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A review on the mechanical behaviour of curvilinear fibre composite laminated panels

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Abstract

A review on works that investigate the mechanical behaviour of variable stiffness composite laminated panels is carried out in this paper. The review mostly focuses on buckling, failure and vibrations in laminates reinforced by curvilinear fibres, although other issues related to variable stiffness laminates are also addressed. The peculiarities in the formulation of curvilinear fibre reinforced plates are briefly described. As an illustration, the natural frequencies of vibration of variable stiffness composite laminated plates with curvilinear fibres are computed by an *h*-version type finite element code and are compared with the ones calculated using another model, based on a Third-order Shear Deformation Theory. Areas of research to explore on variable stiffness composite laminates are suggested.

Keywords

Variable stiffness composite laminates, review, buckling, failure, vibration

Introduction

Composite materials are made of at least two constituent materials that remain separated in the finished structure. A popular type of composites are laminated fibre-reinforced composites,[1–3] which have a number of advantages in comparison to metal-based structures, including the facts that they offer lightweight and stiff surfaces, which resist well to corrosion and are also believed to have a relatively long fatigue life.[4] Most commonly, fibre reinforced composite materials have straight and unidirectional fibres, which are homogeneously distributed in each lamina, hence, in a macroscopic sense, it can be considered that the stiffness does not vary in the laminate domain. However, composite materials where the stiffness is purposely made to vary in the laminate have, particularly since the nineties,[5–9] deserved greater interest, because they may lead to more efficient designs.

There are several techniques to obtain variable stiffness composite laminated (VSCL) panels. The main ones, represented in Figure 1, are using curvilinear fibres,[5–74] varying the volume fraction of fibres,[61,62,75–85] and dropping or adding plies to the laminate.[86–90] Attaching discrete stiffeners to the laminate,[91–98] Figure 1(d), can be regarded as a procedure that varies the stiffness[11] and will be briefly addressed later. Functionally graded materials[99] could be also categorised as composite materials with variable stiffness, but in this review we focus on fibrous composites and will not include FGM. It is also worth noting that materials whose stiffness varies in space appear in nature.[100]

This review concentrates on VSCL panels where the fibre orientation angle is not constant in a ply,[5–74] as represented in Figure 1(a), and acronym "VSCL" will be mostly used for this specific type of panel. Not negating that other ways of varying the stiffness may, for technologic or economic reasons, be more adequate in specific situations, varying the fibre orientation has a few advantages. One of them is that the stiffness varies continuously with the membrane coordinates,

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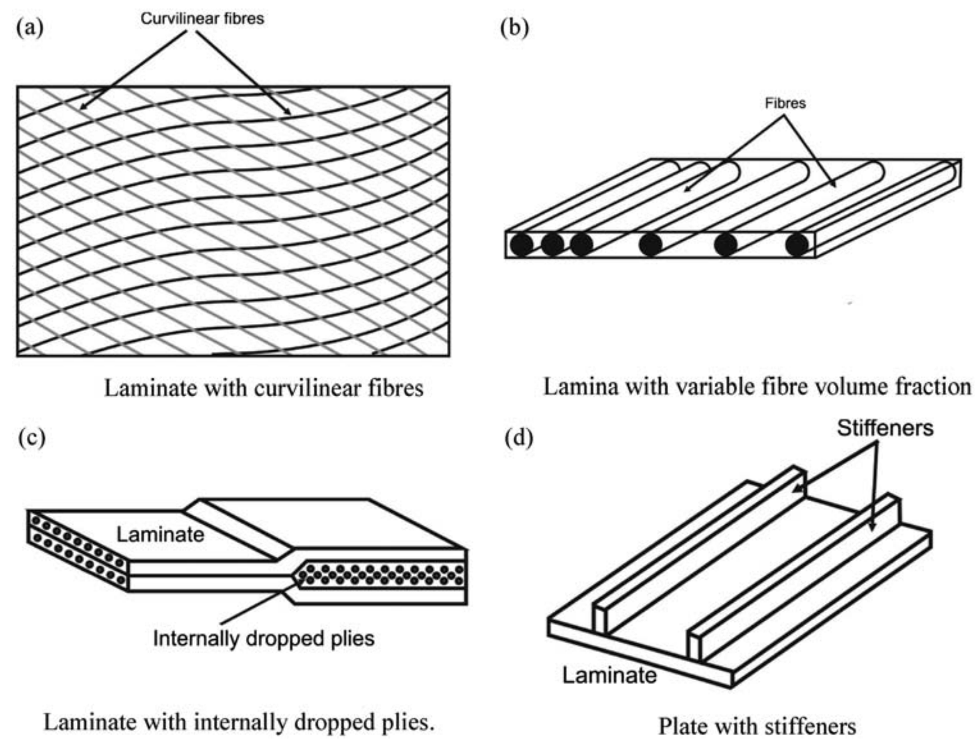

Figure 1. Examples of four types of variable stiffness composite panels.

in opposition to what occurs either when stiffeners are attached to a laminate or when plies are terminated at different locations. The two last options lead to abrupt changes in the thickness direction, which produce stress concentration and out-of-plane, interlaminar, stresses.[86,88] Moreover, unlike stiffeners, curvilinear fibres do not introduce major geometry variations. Curved fibres offer a wider degree of possibilities than variations of rectilinear fibre volume fraction and provide a solution to the problem of continuity when one considers manufacturing a structure with different fibre angles in adjoining elements.[5,6,25] Furthermore, curvilinear fibres also offer a way to diminish stress concentrations that occur, e.g. around holes or cut-outs.

For variable stiffness laminates with curvilinear fibres, a reference fibre path can often be defined as a function of one variable, which can be one of the Cartesian coordinates. Discussions and equations for reference paths on plates can be found in, for example, Ref. 14. Four theoretical paths are introduced for generalised conical shell surfaces in Ref. 59 and more detailed information related to fibre paths in shells can be found in Ref. 60.

Within a specific lamina, there are two common methods to produce other fibres from the reference fibre, using automated tow placement machines: the "parallel fibre" and the "shifted fibre" methods.[10] In the shifted pattern, each additional path replicates the baseline, but shifted in a direction perpendicular to the

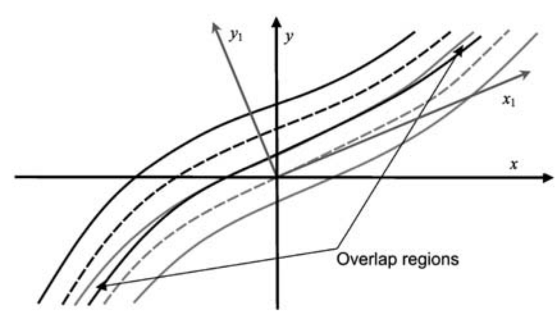

Figure 2. Representation of shifted fibre method.

one used to define the fibre path and the variation in stiffness. For example, the fibre path is a function of the horizontal coordinate and the shift is applied in the vertical direction. In Figure 2, a more general case is portrayed, where the reference fibre path is a function of coordinate x_1, rotated in relation to x. The dashed lines represent the centre line of each strip of fibres and the solid lines denote the edges of the width strip. Note that this procedure results in overlaps and/or in gaps. In the parallel fibre method, the adjacent fibres are placed so that they are parallel to the reference path, i.e. their points lay at a fixed distance measured along the normal to the reference path.[10] This actually makes the fibre orientation change from fibre to fibre, both in the vertical and the horizontal directions. In this

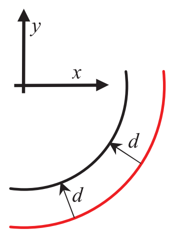

Figure 3. Two parallel fibres.

case, the baselines do not obey to the same analytical expression and one may use a numerical scheme to generate the parallel fibre paths.[10] A simple example is shown by the two curves in Figure 3, which are parallel, since the distance d measured in the normal direction is constant, but obviously cannot result from shifting in any direction, as their curvature radii are different.

One of the reasons behind the increasing availability of curved fibre laminates is that present tow-placement machines are capable of controlling fibre tows individually. Tow-placement machines have a computer controlled robotic arm with a fibre placement head, allowing for a precise control of the fibre orientation and providing the possibility to curve the fibres within the plane of the laminate; they also have cut/restart capabilities.[13,14] This technology opened the door to an enlarged design space, with respect to the traditional straight-fibre reinforced composites, offering new possibilities for weight reduction and better performance, which is particularly important in means of transport.[57] The application of this technology to aircraft fuselage regions dominated by bending, linked to regions where shear deformation mostly affects the response, is suggested in Ref. 58. In such a case, plies with fibres aligned along a certain axis should turn to plies where the fibres make 45° with relation to the same axis. With curvilinear fibres, these two regions can be connected in a continuous fashion.

Automated fibre placement (AFP) with advanced machines is the chief procedure to obtain laminates with curvilinear fibres and – as written above – benefits from the machines' capability to control tow-placement; but it also induces defects.[16,48] Tow overlapping and tow dropping result in fibre and resin rich or poor regions. A novel fibre placement technique, designated as ''Continuous Tow Shearing (CTS)'' and which uses shear deformation characteristics of dry tows, has been presented in Ref. 101. Tests indicate that CTS can decrease process-induced defects such as fibre wrinkling, resin rich areas and fibre discontinuity.

Another type of advanced fibre placement – different from the one mentioned in a previous paragraph and, for example, in Refs 13 and 14 – is tailored fibre placement (TFP), an automated textile process for the production of reinforcing structures.[102,103] This process apparently allows manufacturing textile preforms for composite parts with fibre layouts of arbitrary direction using embroidery technology; the desired fibre quantities and orientations can be transferred into fibre preforms. But it was not possible to find many works on failure, buckling or vibrations of composites with curvilinear fibres manufactured via this technology. An exception is Ref. 104, where a bladed rotor made using tailored fibre placement is analysed and manufacturing aspects, stresses and vibration modes are discussed.

