_d^T, \ddot{\eta}_d^T]^T \| \le a\} \tag{22}$$

where $\mu = \sqrt{\frac{\max\{\lambda(I_3), \lambda(M)\}}{\min\{\lambda(I_3), \lambda(M)\}} b^2}$, $\lambda(\cdot)$ represents the eigenvalues of matrix, $I_3$ is a 3 order unit matrix.

Let the Lipschitz constant of the control input as Eq. (14) on the compact sets $S$ be $L$, we have:

$$\| \tau(\zeta(t) + e(t)) - \tau(\zeta(t)) \| \le L \| e(t) \|, \quad \forall\ \zeta, (\zeta + e) \in S \tag{23}$$

Define the minimum eigenvalue $\gamma = \min\{\lambda(\Lambda), \lambda(\Gamma)\} > 0$. Thus Eq. (21) can be changed to:

$$\dot{V}_2 \le -\gamma \| [\tilde{\eta}^T, \tilde{v}^T]^T \|^2 + \| \tilde{v} \| L \| e(t) \|, \quad \forall\ \zeta, (\zeta + e) \in S \tag{24}$$

Then Eq. (24) suggests a driven condition. The event driven tracking controller is updated when the driven condition satisfied. The time $t_i$ of control input updated is calculated based on the last time $t_{i-1}$. Therefore, the initial time $t_0$ should be described separately. Meanwhile, $t_0$ does not need to be equal to 0.

According to actual operation demand, the allowable position tracking error range can be determined. The error range can be represented as $\| \tilde{\eta} \| \le p$, $p \ge 0$. Then the velocity tracking error should be ultimate bounded, that is $\| \tilde{v} \| \le q$, $q \ge 0$. Overall, $\| [\tilde{\eta}^T, \tilde{v}^T]^T \| \le \delta$, $\delta > 0$.

We assume that $\tau = 0$ when $0 \le t < t_0$. The initial event driven time could defined as:

$$t_0 = \min\left\{ t \ge 0\colon \| [\tilde{\eta}^T, \tilde{v}^T]^T \| \ge \delta \right\} \tag{25}$$

In order to guarantee the tracking error is ultimately bounded only when $\| \tilde{v} \| L \| e(t) \| - \gamma \| [\tilde{\eta}_p^T, \tilde{v}^T]^T \|^2 \le 0$. So each event driven time could defined as:

$$t_i = \min\left\{ t \ge t_{i-1}\colon \begin{array}{c} \tilde{v}Le(t) - \sigma(\gamma[\tilde{\eta}_p^T, \tilde{v}^T]^{T_2}) \ge 0, \\ [\tilde{\eta}^T, \tilde{v}^T]^T \ge \delta \end{array} \right\} \tag{26}$$

where $0 < \sigma < 1$ is a design parameter.

For $\tilde{v} \ne 0$, Eq. (26) can be represented as:

$$t_i = \min\left\{ t \ge t_{i-1}\colon \begin{array}{c} e(t) \ge \dfrac{\sigma(\gamma[\tilde{\eta}_p^T, \tilde{v}^T]^{T_2})}{\tilde{v}L}, \\ [\tilde{\eta}^T, \tilde{v}^T]^T \ge \delta \end{array} \right\} \tag{27}$$

From the above definition of event driven time, we know that the control input is not permitted to update whenever $\| [\tilde{\eta}^T, \tilde{v}^T]^T \| < \delta$.

Motivated the theoretical analysis method and the results in [27], the following theorem is obtained and the detailed proof is presented below.

**Theorem 3.1.** *Considering the marine vessel requires performing the tracking task, the initial position and velocity satisfy the following condition* $\| [\tilde{\eta}(0)^T, \tilde{v}(0)^T]^T \| \le b$. *Let the allowable tracking error range be* $\| [\tilde{\eta}^T, \tilde{v}^T]^T \| \le \delta \le b$. *The proposed event driven tracking control algorithm (17), (25), and (27) can ensure that the tracking error is uniform ultimately bounded, the bounded is denoted as* $\rho = \sqrt{\frac{\max\{\lambda(I_3), \lambda(M)\}}{\min\{\lambda(I_3), \lambda(M)\}} \delta^2}$, *and no Zeno behaviors occur.*