Due to the large number of possibilities offered by curvilinear fibres, optimisation procedures are obviously interesting in VSCL design.[105,106] In Ghiasi et al.,[105] published in 2010, different parameterisation and optimisation algorithms used in VSCL plates are reviewed. For optimisation purposes, often the plate is discretised into a number of small elements, then the fibre orientation angle, or a parameter related to it, is obtained in each element so that a certain objective is achieved. An application of the Rayleigh–Ritz method for optimal design of VSCL panels is investigated in Muc and Ulatowska.[106] It is almost inevitable to refer to optimisation when addressing VSCL, but the aim of this literature review is not to discuss optimisation techniques; instead, it will give more focus to the mechanical behaviour of these panels.

The remainder of the paper is constituted by three sections. The following section addresses the use of curvilinear fibres and is divided in three subsections. In sub-section ''*Buckling and static response*'' works devoted to the application of curvilinear fibres to maximise buckling loads on composite structures, a main challenge in the design of compressed elements,[107] are reviewed. Works that address aspects on static behaviour, other than failure, are also included in this subsection. Failure is naturally and for obvious reasons a major concern[108–111] and research on failure on VSCL is addressed in sub-section ''*Failure*''. Clearly, there is a relation between buckling and failure, and some works are common to the latter two sections. The potentiality of application of VSCL is large in lightweight structures, where vibrations can cause failure, instability, noise and life reduction due to fatigue.[112–115] Sub-section ''*Vibration analysis*'' reviews works on vibrations of VSCL. In Section ''*Variable stiffness laminates with rectilinear fibres*'', a partial review is carried out on works where other means of varying the stiffness – not curvilinear fibres – are employed in fibre reinforced composites. After the sections devoted to review, a model for VSCL with curvilinear fibres is presented, illustrating the chief differences in formulation between these and traditional laminates. A short explanation on how to implement a finite element model using commercially available software is also written.

The same section presents, as an application, the computation of modes of vibration. The final section is dedicated to a concluding summary and suggestions for future research.

Laminates reinforced with curved fibres

Buckling and static response

There is a large body of work on buckling of VSCL panels and one can say it is now established that significant increase in the buckling loads can be obtained using VSCL, instead of traditional straight fibres composite panels.

Early works that addressed the gains that can be achieved in buckling performance employing curvilinear fibre paths were published by Hyer et al.[5–8] A sensitivity analysis and a gradient-search technique were used to select fibre orientations that increase the buckling load. Improvements in buckling resistance of plates with a hole were obtained. Another early study was published by Gürdal and Olmedo,[9] where the in-plane elastic response of panels with curvilinear fibres was modelled. The stress distribution was investigated, illustrating the different stress variations that occur in VSCL and in traditional, straight fibre laminates. Still in the 1990s, Refs. 10–12 included more analysis on buckling of VSCL plates. In Ref. 10, examples of changes in the in-plane response and buckling of variable stiffness plates, manufactured by the parallel and shifting methods, are presented in detail. Ref. 12 gives some examples of using a commercial finite element and optimisation program, GENESIS,[116] to achieve variable stiffness plates with larger in-plane stiffness and higher buckling load.

In ensuing publications, after 2000, Tatting, Gürdal and co-workers[13–31] continued exploring the extra freedom that is provided by curvilinear fibre paths to optimise flat and curved panels for maximum buckling load, taking into account manufacturing limits of tow-placement machines and exploring the change of other properties, as for example the overall in-plane stiffness, with the fibre path. Increases of the buckling load up to twice the value of conventional straight-fibre panels were achieved. In one of these references,[17] the response of plates is analysed with STAGS code[117] including both geometric and material non-linearities. The results indicate that assuming linear material behaviour is adequate to analyse prebuckling and near-postbuckling behaviour of these plates, but non-linear material behaviour is an increasingly critical part of the structural response during deep postbuckling and up to failure. Another example of the tailoring possibilities opened by VSCL is offered by Setoodeh et al.,[21] where VSCL were designed in such a way that the majority of the load was carried by the region near the edges of the plate where the boundaries suppress the displacements, while reducing the compressive load at the centre. Ref. 24 reminds us that laminated panels in aircraft and aerospace structures endure vibration and buckling in addition to static deformations and that, therefore, those structures can benefit from VSCL. This is in particular so because filament winding and tow-placement, techniques well suited to implement curvilinear fibre paths, are established procedures used to manufacture composite shells.

In Refs 25 and 26, design tailoring for the pressure pillowing problem of VSCL fuselage panels was addressed using finite element software ABAQUS[118] and Ritz method, with the target of improving simultaneously the buckling load and the failure load. The analyses includes perfect plates (studied using Ritz method) and plates with cut-outs (studied using the finite element method). It was found that designing laminates for maximum strength may be a better choice to satisfy the structural and manufacturing requirements than trying to optimise for maximum buckling load. Ref. 29 had the aim of studying the prebuckling and postbuckling behaviour, including buckling modes, of optimised plates and using a perturbation method that had been developed within a finite element environment. An improvement in stiffness was also found, before and after buckling, for variable stiffness laminates in comparison with conventional laminates. The authors found that gaps deteriorated both in-plane stiffness and buckling load, whereas overlaps improved the structural performance.

A study, by the classical lamination theory (CLT), on the effect of thermal residual stresses and on the buckling performance of thin VSCL again indicates that these panels can perform better than constant stiffness ones[28]; residual thermal stress helps to reduce the stress resultant distribution near the centre of the plate. Buckling, post-buckling and thermo-mechanical buckling of VSCL plates and shells are given with an optimisation method on lamination parameters in Ref. 30. Analysis of thermal performance of VSCL is an important part of Ref. 18.

Improving the buckling load by using VSCL plates instead of traditional plates is also achieved in Ref. 31, where the distribution of four spatial lamination parameters is converted to a realistic variable stiffness design in terms of fibre orientations. These lamination parameters are integrals through-the-thickness of the fibre orientation angles and are connected with matrices **A** (extensional stiffness), **B** (bending-extensional coupling stiffness) and **D** (bending stiffness) matrices.[119] In Ref. 33, the Rayleigh–Ritz method was applied for the prebuckling and buckling analysis of VSCL. Airy's stress function is employed in the formulation which deals well with several types of boundary conditions. A procedure to consider non-linear variation of the fibre orientation is proposed. In Ref. 34, a methodology based on the Differential Quadrature Method is developed for buckling analysis of panels modelled using classical laminated plate theory; Airy's stress function is used to perform the prebuckling analysis of anisotropic VSCL. The formulation is thought to facilitate efficient consideration of pure stress and so-called mixed in-plane boundary conditions (as uniform axial compression). In Ref. 35, the simultaneous optimisation of stiffness and buckling load of a composite laminate plate with curvilinear fibre paths was examined. An investigation by the same authors was performed in Ref. 36, on the effect of gaps or overlaps, captured by a so-called defect layer method implemented by a MATLAB subroutine, on the in-plane stiffness and buckling load of VSCLs. Tow-drop areas caused by the individual cutting of tows reduce stiffness and strength. It is reported that when the tows are wider, the tow-drop areas are larger and the strength of the design is lower.

Works where static regime was considered, but not focusing on buckling or failure, can be found in Refs 37–44. A cellular automata method was presented in Ref. 37, where it is suggested to improve stiffness and displacements in bending problem of VSCL plates with cut-outs. In Ref. 38, the optimal design of fibre reinforced rectangular composite plates for minimum compliance is investigated and it is concluded that significant improvements in stiffness can be gained by using variable–stiffness design. In Ref. 39, the possibility of creating bend-free composite shells by steering the fibre-tows is investigated. In Ref. 40, the use of curvilinear fibres in the local reinforcement of a circular hole is analysed, while in reference Ref. 41 a concept for morphing composite structures based on variable stiffness composite plates is presented. The stress concentration due to lay-up mismatch between two regions is mitigated by using fibres which vary smoothly from one region to the other.

The effect of transverse shear stresses on the bending deflection of variable angle tow (VAT) laminates is assessed in Ref. 42. The formulation uses multiple shear correction factors. Large deflection and stresses of laminated plates with curvilinear fibres are studied in Ref. 43, using a p-version finite element, which follows Third-order Shear Deformation Theory. Since an equivalent single layer approach was followed, very accurate computation of interlaminar shear stresses required resorting to the equilibrium equations. It was found that VSCL plates can lead to significantly smaller deflections and to important changes in the stress field, as altering the position of maximum stress at the plate. But the study does not indicate that VSCL with a linear variation of the fibre angle always outperform significantly CSCL in similar materials. The use of curvilinear fibres in rectangular plates without holes or cut-outs and mainly subjected to static bending appears to be substantially advantageous only for certain loading conditions.

In Ref. 44, the effect of curvilinear fibres on the in-plane flexibility and out-of-plane bending stiffness of composite plates, representing a morphing wing skin, was examined. A flexibility ratio was defined to assess the in-plane and out-of-plane deformation simultaneously. This ratio was maximised via a multi-objective optimisation strategy. The study indicates that plies away from a neutral axis tend to be curved while the plies near the same axis tend to be straight fibres. The numerical results also show that curvilinear fibres can be an answer to conflicting design requirements of morphing wing skins.

Failure

Composite laminates may suffer damage in service or during maintenance, due, for example, to low velocity impacts or fatigue. The laminate mechanical properties can be significantly reduced by damage, which can occur due to fibre rupture, matrix cracking, fibre matrix debonding, transverse-ply cracking and delamination.[108–111] Published works indicate that tow-steered composite panels can be more resistant to the onset of damage than straight-fibre laminates. Usually, the criteria used to predict failure on-set are similar to the ones employed for other fibre-reinforced composites; for example Tsai-Wu criterion still determines that, in the case of plane stress, a laminate has not failed if at every point

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_{66}\sigma_{12}^2 < 1 \tag{1}$$

where F_i and F_{ij}, $i,j = 1,\ldots,6$, are the second- and fourth-order strength tensors that may be found in a number of references, as, for example, in Ref. 110, σ_1, σ_2 and σ_{12} are the in-plane stresses in the principal material directions. These are the fibre direction and the direction perpendicular to it, but in the plane; therefore, these directions are not constant in a lamina with curvilinear fibres. Doubts may arise about parameters necessary to implement some failure criteria in VSCL laminates, although often (for example in the case of failure stresses in material directions) these are common to the straight fibre case.

In an early work,[5] two failure criteria were applied to predict the tensile load capacity of various VSCL designs: the maximum strain criterion, which identifies three failure modes (longitudinal failure, transverse failure, shear failure), and the Tsai-Wu criterion. Waldhart[10] dedicated part of his work to the analysis of VSCL plates with Tsai-Hill failure criterion. Tatting[11] evaluated failure of VSCL cylinders under different types of loading as bending moment, internal pressure, torsion and axial compression. Langley[12] used GENESIS to design VSCL plates subjected to in-plane loading. Different methods of finding failure indexes – including maximum strain, maximum stress, Tsai-Hill, Hoffman and Tsai-Wu failure theories – can be used in GENESIS. More work on failure can be found in Ref. 17, where a computational code named STAGS[117] was used to predict the progressive failure analysis of variable stiffness laminates, including geometric and material non-linearities. The importance of material non-linearity in the stresses and strains was highlighted. This work also includes a few comparisons between predicted and experimental tests.

In Ref. 45, a discussion regarding the design of fibre-reinforced composites is carried out. The analysis takes into consideration the stress field and failure. In Ref. 46, the optimum design of laminated composites with curved fibres and variable thickness is addressed, attempting to minimise weight under stress constraints. Tsai-Hill criterion is employed and it is apparent that curvilinear fibres allow for better performance.

In Refs 19 and 47, simulations in finite element software ABAQUS were carried out to investigate the advantages in terms of resistance to damage of curvilinear over straight-fibre laminates, under compressive loads. More specifically, in Ref. 19, numerical simulations were carried out to analyse resistance to the onset of damage, and in Ref. 47 the former analysis was extended into the postbuckling progressive damage behaviour and final structural failure due to accumulation of damage. The buckling and postbuckling first-ply failure response characteristics of variable-stiffness panels, with a central hole, fabricated by AFP Technology, were analysed in Ref. 27. Both in terms of buckling and first-ply failure, important advantages in comparison with constant-stiffness formats were demonstrated via an ABAQUS-based numerical analysis. The works in Refs 19, 27 and 47 can also be found in Ref. 48, where it is written that VSCL perform better than conventional laminates due to favourable residual thermal stresses developed during the laminate curing process, due to the curvature of the fibres. It is reinforced that one can design VSCL panels that are insensitive to central holes. In Refs 49 and 50, failure and interlaminar stresses of VSCL plates were investigated, again using ABAQUS. Design of VSCL for maximum strength was investigated in Ref. 51, adopting failure envelopes, suggested in Ref. 52, which represent a conservative approximation of Tsai-Wu failure criterion. Numerical results indicate that VSCL outperform quasi-isotropic laminates, particularly if strength-optimal designs are adopted.

The simultaneous improvement of failure load and buckling load of a VSCL fuselage is explored in Refs 25 and 26 using ABAQUS and Raleigh-Ritz method, as already referred to in the previous section. The failure load was defined as the load level at which first failure occurs, predicted by Tsai-Wu failure criterion. It was concluded that laminates optimised for maximum failure loads have buckling loads that are also significantly higher than those for quasi-isotropic laminates.

In Ref. 53, the plate thickness and the fibre angles are optimised, with the objective of minimising the weight of the composite panel subject to Tsai-Hill failure criterion. The adverse effect of ply drops and splicing in the structure's strength was not considered.

Although tests on panels and aerospace parts made by AFP machines indicate that they perform similarly or better than those made by hand layup,[54] defects as twisted rows, gaps and overlaps occur. These defects are more pronounced in curvilinear than straight fibre laminates. In Ref. 54, experiments, essentially on laminates reinforced by rectilinear fibres, were performed to study the effect of gaps, overlaps, half gaps/overlaps and twisted tows on the ultimate strength. Amongst other outcomes, it was found that the fibre microbuckling influences plate failure. Impact and compression after impact of VSCL panels is simulated in ABAQUS in Ref. 55, considering fibre orientation, matrix crack growth and delaminations propagation. Bilinear cohesive law-based interface elements and cohesive contacts were used to model the delamination and crack growth. Ref. 56 showed an optimisation method for VSCL plates to maximise in-plane strength represented by the Tsai-Wu failure index. In Ref. 30, Tsai-Wu criterion is developed in terms of two strain invariants and applied to VSCL plates under in-plane loads. In a similar way, but with stress invariants, Ref. 29 compares failure among VSCL plates. Better results were found for stress distribution in VSCL plates that lead later to ultimate failure in these plates. In Ref. 62, a cellular automata-based methodology is demonstrated for in-plane analysis of VSCLs with Tsai-Hill failure onset criterion. This cellular automata method was presented in Ref. 37.

Vibration analysis

In the usual applications of laminated panels, the main vibration-related issues tend to be fatigue, failure and noise. Although these problems generally develop under the action of forces, knowledge on the modes of vibration is important to understand the dynamic characteristics of the VSCL panels, justifying that free oscillations are also studied.

The number of publications that involve vibrations of laminated fibre panels with curvilinear fibre paths is still somewhat small, although recently increasing. In Ref. 63, the maximisation of the fundamental frequency of variable stiffness composite thin plates was treated by CLT; this design is conducted by means of four lamination parameters depending on the fibre orientation. Numerical results showed that a significant increase is achievable in the fundamental frequency using VSCL instead of constant stiffness panels. In Ref. 64, a study is conducted with the same goal – maximising the fundamental frequency – but this turn on VSCL conical shells. Also in this case a higher fundamental frequency is found in VSCL than in constant stiffness shells. In Ref. 65, a study is performed to determine how a variable fibre orientation format influences the fundamental frequency of elliptical and circular cylinders. It was concluded that using a variable fibre orientation format which improves the axial buckling load[32] does not have a significant effect on the fundamental vibration frequency. It was also found that the circumferential wave number is not too sensitive to the variable fibre paths considered. In Ref. 66 an optimisation method was applied to find the maximum frequency for different shapes and boundary conditions; curvilinear fibre laminates provided higher or equal fundamental frequencies compared to straight fibre laminates. In Refs 67 and 68, the modes of vibration of VSCL plates were investigated in the linear regime using classical (thin) plate theory. It was found that plates with curvilinear fibres can have mode shapes that markedly differ from the ones of conventional plates, with vibration mode shapes resembling the fibre shapes. It was also found that curvilinear fibres affect the natural frequencies.

In Ref. 69, a new *p*-version finite element that uses a Third-order Shear Deformation Theory was presented, still for linear vibrations of VSCL. The third-order theory followed leads to zero transverse shear stress at the free surface and, in this sense, does not require a shear correction factor (but the true shear stress is not perfectly approached). It was shown that by using curvilinear fibres, and keeping all other properties fixed, important changes in mode shapes and natural frequencies can be achieved. The *p*-version Third-Order Shear Deformation element of Ref. 69 was extended to include geometrical non-linearity, and to investigate deflections, stresses and failure in the static and dynamic regimes.[70]

In Refs 71 and 72, the differences in the geometrically non-linear vibrations of VSCL and traditional plates were explored. It was found that plates with curvilinear and straight fibres that had essentially the same dynamic characteristics at low vibration amplitudes can behave quite differently at large vibration amplitudes: one can avoid/encounter resonance in the non-linear regime by curving fibres, even when the linear behaviour does not change much with respect to a reference traditional plate. In free vibrations, different internal resonances appeared due to the curvilinear fibres.[71] In forced vibrations,[72] it was found that, at least in some situations, curvilinear fibres lead to lower vibration amplitudes. The modes of vibration of composite laminated plates with curvilinear fibres are investigated in the linear and non-linear regimes in Ref. 73. The model proposed is based on thin plate theory with Von Kármán strain-displacement relations and employs a hierarchical set of basis functions. Symmetric and antisymmetric laminates are analysed, and a few conclusions are taken regarding the natural frequencies of vibration of VSCL.

Variable stiffness laminates with rectilinear fibres

In this section, we consider ways of making variable stiffness panels using composites reinforced by rectilinear, long fibres. It is not intended to provide a complete review, but to alert that there are alternatives to curvilinear fibres and to provide some information on those. We briefly address: varying the volume fraction of fibres, dropping or adding plies, and attaching discrete stiffeners.

In what concerns plates with variable fibre content (Figure 1(b)), as early as 1989[75] a plane elasticity problem was solved to determine the in-plane stresses caused by external loads. Ritz method was used. In a follow-up publication, Ref. 76, the in-plane stresses were used to compute the elastic potential energy, which was then applied in vibration and buckling problems. The latter were again dealt with by means of Ritz method and only single-layered plates were addressed. Biggers and co-workers[77,78] redistributed material across the plate width and through the plate thickness to improve the design.