**Proof.** *Step* 1: we will prove that $\| [\tilde{\eta}^T, \tilde{v}^T]^T \|$ is uniform ultimately bounded.

According the definition of the initial event driven time, for $t \in [0, t_0)$, we have:

$$\| [\tilde{\eta}^T, \tilde{v}^T]^T \| \le \delta \le b \tag{28}$$

and

$$\| [\tilde{\eta}(t_0)^T, \tilde{v}(t_0)^T]^T \| \le b \tag{29}$$

Considering the definition of Lyapunov function $V_2$, we obtain that:

$$V_2(t_0) = \frac{1}{2}\tilde{\eta}^T(t_0)\tilde{\eta}(t_0) + \frac{1}{2}\tilde{v}^T(t_0)M\tilde{v}(t_0) \le \frac{1}{2}\max\{\lambda(I_3), \lambda(M)\}b^2 \tag{30}$$

Assuming that for every $t \in [t_0, t_i]$, $i = \{0, 1, 2, \ldots\}$,

$$V_2(t) \le \frac{1}{2}\max\{\lambda(I_3), \lambda(M)\}b^2 \tag{31}$$

Note that for $t \in [t_j, t_{j+1}), j \in \{0, 1, \ldots, i\}$, $(\zeta(t) + e(t)) \in S$. Then $(\zeta(t) + e(t)) \in S$ for every $t \in [t_0, t_{i+1})$.

According to the definition of event driven time (27) and inequality (24), for each $t \in [t_0, t_{i+1})$, for all $[\tilde{\eta}^T, \tilde{v}^T]^T$ satisfies $\delta \le \| [\tilde{\eta}^T, \tilde{v}^T]^T \| \le \mu$, we can obtain:

$$\dot{V}_2 \le -(1-\sigma)\gamma \| [\tilde{\eta}^T, \tilde{v}^T]^T \|^2 \tag{32}$$

where $(1-\sigma)\gamma \| [\tilde{\eta}^T, \tilde{v}^T]^T \|^2$ is a class $\mathcal{K}_\infty$ function. Then inequality (31) is true for all $t \in [t_0, t_{i+1}]$ due to that tracking error is continuous and the inequality (32) holds.

Therefore, the inequality (31) holds for all $t \in [t_0, t_i]$, $\{i = 0, 1, 2, \ldots\}$.

Define two sets as the following:

$$\Phi = \{[\tilde{\eta}^T, \tilde{v}^T]^T : \delta \le \| [\tilde{\eta}^T, \tilde{v}^T]^T \| \le \mu\} \tag{33}$$

$$\Psi = \{[\tilde{\eta}^T, \tilde{v}^T]^T : V_2 \le \frac{1}{2}\max\{\lambda(I_3), \lambda(M)\}\delta^2\} \tag{34}$$

Let $\lim_{i\to\infty} t_i = \infty$, according to the inequality (32), for $[\tilde{\eta}_p^T, \tilde{v}^T]^T \in \Phi$ and $t \ge t_0$, we have that:

$$\dot{V}_2 \le -(1-\sigma)\gamma\delta^2 \le 0 \tag{35}$$

The set $\Phi \bigcap \Psi$ is non-empty, and the level set satisfies:

$$\Delta\Psi = \left\{[\tilde{\eta}^T, \tilde{v}^T]^T : V_2 = \frac{\max\{\lambda(I_3), \lambda(M)\}\delta^2}{2}\right\} \subset \Phi \tag{36}$$

Thus the tracking error finally converges the set $\Psi$ in a limited time when the initial tracking error is bounded.

Considered the definition of Lyapunov function $V_2$, we have:

$$\frac{1}{2}\min\{\lambda(I_3), \lambda(M)\}[\tilde{\eta}^T, \tilde{v}^T]^{T2} \le V_2 \le \frac{1}{2}\max\{\lambda(I_3), \lambda(M)\}[\tilde{\eta}^T, \tilde{v}^T]^{T2} \tag{37}$$

Obviously, $\frac{1}{2}\min\{\lambda(I_3), \lambda(M)\}\| [\tilde{\eta}^T, \tilde{v}^T]^T \|^2$ is a class $\mathcal{K}_\infty$ function. There is a constant $\rho$ that

$$\Psi \subseteq \{[\tilde{\eta}^T, \tilde{v}^T]^T : \| [\tilde{\eta}^T, \tilde{v}^T]^T \| \le \rho\} \tag{38}$$

where $\rho = \sqrt{\frac{\max\{\lambda(I_3), \lambda(M)\}}{\min\{\lambda(I_3), \lambda(M)\}}\delta^2}$.

Therefore, $\| [\tilde{\eta}^T, \tilde{v}^T]^T \|$ is uniform ultimately bounded and the bounded is $\rho$.

*Step* 2: We will prove no Zeno behaviors occur in the execution times. Define the minimum inter-update time $t_{i+1} - t_i$ is denoted as $T$. The following paragraphs show that $T > 0$. According to the front designed tracking controller based on the backstepping method and the extended event driven tracking controller, we can know that:

$$\begin{bmatrix} \dot{\tilde{\eta}} \\ \dot{\tilde{v}} \end{bmatrix} = \begin{bmatrix} -\Lambda & R(\psi) \\ -M^{-1}R^T(\psi) & -M^{-1}\Gamma \end{bmatrix}\begin{bmatrix} \tilde{\eta} \\ \tilde{v} \end{bmatrix} + \begin{bmatrix} 0 \\ \tau(\zeta + e) - \tau(\zeta) \end{bmatrix} \tag{39}$$

Based on the vessel model parameter and the designed control parameter matrices and Eq. (15), we have:

$$\left\| \begin{bmatrix} -\Lambda & R(\psi) \\ -M^{-1}R^T(\psi) & -M^{-1}\Gamma \end{bmatrix}\begin{bmatrix} \tilde{\eta} \\ \tilde{v} \end{bmatrix} \right\| \le L_1(\| [\tilde{\eta}^T, \tilde{v}^T]^T \| + \| \eta_d \|) \tag{40}$$

where $L_1$ is the Lipschitz constant of above left expression.

Because of $\zeta \in S$, $(\zeta + e) \in S$ for all $t \in [t_0, t_i)$, thus (23) holds. Then:

$$\begin{aligned} \| [\dot{\tilde{\eta}}^T, \dot{\tilde{v}}^T]^T \| &\le L_1(\| [\tilde{\eta}^T, \tilde{v}^T]^T \| + \| \eta_d \|) + L \| e \| \\ &\le L_2(\| [\tilde{\eta}^T, \tilde{v}^T]^T \| + \| \eta_d \| + \| e \|) \end{aligned} \tag{41}$$

where $L_2 > 0$.

Based on the practical marine task, we can know that: $\| [\eta_d^T, \dot{\eta}_d^T, \ddot{\eta}_d^T]^T \| \le a$. Especially, $\| \eta_d \| \le c$, $c > 0$. We have:

$$\| [\dot{\tilde{\eta}}^T, \dot{\tilde{v}}^T]^T \| \le L_2(\mu + c + \| e \|) \le L_2(\mu + a + \| e \|) \tag{42}$$

Define $\dot{e} = -[\dot{\tilde{\eta}}^T, \dot{\tilde{v}}^T, v_d^T, \dot{v}_d^T, \ddot{v}_d^T]^T$. In order to facilitate the representation, define $\dot{\eta}_d = v_d$, then $\ddot{\eta}_d = \dot{v}_d$. Obviously, $\| \ddot{v}_d \| \le d$ is true. Then we have:

$$\frac{\mathrm{d}\| e \|}{\mathrm{d}t} \le \| \dot{e} \| \le L_3\Big(\mu + a + d + \| e \|\Big) \tag{43}$$

$L_3 > 0$. According to comparison lemma [28], we can obtain:

$$\| e \| \le \big(\mu + a + d\big)\big(\mathrm{e}^{L_3(t-t_i)} - 1\big), \quad \text{for } t \ge t_i \tag{44}$$