Examples of more recent works on composite laminated plates with variable fibre spacing can be found in Refs 80–85. Buckling and vibration were studied using the finite element method in Ref. 80. The analysis indicates that more fibres in the plate central area may efficiently, i.e. without employing more fibres in total, increase the buckling load and natural frequencies. It was also concluded that using more fibres in the outer portion of the plate may increase the critical buckling load, but decrease the natural frequencies, because in this case the mass increases more than the stiffness. The flutter of this type of plates was evaluated in Ref. 81. Non-linear free vibration of a composite rectangular plate with variable fibre spacing, where straight and parallel fibres are distributed more densely in the central region, was investigated in Ref. 82. In Ref. 83, the variable fibre distribution concept was employed to increase in-plane flexibility and bending stiffness of a laminate employed in morphing aircraft skins; it was concluded that variable fibre distribution plays a major role in enhancing the characteristics of composite laminates. In Ref. 84, the use of composites with variable volume fraction of fibres to strength reinforced concrete shear walls was investigated. Although it is more general and apparently more interesting to vary the volume fraction of fibres in the in-plane directions, it should be mentioned that it can also be varied in the thickness direction.[85] We finish this short review on plates with parallel but variably spaced rectilinear fibres by recalling that this type of material is still orthotropic, but non-homogeneous. The stress–strain relation contains functions of space variables, functions that are defined from the fibre and matrix constants using the law of mixtures.

Another method of creating a laminate with variable stiffness is to vary the laminate thickness (Figure 1(c)). Short reviews on laminates with dropped or added plies, including discussions on their advantages and disadvantages, are given in Refs 10–12. Tatting[11] provides more information of interest on this type of laminates, including the stiffness terms for the particular case where the laminates are still symmetric about a middle plane. A finite element approach based on a First-order Shear Deformation Theory (FSDT) to include ply drop-offs in analyses of laminates has been presented in Ref. 87, where deflections and stresses on beams with different drop ratios of the ply were studied. Ply drop-off regions introduce a discontinuity within a laminate, and it can be a place of high stress concentration. This may lead to an early failure of resin or to delamination, before the ultimate load-carrying capacity of the laminate is reached. Different laminate configurations, which included both internal and external ply drops, were tested in Ref. 88. Static and fatigues tests were carried out, and the initialisation, propagation and arrest characteristics of delaminations resulting from ply drops were analysed by the finite element method. In Ref. 89, the high stress concentration caused by the ply drop-off which leads to failure was investigated. A combination of analytical and numerical methods is applied to determine singularity characteristics at the corners of the ply drop-off regions arising from the material discontinuity. In Ref. 90, a combination of a genetic algorithm and a finite element programme was introduced to deal with laminates with variable thickness obtained by ply drops.

The inclusion of stiffeners in plates (Figure 1 (d)) and shells is sometimes modelled as if the stiffeners homogeneously change the properties over the structure, and in fact employing formulations developed for unstiffened panels. For example, in Ref. 96, the buckling of thin-walled composite columns with stiffeners is considered. The classical theory of composite plates and the plate model of thin-walled structures and stiffeners is assumed. The non-linear problem is solved by the asymptotic method. Stiffeners can also be taken into account in a detailed fashion, and often this is performed using finite element approaches, where elements are applied specifically for the stiffeners (again Ref. 96 where the FEM is also applied, and Refs 97 and 98, for example). However, as pointed out in Ref. 11, stiffeners locally change the stiffness and can be considered to be another form of obtaining a variable stiffness laminate. Stiffeners provide a way to increase critical buckling loads and natural frequencies of vibration with less weight than the one of a corresponding panel with uniform thickness. A problem that arises in this type of variable stiffness laminate is the possibility of failure at the skin-stiffener interface region. Buckling and failure of laminates with T-shaped stiffeners optimised with a genetic algorithm are addressed in Ref. 91. An optimisation procedure for stiffened panels under mechanical and hygrothermal loads is presented in Ref. 92. In a recent work,[93] the critical local and global buckling loads of so-called advanced grid-stiffened conical shells is evaluated. Different grid patterns, designated as triangle-grid, ortho-grid and angle-grid, are considered. Various optimisation methods which are used for fibrous composite plates with stiffeners are presented in Ref. 94.

A different type of variable stiffness plate is considered in Ref. 95, where an optimisation technique for plates with discrete varying bending and shear stiffness is proposed. The plates have constant thickness, but with discrete variable stiffness where each discrete domain is a sandwich plate. The proposed method consists of three main steps – bending and shear stiffness optimisation, optimal discretisation of plate's internal structure and calculation of optimal geometrical or mechanical properties of the internal structure in each discrete domain.

Although we separated four approaches to achieve variable stiffness laminates, more than one procedure can be used simultaneously. In Ref. 74, a method is suggested to determine the optimal volume fraction and fibre orientation in one layer. The plate is discretised into elements and both the fibre orientation and the fibre volume fraction are assumed to be constant within each element. One should also not forget that, due to the way the fibre paths are defined and to the characteristics of tow placement technologies, in reality the volume fraction of fibres and the laminate thickness change when curvilinear fibres are used.

Demonstrative example – natural modes of vibration

A model for VSCL with curvilinear fibres

With the intention of pointing out where the curvilinear fibres introduce differences in formulation, it is here briefly explained (missing details can be found in Refs 69–71) how to obtain a model representing VSCL plates with curvilinear fibres. If an equivalent single layer, Third-order Shear Deformation approach,[2,69,70] is followed, then the displacement components in the x, y and z directions, respectively, represented by $u(x, y, z, t)$, $v(x, y, z, t)$ and $w(x, y, z, t)$ are given by

$$\begin{aligned} u(x,y,z,t) &= u^0(x,y,t) + z\phi_x(x,y,t) \\ &\quad - c_1 z^3\left(\phi_x(x,y,t) + w_x^0(x,y,t)\right) \\ v(x,y,z,t) &= v^0(x,y,t) + z\phi_y(x,y,t) \\ &\quad - c_1 z^3\left(\phi_y(x,y,t) + w_x^0(x,y,t)\right) \\ w(x,y,z,t) &= w^0(x,y,t) \end{aligned} \tag{2}$$

where $u^0(x, y, t)$, $v^0(x, y, t)$ and $w^0(x, y, t)$ are the values of displacement components u, v and w at the reference plane. Partial derivation is represented by a comma followed by the respective variable.

The origin of the Cartesian coordinate system is located in the geometric centre of the undeformed plate; axis x and y define the plate's middle reference plane (Figure 4).

If the fibre angle with respect to axis x changes linearly with x, and in a symmetric way with respect to $x = 0$, then the fibre angle in lamina k is defined as

$$\theta^k(x) = \frac{2\left(\Theta_1^k - \Theta_0^k\right)}{a}|x| + \Theta_0^k \tag{3}$$

Θ_0^k gives the angle between the fibre and the x axis at $x = 0$, and Θ_1^k gives this angle at the panel ends ($x = \pm a/2$). The fibre path that corresponds to equation (3) is represented by $<\Theta_0^k|\Theta_1^k>$.

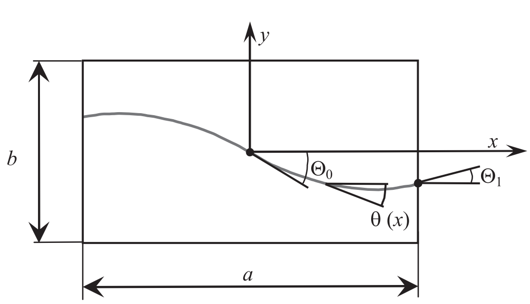

Figure 4. Curved fibre and plate dimensions. Angles Θ_0 and $\theta(x)$ have negative values in this figure; they are defined as positive in the counter-clockwise direction.

In displacement-based approaches, as in the p-version finite element method of Refs 69–72, the displacement components are written as products of shape functions and generalised coordinates. In this expansion of the displacement components, there is no difference between straight[120,121] and curvilinear fibre[69–72] laminates.

The strain displacement relations are also not different from the relations in isotropic or constant stiffness laminates. In the TSDT formulation which can be found, for example, in Ref. 2, these relations are:

$$\begin{aligned} \begin{Bmatrix} \varepsilon_x(x,y,z,t) \\ \varepsilon_y(x,y,z,t) \\ \gamma_{xy}(x,y,z,t) \end{Bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & z & 0 & 0 & -c_1z^3 & 0 & 0 \\ 0 & 1 & 0 & 0 & z & 0 & 0 & -c_1z^3 & 0 \\ 0 & 0 & 1 & 0 & 0 & z & 0 & 0 & -c_1z^3 \end{bmatrix} \\ &\quad \times \left(\begin{Bmatrix} \varepsilon_0^p(x,y,t) \\ \varepsilon_0^b(x,y,t) \\ \varepsilon_0^b(x,y,t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \varepsilon_0^{b1}(x,y,t) \end{Bmatrix} \right) \end{aligned} \tag{4}$$

$$\varepsilon_0^p(x,y.z,t) = \begin{Bmatrix} u_{,x}^0(x,y,t) \\ v_{,y}^0(x,y,t) \\ u_{,y}^0(x,y,t) + v_{,x}^0(x,y,t) \end{Bmatrix}, \tag{5}$$

$$\begin{aligned} \varepsilon_0^b(x,y,z,t) &= \begin{Bmatrix} \phi_{x,x}(x,y,t) \\ \phi_{y,y}(x,y,t) \\ \phi_{x,y}(x,y,t) + \phi_{y,x}(x,y,t) \end{Bmatrix}; \\ \varepsilon_0^{b1}(x,y,z,t) &= \begin{Bmatrix} w_{,xx}^0(x,y,t) \\ w_{,yy}^0(x,y,t) \\ 2w_{,xy}^0(x,y,t) \end{Bmatrix} \end{aligned} \tag{6}$$

$$\begin{aligned} \begin{Bmatrix} \gamma_{yz}(x,y,z,t) \\ \gamma_{xz}(x,y,z,t) \end{Bmatrix} &= \begin{bmatrix} \left(1 - 3c_1z^2\right) & 0 \\ 0 & \left(1 - 3c_1z^2\right) \end{bmatrix} \\ &\quad \times \begin{Bmatrix} w_{,y}^0(x,y,t) + \phi_y(x,y,t) \\ w_{,x}^0(x,y,t) + \phi_x(x,y,t) \end{Bmatrix}, \end{aligned} \tag{7}$$

Although the fibre orientation varies, principal stresses and strains are still related by the equation that applies in traditional orthotropic lamina:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}^{(k)}, \tag{8}$$

with numbers 1, 2 and 3 indicating the principal material axes x_1, x_2 and x_3.[2,3] But curvilinear fibre laminates differ from straight fibres laminates, precisely in the fact that the orientation of the principal material axis is not constant in the lamina domain.

Refs 2 and 3 give the plane-stress reduced stiffnesses Q_{ij} as functions of the longitudinal and transverse moduli of elasticity (E_1^k, E_2^k, respectively), Poisson ratios (ν_{12}^k, ν_{21}^k) and shear moduli (G_{12}^k, G_{13}^k, G_{23}^k). The same references also give transformed stress–strain relations for each lamina, in a form similar to:

$$\begin{Bmatrix} \sigma_x(x,y,t) \\ \sigma_y(x,y,t) \\ \tau_{xy}(x,y,t) \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11}(\theta^k(x)) & \overline{Q}_{12}(\theta^k(x)) & \overline{Q}_{16}(\theta^k(x)) \\ \overline{Q}_{12}(\theta^k(x)) & \overline{Q}_{22}(\theta^k(x)) & \overline{Q}_{26}(\theta^k(x)) \\ \overline{Q}_{16}(\theta^k(x)) & \overline{Q}_{26}(\theta^k(x)) & \overline{Q}_{66}(\theta^k(x)) \end{bmatrix}^{(k)} \times \begin{Bmatrix} \varepsilon_x(x,y,t) \\ \varepsilon_y(x,y,t) \\ \gamma_{xy}(x,y,t) \end{Bmatrix}^{(k)},$$

$$\begin{Bmatrix} \tau_{yz}(x,y,t) \\ \tau_{zx}(x,y,t) \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{44}(\theta^k(x)) & \overline{Q}_{45}(\theta^k(x)) \\ \overline{Q}_{45}(\theta^k(x)) & \overline{Q}_{55}(\theta^k(x)) \end{bmatrix}^{(k)} \times \begin{Bmatrix} \gamma_{yz}(x,y,t) \\ \gamma_{zx}(x,y,t) \end{Bmatrix}^{(k)} \tag{9}$$

However, the transformed reduced stiffnesses $\overline{Q}_{ij}^k(x,y)$ are constant in a straight fibre panel and become functions of x and/or y in a curvilinear fibre panel. Often it is assumed that the fibre orientation is just a function of x, as in equation (3), and the following relations hold:

$$\begin{aligned}
\overline{Q}_{11}^k(\theta^k(x)) &= U_1 + U_2\cos(2\theta^k(x)) + U_3\cos(4\theta^k(x)) \\
\overline{Q}_{12}^k(\theta^k(x)) &= U_4 - U_3\cos(4\theta^k(x)) \\
\overline{Q}_{22}^k(\theta^k(x)) &= U_1 - U_2\cos(2\theta^k(x)) + U_3\cos(4\theta^k(x)) \\
\overline{Q}_{16}^k(\theta^k(x)) &= \frac{1}{2}U_2\sin(2\theta^k(x)) + U_3\sin(4\theta^k(x)) \\
\overline{Q}_{26}^k(\theta^k(x)) &= \frac{1}{2}U_2\sin(2\theta^k(x)) - U_3\sin(4\theta^k(x)) \\
\overline{Q}_{66}^k(\theta^k(x)) &= U_5 - U_3\cos(4\theta^k(x)) \\
\overline{Q}_{44}^k(\theta^k(x)) &= U_6 + U_7\cos(2\theta^k(x)) \\
\overline{Q}_{45}^k(\theta^k(x)) &= -U_7\sin(2\theta^k(x)) \\
\overline{Q}_{55}^k(\theta^k(x)) &= U_6 - U_7\cos(2\theta^k(x))
\end{aligned} \tag{10}$$

where U_i are constant and defined as follows:

$$\begin{aligned}
U_1 &= \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}) \\
U_2 &= \frac{1}{2}(Q_{11} - Q_{22}) \\
U_3 &= \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}) \\
U_4 &= \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}) \\
U_5 &= \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) \\
U_6 &= \frac{1}{2}(Q_{44} + Q_{55}) \\
U_7 &= \frac{1}{2}(Q_{44} - Q_{55})
\end{aligned} \tag{11}$$

The stress resultants are defined by the constitutive relations given in Ref. 2, but now the matrices relating the stress resultants to the strains are functions of x. For example, the extensional stiffness matrix is

$$\begin{aligned}
\mathbf{A}(x) = \sum_{k=1}^{n} h_k \Bigg(& \begin{bmatrix} U_1(\theta^k(x)) & U_4(\theta^k(x)) & 0 \\ U_4(\theta^k(x)) & U_1(\theta^k(x)) & 0 \\ 0 & 0 & U_5(\theta^k(x)) \end{bmatrix} \\
& + U_2(\theta^k(x)) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cos(2\theta^k(x)) \\
& + U_3(\theta^k(x)) \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cos(4\theta^k(x)) \\
& + \frac{1}{2}U_2(\theta^k(x)) \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \sin(2\theta^k(x)) \\
& + U_3(\theta^k(x)) \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \sin(4\theta^k(x)) \Bigg)
\end{aligned} \tag{12}$$

Further matrices that relate stress resultants with strains are given in Ref. 71, for FSDT, and in Ref. 70 for Third-Order Shear Deformation Theory. If the thin, or Kirchhoff, plate theory is employed, the necessary matrices are also given in Ref. 71, because they are common to FSDT matrices.

Only when integration over the domain is performed, as occurs in weight-residual methods or by application of the virtual work principle, are the fibre angle functions evaluated. The integration over the domain leads to equilibrium equations (in statics) or

Table 1. Natural frequencies (rad/s) of simply-supported variable stiffness composite laminated (VSCL) plates by *h* and *p*-version FEM.

	[<0°,45°>,<−45°,−60°>, <0°,45°>]		[<30°,0°>,<45°,90°>, <30°,0°>]	
Mode	*h*-version FEM	*p*-version FEM[69]	*h*-version FEM	*p*-version FEM[69]
1	355.41	358.49	309.06	308.80
2	600.50	589.90	503.28	503.80
3	986.65	960.36	852.06	845.51
4	1027.55	1075.21	1143.48	1131.31
5	1309.92	1327.88	1297.29	1279.85
6	1506.90	1474.67	1309.60	1307.40
7	1743.33	1726.71	1696.40	1701.66
8	2106.31	2137.13	1779.78	1758.95
9	2171.03	2262.35	2331.44	2342.00

to equations of motion (in dynamics). In the particular case of free vibrations, the later look like:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\,\mathbf{q}(t) = 0 \tag{13}$$

with **M** and **K** representing, respectively, the mass and stiffness matrices and **q** the generalised coordinates, which are functions of time (t).

The formulation of the previous paragraphs took into consideration the analytic definition of the fibre path, with this varying in a continuous way. But when commercial finite element software is applied, it becomes easier to assume that the fibre orientation is constant in each element, actually turning a curve into a number of rectilinear segments. It would also be adequate to fix the orientation at each Gauss point, but this option may not be feasible in commercial finite element software. When the fibres are placed by shifting them, a way to approximately define their orientation in the finite element model is to divide the panels in narrow stripes, with each stripe running along the shifting direction, and in each of these stripes assume that the fibres are straight with an average orientation. As the number of stripes increases, one tends to the true path. FEM also allows one to vary the fibre path with x and y, although this is more involved, requiring longer pre-processing.

Modes of vibration

Modes of vibration that result from numerical calculations performed with Abaqus software package are now shown and compared with the ones from Ref. 69 where the more continuous model outlined in the previous section was applied. Multi-layered S4R shell element with four nodes was employed in Abaqus. This element follows a FSDT, has six degrees of freedom in each node and there is one integration location per element. The FEM implemented in Abaqus is of the *h*-version type, whilst in Ref. 69 a *p*-version FEM is adopted.

Table 2. Natural frequencies (rad/s) of clamped variable stiffness composite laminated (VSCL) plates by *h* and *p*-version FEM.

	[<0°,45°>,<−45°,−60°>, <0°,45°>]		[<30°,0°>,<45°,90°>, <30°,0°>]	
Mode	*h*-version FEM	*p*-version FEM[69]	*h*-version FEM	*p*-version FEM[69]
1	567.56	579.40	668.09	667.18
2	831.39	821.53	864.38	862.92
3	1253.18	1225.79	1244.57	1234.64
4	1448.46	1493.76	1729.13	1701.04
5	1719.96	1726.96	1799.13	1775.56
6	1818.98	1775.16	1918.95	1902.48
7	2175.80	2135.76	2282.05	2269.83
8	2505.73	2443.53	2357.20	2310.69
9	2750.46	2706.78	2908.17	2879.58

Table 3. Natural frequencies (rad/s) of free variable stiffness composite laminated (VSCL) plates by *h* and *p*-version FEM.