On the compact set $S = \{\zeta: \| [\tilde{\eta}_p^T, \tilde{v}^T]^T \| \le \mu, \| [\eta_d^T, \dot{\eta}_d^T, \ddot{\eta}_d^T]^T \| \le a\}$, if the initial velocity error satisfies $\| \tilde{v}(0) \| \le b_1$, we can obtain that $\| \tilde{v} \| \le \sqrt{\frac{\max\{\lambda(M)\}}{\min\{\lambda(M)\}}b_1^2} = \mu_1$. $e_{min} = \frac{\sigma\gamma\delta^2}{L\mu_1}$. Then the inter-update times are uniformly lower bounded by $T$, and $T$ satisfies the following inequality:

$$T \ge \frac{1}{L_3}\log\left(1 + \frac{\sigma\gamma\delta^2}{L\mu_1(\mu + a + d)}\right) \tag{45}$$

Because $L_3$ and $L$ are finite, we can obtain that the minimum inter-update time $T$ is greater than 0. □

## 4. Simulation analysis

The performance of the developed event driven tracking control algorithm is validated though simulation experiment. The Cybership II of the NTNU [1] is used to carry out the experiment. The initial location and the initial velocity of vessel are $\eta(0) = [0,\ 0,\ 0]^T$ and $v_0 = [0,\ 0,\ 0]^T$, respectively. The control parameters of the tracking controller were chosen as $\Lambda = \mathrm{diag}\{4,\ 4,\ 3\}$, $\Gamma = \mathrm{diag}\{3,\ 3,\ 3\}$. The parameter of the event driven condition is chosen as $\sigma = 0.55$. Set the simulated time to be 100 s. The simulation results for straight line tracking and curve tracking are shown as the following, respectively.

*For straight line tracking*: The desired straight line trajectory is defined as $\eta_d = [t,\ 0,\ 0]^T$. The allowed error parameter $\delta$ is chosen as $\delta = 0.3$. The simulation results are shown in Figs. 1–4. Though the event driven condition, the event driven times of the straight line tracking is presented in Fig. 1, we can see that the event number of the event driven tracking control law is 272. In order to guarantee the tracking error, the sampling period of the normal discrete-time control law is set to be ≤0.1 s. Then the number of the control executions is ≥1000. Obviously, within the allowable error range, the event number is fewer than the periodic sampling of the normal control law. The control inputs corresponding to the event time for straight line tracking are shown in Fig. 2. Fig. 3 shows the position tracking error for straight line tracking. The tracking errors are ultimately limited in one interval that below the desired bound. Thus the amount of computation and communication will be reduced and results in little the executions of actuator while ensuring the tracking performances. In addition, the inter-execution time $(t_{i+1} - t_i)$ of the straight line tracking controller is shown in Fig. 4. Though calculation and comparison, the minimum inter-execution time is 0.1 s, which is greater than

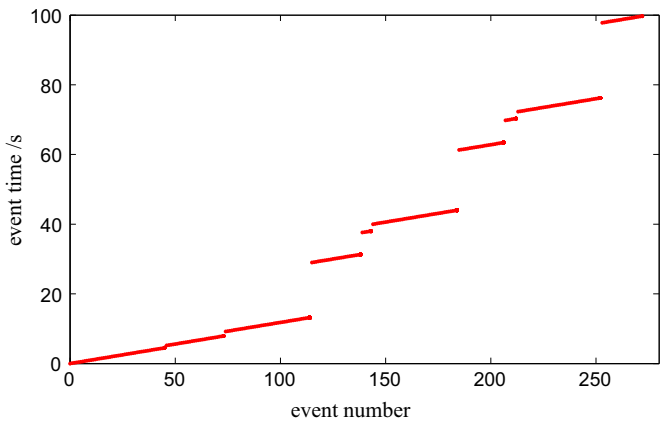


**Fig. 1.** The event times of straight line tracking controller.

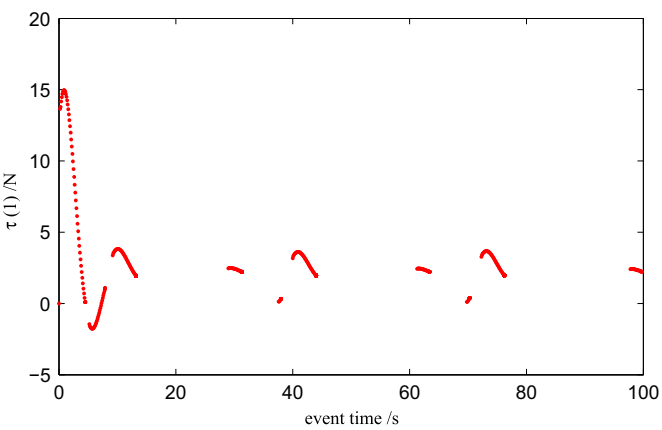


**Fig. 2.** The longitudinal force input for straight line tracking.

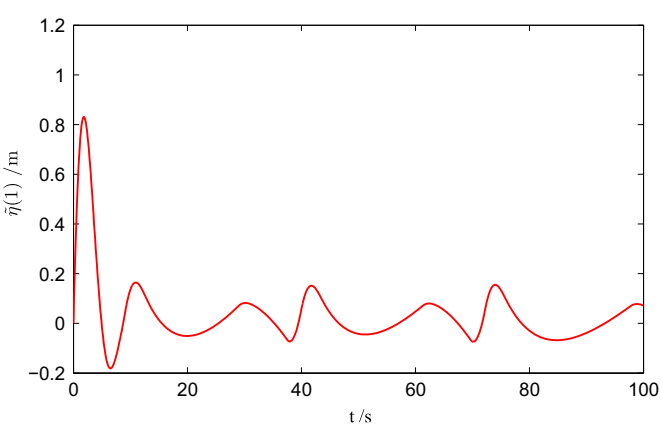


**Fig. 3.** The tracking error for straight line tracking.

zero. In other words, no Zeno behaviors occur in the execution times.

*For curve tracking*: The desired curve trajectory is defined as $\eta_d = [t,\ \ 10\sin(\frac{t}{10}),\ \ 0]^T$. The allowed error parameter $\delta$ is chosen as $\delta = 0.9$. The simulation results are shown in Figs. 5–8. Though the event driven condition, the event driven times for the curve tracking is shown in Fig. 5. According to Fig. 5, we can see that the event number of the event driven tracking control law is 606. In order to guarantee the tracking error, the sampling period of the normal discrete-time control law is set to be $\leq$0.1 s. Then the number of the control executions is $\geq$1000. Obviously, within the allowable error range, the event number is fewer than the periodic sampling of the normal control law. The control inputs