	[<0°,45°>,<−45°,−60°>, <0°,45°>]		[<30°,0°>,<45°,90°>, <30°,0°>]	
Mode	*h*-version FEM	*p*-version FEM[69]	*h*-version FEM	*p*-version FEM[69]
1	144.39	140.95	118.60	110.44
2	175.18	170.21	180.60	177.48
3	348.00	344.57	308.50	266.53
4	495.15	477.56	469.71	459.73
5	580.65	592.53	582.38	468.74
6	710.88	715.99	630.96	618.25
7	718.29	718.89	-	658.13
8	865.70	872.20	757.00	771.55
9	1066.63	1007.90	872.42	856.68

The geometric and material properties of the plates are the ones of Plate 1 of Ref. 69: $a = b = 1$ m, $h = 0.01$ m, $E_1 = 173$ GPa, $E_2 = 7.2$ GPa, $G_{12} = G_{13} = G_{23} = 3.76$ GPa, $\nu_{12} = 0.29$, $\rho = 1540$ kg/m^3. This plate has three layers and two lay-ups are considered: [<0°,45°>,<−45°,−60°>,<0°,45°>] and [<30°,0°>, <45°,90°>,<30°,0°>] (see Figure 4).

An eigenproblem was solved to determine the natural frequencies and mode shapes of vibration. The h and p version FEM results are compared in Tables 1–3, which show the first nine frequencies for simply-supported with movable edges, clamped and

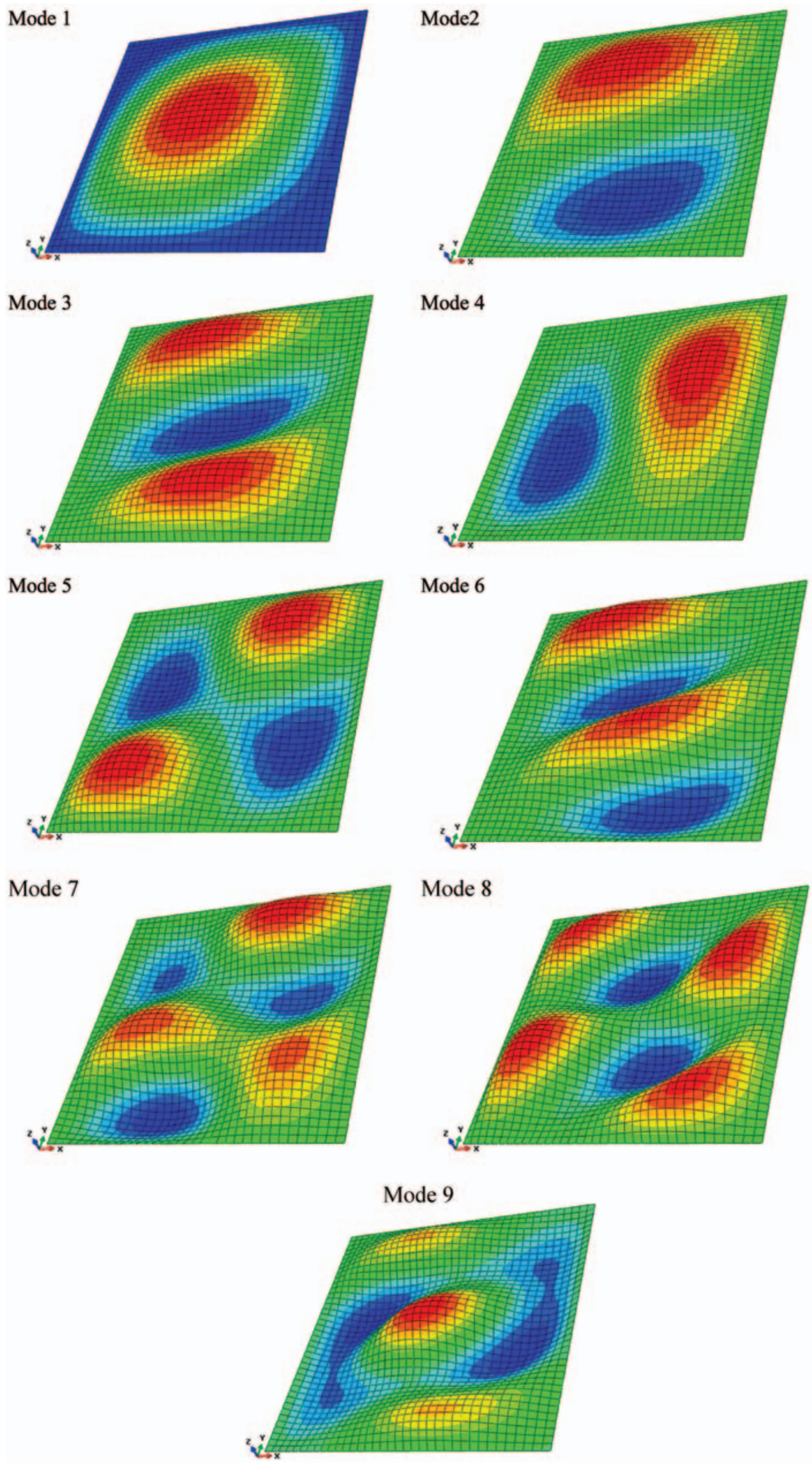

Figure 5. Vibration mode shapes of plate with [<0°,45°>,<−45°,−60°>,<0°,45°>], simply-supported case.

free boundary conditions, respectively. A reasonable good agreement between the results obtained by Abaqus and by the *p*-version finite element was achieved, with the small differences justified by the difference in the formulation and discretisation procedures.

The mode shapes, computed by Abaqus, of simply supported plate [<0°,45°>,<−45°,−60°>,<0°,45°>] are shown in Figure 5. Due to the fibre paths, planes $x = 0$ and $y = 0$ are not planes of symmetry and the shapes are somewhat warped. The effect of the curvilinear paths of this plate become more visible in higher order modes, where the ''twisting'' of the lobes is more pronounced.

Closing comments

The use of curvilinear fibres paths to tailor fibre-reinforced laminated composites so that the stiffness varies as a function of position is an increasingly available option, essentially turned possible by developments on tow placement technology. The practical interest of variable stiffness composite laminates with curvilinear fibres (VSCL) is large, particularly in aeronautics, because the expanded design space can lead to weight reductions and performance improvements. Provided that a decrease in manufacturing costs is achieved, VSCL laminates may well become more common in other applications.

The works addressed in the literature review demonstrate that variable stiffness laminates with curvilinear fibres introduce new design possibilities and can be used to improve the structural response in comparison to the conventional straight-fibre, constant-stiffness laminates. Permitting the fibres to curve within the surface of the laminate allows, for example, the designer to move higher stresses away from a less-supported area to the plate boundaries, thus increasing the buckling capacity. Curvilinear fibres also permit one to achieve less failure-prone designs or lead to changes in natural frequencies and mode shapes of vibration. But, on the other hand, this concept results in a more difficult design, increases the complexity of mechanical analysis and eventually requires the application of optimisation techniques.

The literature review reveals that advances have been achieved in the formulation, in the understanding of the mechanical behaviour of VSCL and about the possibilities offered by this technology. But there are still many unexplored issues regarding VSCL. In particular, more studies addressing failure under dynamic loads, fatigue and non-linear vibrations would be welcome, as well as more experimental tests. There appear to be no work on acoustic-structure interaction, fluid-structure interaction, active or passive control on VSCL. Tow placement results in gaps, in overlapping of some fibres and/or on fibre cutting. Although some works have included the effect of gaps and overlaps, defects or manufacturing issues associated with tow placement technology have been often overlooked in formulations. For example, a few formulations were presented for the shifted fibre case where the associated variation of the fibre volume fraction with the location in the panel has been neglected. Similarly, the thickness variation of the panel thickness was quite often not considered. There are a number of studies where the curvilinear fibre angle is a linear function of one parameter; but formulations that consider, via analytical expressions, a general variation of the stiffness properties over the domain appear to be lacking.

Conflict of interest

None declared.

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References

1. Altenbach H. Mechanics of advanced materials for lightweight structures. *J Mech Eng Sci* 2011; 225: 2481–2496.
2. Reddy JN. *Mechanics of laminated composite plates and shells: theory and analysis*. Boca Raton, FL: CRC Press, 2004.
3. Qatu MS. *Vibration of laminated shells and plates*. Amsterdam, The Netherlands: Elsevier, 2004.
4. Tomblin J and Seneviratne W. *Determining the fatigue life of composite aircraft structures using life and load-enhancement factors*. Technical Report, Federal Aviation Administration William J. Hughes Technical Centre, 2011.
5. Hyer MW and Charette RF. *Innovative design of composite structures: the use of curvilinear fiber format in composite structure design*. NASA/CR-186453, 1990.
6. Hyer MW and Lee HH. *Innovative design of composite structures: the use of curvilinear fiber format to improve buckling resistance of composite plates with central circular holes*. NASA/CR-186454, 1990.
7. Hyer MW and Charette RF. Use of curvilinear fiber format in composite structure design. *AIAA J* 1991; 29: 1011–1015.
8. Hyer MW and Lee H. The use of curvilinear fiber format to improve buckling resistance of composite plates with central circular holes. *Compos Struct* 1991; 18: 239–261.

9. Gürdal Z and Olmedo R. In-plane response of laminates with spatially varying fiber orientations: variable stiffness concept. *AIAA J* 1993; 31: 751–758.
10. Waldhart C. *Analysis of tow-placed, variable-stiffness laminates*. MSc Thesis, Virginia Tech, Blacksburg, 1996.
11. Tatting BF. *Analysis and design of variable stiffness composite cylinders*. PhD Thesis, Virginia Tech, Blacksburg, 1998.
12. Langley T. *Finite element modeling of tow-placed variable-stiffness composite laminates*. MSc Thesis, Virginia Tech, Blacksburg, 1999.
13. Tatting BF and Gürdal Z. *Design and manufacture of elastically tailored tow placed plates*. NASA/CR-2002-211919, August, 2002.
14. Tatting BF and Gürdal Z. *Automated finite element analysis of elastically- tailored plates*. NASA/CR-2003-212679, December, 2003.
15. Jegley DC, Tatting BF and Gürdal Z. Optimization of elastically tailored tow-placed plates with holes. In: *Proceedings of the 44th AIAA/ASME/ASCE /AHS/ASC Structures, Structural Dynamics, and Materials Conference (SDM)*, Norfolk, VA, April, 2003.
16. Gürdal Z Tatting BF and Wu KC. Tow-placement technology and fabrication issues for laminated composite structures. In: *46th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, Austin, TX, April, 2005.
17. Wu KC and Gürdal Z. Variable Stiffness Panel Structural Analyses with Material Nonlinearity and Correlation with Tests. In: *47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Newport, RI, May, 2006.
18. Wu KC. *Thermal and structural performance of tow-placed, variable stiffness panels*, PhD Thesis, Delft University of Technology, The Netherlands, 2006.
19. Lopes CS, Gürdal Z and Camanho PP. Variable-stiffness composite panels: Buckling and first-ply failure improvements over straight-fibre laminates. *Comput Struct* 2008; 86: 897–907.
20. Gürdal Z, Tatting BF and Wu CK. Variable stiffness composite panels: effects of stiffness variation on the in-plane and buckling response. *Compos Part A* 2008; 39: 911–922.
21. Setoodeh S, Abdalla MM, Ijsselmuiden ST, et al. Design of variable-stiffness composite panels for maximum buckling load. *Compos Struct* 2009; 87: 109–117.
22. Blom AW, Stickler PB and Gürdal Z. Optimization of a composite cylinder under bending by tailoring stiffness properties in circumferential direction. *Compos B Eng* 2010; 41: 157–165.
23. Khani A, Abdalla MM and Gürdal Z. Circumferential stiffness tailoring of general cross section cylinders for maximum buckling load with strength constraints. *Compos Struct* 2012; 94: 2851–2860.
24. Setoodeh S. *Optimal design of variable-stiffness fiber-reinforced composites using cellular automata*. PhD Thesis, Delft University of Technology, The Netherlands, 2005.
25. Alhaj Ahmad A. *Design tailoring of panels for pressure pillowing using tow-placed steered fibers*. PhD Thesis, Faculty of Aerospace Engineering, Delft University of Technology, Delft, The Netherlands, 2008.
26. Alhajahmad A, Abdalla MM and Gürdal Z. Optimal design of tow-placed fuselage panels for maximum strength with buckling considerations. *J Aircraft* 2010; 47: 775–782.
27. Lopes CS, Gürdal Z and Camanho PP. Tailoring for strength of composite steered fibre panels with cut outs. *Compos Part A* 2010; 41: 1760–1767.
28. Abdalla MM, Gürdal Z and Abdelal GF. Thermomechanical response of variable stiffness composite panels. *J Thermal Stress* 2009; 32: 187–208.
29. Rahman T, Ijsselmuiden ST, Abdalla MM and Jansen EL. Postbuckling analysis of variable stiffness composite plates using a finite element-based perturbation method. *Int J Struct Stabil Dynam* 2011; 11: 735–775.
30. IJsselmuiden ST. *Optimal design of variable stiffness composite structures using lamination parameters*. PhD Thesis, Faculty of Aerospace Engineering, Delft University of Technology, Delft, The Netherlands, 2011.
31. Van Campen JMJF, Kassapoglou C and Gürdal Z. Generating realistic laminate fiber angle distributions for optimal variable stiffness laminates. *Compos B Eng* 2012; 43: 354–360.
32. Sun M and Hyer MW. Use of material tailoring to improve buckling capacity of elliptical composite cylinders. *AIAA J* 2008; 46: 770–782.
33. Wu Z, Weaver PM, Raju G, et al. Buckling analysis and optimisation of variable angle tow composite plates. *Thin-Walled Struct* 2012; 60: 163–172.
34. Raju G, Wu Z, Kim BC, et al. Prebuckling and buckling analysis of variable angle tow plates with general boundary conditions. *Compos Struct* 2012; 94: 2961–2970.
35. Nik MA, Fayazbakhsh K, Pasini D, et al. Surrogate-based multi-objective optimization of a composite laminate with curvilinear fibers. *Compos Struct* 2012; 94: 2306–2313.
36. Fayazbakhsh K, Nik MA, Pasini D, et al. Defect layer method to capture effect of gaps and overlaps in variable stiffness laminates made by automated fiber placement. *Compos Struct* 2013; 97: 245–251.
37. Setoodeh S and Gürdal Z. Design of composite layers with curvilinear fiber paths using cellular automata. In: *Proceedings of the 44th AIAA/ASME/ASCE/AHS conference on structures, structural dynamics, and materials*, Norfolk, VA, April, 2003.
38. Setoodeh S, Abdalla MM and Gürdal Z. Design of variable–stiffness laminates using lamination parameters. *Composites Part B* 2006; 37: 301–309.
39. White S and Weaver PM. Bend-free shells under uniform pressure with variable-angle tow derived anisotropy. *Compos Struct* 2012; 94: 3207–3214.
40. Muc A and Ulatowska A. Local fibre reinforcement of holes in composite multilayered plates. *Compos Struct* 2012; 94: 1413–1419.
41. Sousa CS, Camanho PP and Suleman A. Analysis of multistable variable stiffness composite plates. *Compos Struct* 2013; 98: 34–46.
42. Groh RMJ, Weaver PM, White S, et al. A 2D equivalent single-layer formulation for the effect of transverse shear

on laminated plates with curvilinear fibres. *Compos Struct* 2013; 100: 464–478.

43. Akhavan H, Ribeiro P and Moura MFSF. Large deflection and stresses in variable stiffness composite laminate with curvilinear fibres. *Int J Mech Sci* 2013; 73: 14–26.
44. Murugan S and Friswell MI. Morphing wing flexible skins with curvilinear fiber composites. *Compos Struct* 2013; 99: 69–75.
45. Jones SE and Platts MJ. Practical matching of principal stress field geometries in composite components. *Composites Part A* 1998; 29A: 821–828.
46. Parnas L, Oral S and CeyhanÜ. Optimum design of composite structures with curved fiber courses. *Compos Sci Technol* 2003; 63: 1071–1082.
47. Lopes CS, Camanho PP, Gürdal Z, et al. Progressive failure analysis of tow-placed, variable-stiffness composite panels. *Int J Solid Struct* 2007; 44: 8493–8516.
48. Lopes C. *Damage and failure of non-conventional composite laminates*. PhD Thesis, Faculty of Aerospace Engineering, Delft University of Technology, Delft, The Netherlands, 2009.
49. Blom AW, Lopes CS, Kromwijk PJ, et al. A theoretical model to study the influence of tow-drop areas on the stiffness and strength of variable-stiffness laminates. *J Compos Mater* 2009; 43: 403–425.
50. Díaz J, Fagiano C, Abdalla MM, et al. A study of interlaminar stresses in variable stiffness plates. *Compos Struct* 2012; 94: 1192–1199.
51. Khani A, IJsselmuiden ST, Abdalla MM, et al. Design of variable stiffness panels for maximum strength using lamination parameters. *Compos B Eng* 2011; 42: 546–552.
52. IJsselmuiden ST, Abdalla MM and Gürdal Z. Implementation of strength-based failure criteria in the lamination parameter design space. *AIAA J* 2008; 46: 1826–1834.
53. Khosravi P and Sedaghati R. Design of laminated composite structures for optimum fiber direction and layer thickness, using optimality criteria. *Struct Multidiscipl Optim* 2008; 36: 159–167.
54. Croft K, Lessard L, Pasini D, et al. Experimental study of the effect of automated fiber placement induced defects on performance of composite laminates. *Compos A Appl Sci Manufact* 2011; 42: 484–491.
55. Dang TD and Hallett SR. A numerical study on impact and compression after impact behaviour of variable angle tow laminates. *Compos Struct* 2013; 96: 194–206.
56. Honda S, Igarashi T and Narita Y. Multi-objective optimization of curvilinear fiber shapes for laminated composite plates by using NSGA-II. *Compos B Eng* 2013; 45: 1071–1078.
57. Lukaszewicz DH-JA, Ward C and Potter KD. The engineering aspects of automated prepreg layup: History, present and future. *Compos B Eng* 2012; 43: 997–1009.
58. http://www.adoptech.com/fibersteering/design.htm (accessed 11 May 2013).
59. Blom AW, Tatting BF, Hol JMAM, et al. Fiber path definitions for elastically tailored conical shells. *Compos B Eng* 2009; 40: 77–84.
60. Blom AW. *Structural performance of fiber-placed, variable-stiffness composite conical and cylindrical shells*. PhD Thesis, Faculty of Aerospace Engineering, Delft University of Technology, Delft, The Netherlands, 2010.
61. Senocak E and Tanriover H. Analysis of composite plates with variable stiffness using Galerkin method. *Aeronaut J* 2007; 111: 247–256.
62. Setoodeh S, Gürdal Z and Watson LY. Design of variable-stiffness composite layers using cellular automata. *Comput Meth Appl Mech Eng* 2006; 195: 836–851.
63. Abdalla MM, Setoodeh S and Gürdal Z. Design of variable stiffness composite panels for maximum fundamental frequency using lamination parameters. *Compos Struct* 2007; 81: 283–291.
64. Blom AW, Setoodeh S, Hol JMAM, et al. Design of variable-stiffness conical shells for maximum fundamental eigenfrequency. *Comput Struct* 2008; 86: 870–878.
65. Lo H-C and Hyer MW. Fundamental frequencies of elliptical composite cylinders with circumferentially varying fiber orientation. *AIAA J* 2012; 50: 928–937.
66. Honda S and Narita Y. Vibration design of laminated fibrous composite plates with local anisotropy induced by short fibers and curvilinear fibers. *Compos Struct* 2011; 93: 902–910.
67. Honda S, Oonishi Y, Narita Y, et al. Vibration analysis of composite rectangular plates reinforced along curved lines. *J Sys Design Dynam* 2008; 2: 76–82.
68. Honda S and Narita Y. Natural frequencies and vibration modes of laminated composite plates reinforced with arbitrary curvilinear fiber shape paths. *J Sound Vibr* 2012; 331: 180–191.
69. Akhavan H and Ribeiro P. Natural modes of vibration of variable stiffness composite laminates with curvilinear fibers. *Compos Struct* 2011; 93: 3040–3047.
70. Akhavan H, Ribeiro P and De Moura MFSF. Composite laminates with linearly varying fiber angles under static and dynamic loads, In: *54th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Co-located Events*, Boston, MA, April, 2013, AIAA 2013-1565.
71. Ribeiro P and Akhavan H. Non-linear vibrations of variable stiffness composite laminated plates. *Compos Struct* 2012; 94: 2424–2432.
72. Ribeiro P. Non-linear free periodic vibrations of variable stiffness composite laminated plates. *Nonlinear Dynam* 2012; 70: 1535–1548.
73. Houmat A. Nonlinear free vibration of laminated composite rectangular plates with curvilinear fibres. *Compos Struct* 2013; 106: 211–224.
74. Duvaut G, Terrel G, Léné F, et al. Optimization of fiber reinforced composites. *Compos Struct* 2000; 48: 83–89.
75. Martin AF and Leissa AW. Application of the Ritz method to plane elasticity problems for composite sheets with variable fiber Spacing. *Inter J Numer Meth Eng* 1989; 28: 1813–1825.
76. Leissa AW and Martin AF. Vibration and buckling of rectangular composite plates with variable fiber spacing. *Compos Struct* 1990; 14: 339–357.
77. Biggers SB and Srinivasan S. Compression buckling response of tailored rectangular composite plates. *AIAA J* 1993; 31: 590–596.