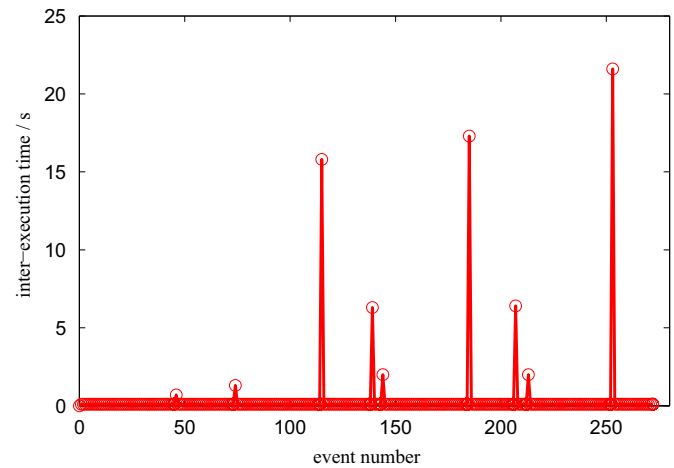


**Fig. 4.** The inter-execution time of straight line tracking controller.

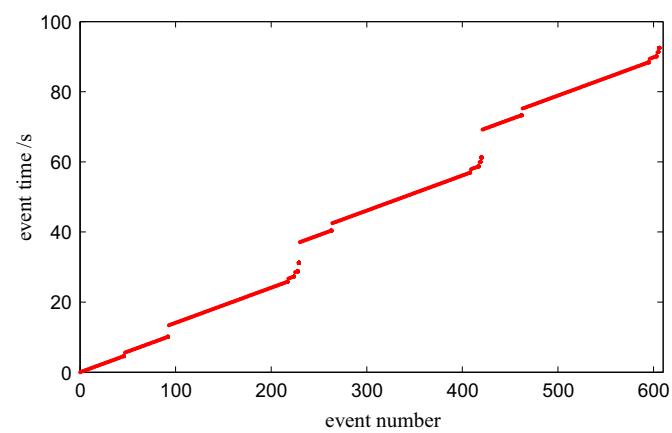


**Fig. 5.** The event times of curve tracking controller.

corresponding to the event time for curve tracking are shown in Fig. 6. Corresponding to each event time, Fig. 6(a) shows the longitudinal force, Fig. 6(b) shows the lateral force, and Fig. 6 (c) shows the rotary torque, respectively. The tracking errors for curve tracking are shown in Fig. 7. Fig. 7(a) shows the tracking errors of north position and east position. In Fig. 7(a), the redline denotes the tracking errors of north position, the blue line denotes the tracking errors of east position. Fig. 7(b) shows the tracking errors of heading. The tracking errors are ultimately limited in one interval that below the desired bound. Thus the amount of computation and communication will be reduced and results in little the executions of actuator while ensuring the tracking performances. In addition, the inter-execution time $(t_{i+1} - t_i)$ of the curve tracking controller is shown in Fig. 8. Though calculation and comparison, the minimum inter-execution time is 0.1 s, which is greater than zero. In other words, no Zeno behaviors occur in the execution times.

## 5. Conclusion

This paper has been proposed an event driven tracking control algorithm for marine vessel based on the backstepping method. The proposed tracking control algorithm is derived from the continuous-time tracking controller. Specifically, the tracking controller is designed based on the backstepping method, and the event driven conditions is introduced to determine the updated time of the controller. Theoretical analysis show that the error of

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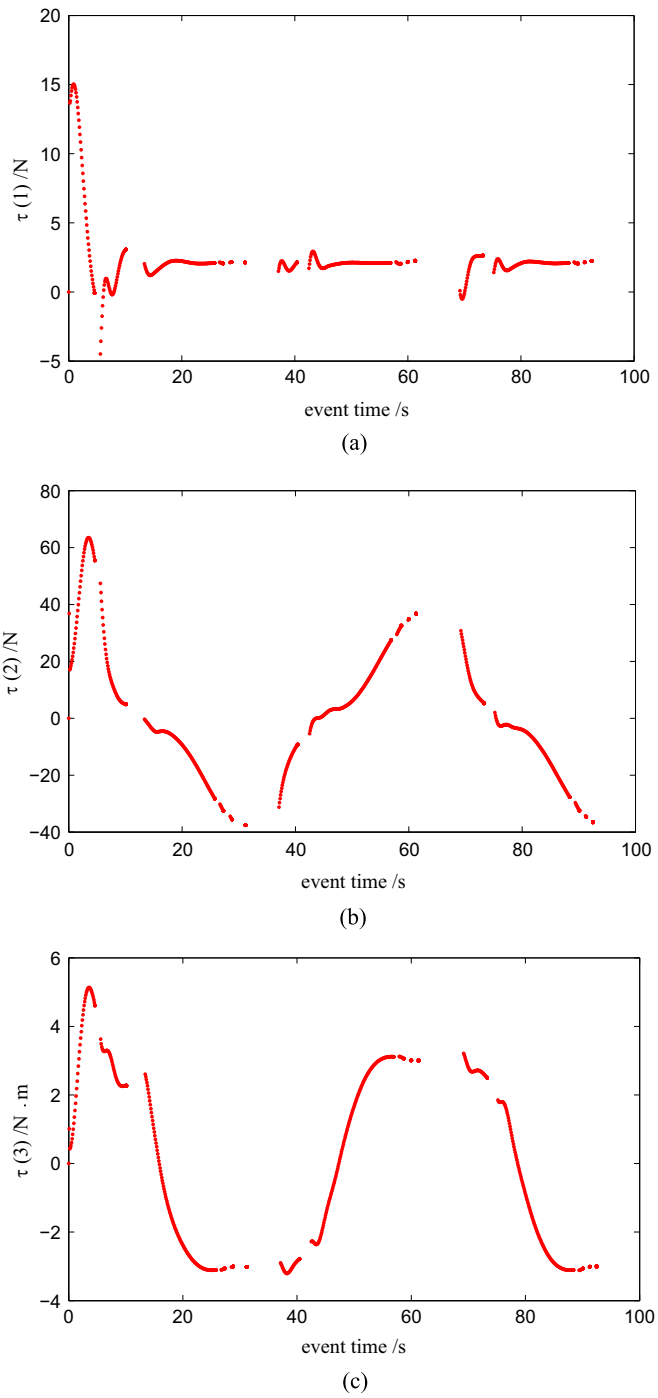


**Fig. 6.** The control input for curve tracking.

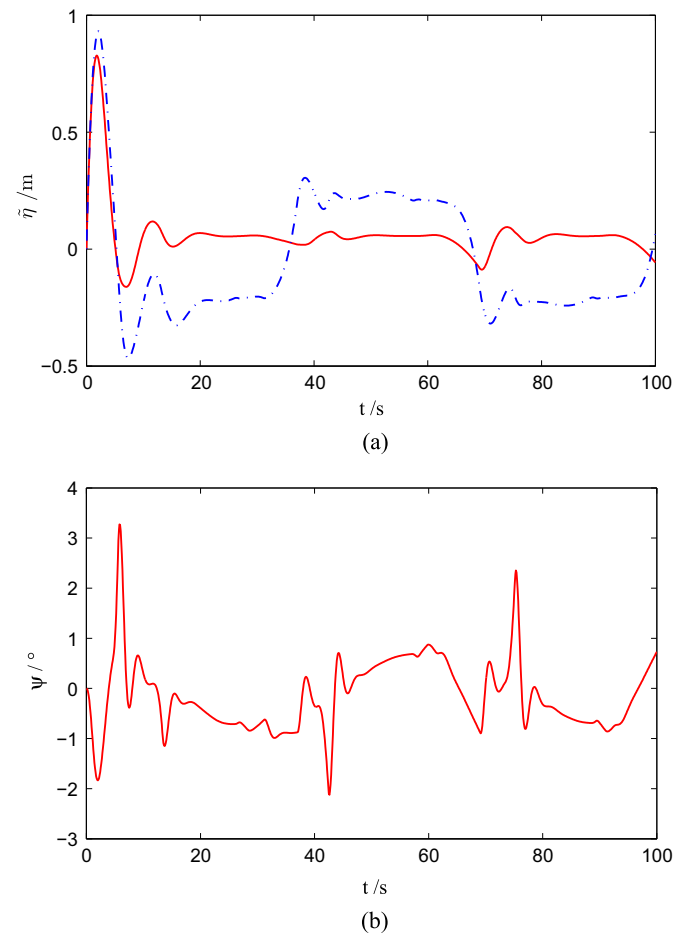


**Fig. 7.** The tracking error for curve tracking. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

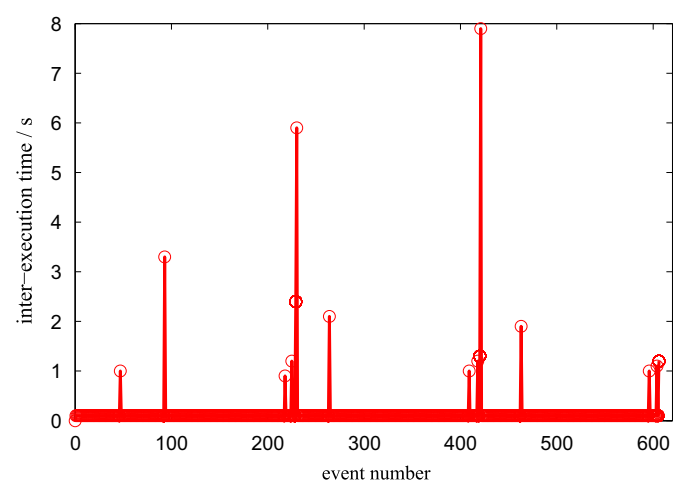


**Fig. 8.** The inter-execution time of curve tracking controller.

tracking is uniform ultimately bounded and no Zeno behavior. Finally, the simulation results have illustrated the tracking task can be performed within the allowed error range, what's more, the communication quantity and the executions of actuator are significantly reduced.

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