78. Biggers SB and Browder TM. Buckling load interaction in tailored composite plates. *Compos Eng* 1994; 4: 745–761.
79. Baranski AT and Biggers SB. Postbuckling analysis of tailored composite plates with progressive damage. *Compos Struct* 1999; 46: 245–255.
80. Kuo SY and Shiau LC. Buckling and vibration of composite laminated plates with variable fiber spacing. *Compos Struct* 2009; 90: 196–200.
81. Kuo SY. Flutter of rectangular composite plates with variable fiber spacing. *Compos Struct* 2011; 93: 2533–2540.
82. Houmat A. Nonlinear free vibration of a composite rectangular specially-orthotropic plate with variable fiber spacing. *Compos Struct* 2012; 94: 3029–3036.
83. Murugan S, Flores EIS, Adhikari S, et al. Optimal design of variable fiber spacing composites for morphing aircraft skins. *Compos Struct* 2012; 94: 1626–1633.
84. Meftah SA, Yeghnem R, Tounsi A, et al. Lateral stiffness and vibration characteristics of composite plated RC shear walls with variable fibres spacing. *Mater Design* 2008; 29: 1955–1964.
85. Fu Y, Zhang P and Yang F. Interlaminar stress distribution of composite laminated plates with functionally graded fiber volume fraction. *Mater Design* 2010; 31: 2904–2915.
86. Curry JM, Johnson ER and Starnes JH. Effect of dropped plies on the strength of graphite–epoxy laminates. *AIAA J* 1992; 30: 449–456.
87. Mukherjee A and Varughese B. A ply drop-off element for inclusion of drop-off in the global analysis of layered composite structures. *Comput Struct* 1995; 54: 865–870.
88. Cairns DS, Mandell JF, Scott ME, et al. Design and manufacturing considerations for ply drops in composite structures. *Compos B Eng* 1999; 30: 523–534.
89. Her SC. Stress analysis of ply drop-off in composite structures. *Compos Struct* 2002; 57: 235–244.
90. Paluch B, Grédiac M and Faye A. Combining a finite element programme and a genetic algorithm to optimize composite structures with variable thickness. *Compos Struct* 2008; 83: 284–294.
91. Herencia JE, Weaver PM and Friswell MI. Optimization of anisotropic composite panels with T-shaped stiffeners including transverse shear effects and out-of-plane loading. *Struct Multidiscipl Optim* 2008; 37: 165–184.
92. Marín L, Trias D, Badalló P, et al. Optimization of composite stiffened panels under mechanical and hygrothermal loads using neural networks and genetic algorithms. *Compos Struct* 2012; 94: 3321–3326.
93. Shi S, Sun Z, Ren M, et al. Buckling resistance of grid-stiffened carbon-fiber thin-shell structures. *Compos B Eng* 2013; 45: 888–896.
94. Awad ZK, Aravinthan T, Zhuge Y, et al. A review of optimization techniques used in the design of fibre composite structures for civil engineering applications. *Mater Design* 2012; 33: 534–544.
95. Sliseris J and Rocens K. Optimal design of composite plates with discrete variable stiffness. *Compos Struct* 2013; 98: 15–23.
96. Teter A and Kolakowski Z. Buckling of thin-walled composite structures with intermediate stiffeners. *Compos Struct* 2005; 69: 421–428.
97. Das HS and Chakravorty D. Bending analysis of stiffened composite conoidal shell roofs through finite element application. *J Compos Mater* 2011; 45: 525–542.
98. Li D, Qing G and Liu Y. A layerwise/solid-element method for the composite stiffened laminated cylindrical shell structures. *Compos Struct* 2013; 98: 215–227.
99. Jha DK, Kant T and Singh RK. A critical review of recent research on functionally graded plates. *Compos Struct* 2013; 96: 833–849.
100. Saavedra Flores EI, Friswell MI and Xia Y. Variable stiffness biological and bio-inspired materials. *J Intelligent Mater Sys Struct* 2012; 24: 529–540.
101. Kim BC, Potter K and Weaver PM. Continuous tow shearing for manufacturing variable angle tow composites. *Compos Part A* 2012; 43: 1347–1356.
102. Mattheij P, Gliesche K and Feltin D. Tailored fiber placement-mechanical properties and applications. *J Reinf Plast Compos* 1998; 17: 774–786.
103. Spickenheuer A, Schulz M, Gliesche K, et al. Using tailored fibre placement technology for stress adapted design of composite structures. *Plast, Rubber Compos* 2008; 37: 227–232.
104. Uhlig K, Spickenheuer A, Bittrich L, et al. Development of a highly stressed bladed rotor made of a CFRP using the tailored fiber placement technology. Mechanics of a highly stressed bladed rotor made of a CFRP using the tailored fiber placement technology. *Mech Compos Mater* 2013; 49: 201–210.
105. Ghiasi H, Fayazbakhsh K, Pasini D, et al. Optimum stacking sequence design of composite materials Part II: Variable stiffness design. *Compos Struct* 2010; 93: 1–13.
106. Muc A and Ulatowska A. Design of plates with curved fibre format. *Compos Struct* 2010; 92: 1728–1733.
107. Turvey GJ and Marshall IH. *Buckling and postbuckling of composite plates*. London: Chapman & Hall, 1995.
108. París F. *A study of failure criteria of fibrous composite materials*. NASA/CR- 2001-210661, 2001.
109. Hinton MJ, Kaddour AS and Soden PD (eds) *Failure criteria in fibre reinforced polymer composites: The World-Wide Failure Exercise*. Amsterdam, The Netherlands: Elsevier, 2004.
110. Daniel M and Ishai O. *Engineering mechanics of composite materials*. Oxford: Oxford University Press, 1994.
111. Sridharan S (ed.) *Delamination behaviour of composites, Series in Composites Science and Engineering No. 25*, Woodhead Publishing, Washington University in St. Louis, USA, 2008.
112. Locke J. Nonlinear random response of angle-ply laminates with static and thermal preloads. *AIAA J* 1991; 29: 1480–1487.
113. Xue DY and Mei C. Finite element nonlinear panel flutter with arbitrary temperatures in supersonic flow. *AIAA J* 1993; 31: 154–162.
114. Chemoul B and Louaas E. Vibration and acoustic dimensioning of Ariane launchers. In: Sas P and Moens D (eds) *Keynote lecture, Proceedings of the 25th*

Int. Conf. on Modal Analysis Noise and Vibration Engrg. Leuven, Belgium: ISMA, 2000, pp.1–22.
115. Zhou RC, Xue Y and Mei C. Finite element time domain-modal formulation for nonlinear flutter of composite panels. *AIAA J* 1994; 32: 2044–2052.
116. http://www.vrand.com/Genesis.html (accessed 24 March 2013).
117. Rankin CC, Loden WA and Brogan FA. *STAGS user manual*. NASA/CR-2009-000000, Langley Research Center, National Aeronautics and Space Administration.
118. http://www.3ds.com/products/simulia/portfolio/abaqus (accessed 24 March 2013).
119. Tsai SW and Hahn HT. *Introduction to composite materials*. Westport, CT: Technomic Publishing Co., Inc., 1980.
120. Szabó BA and Babuška I. *Finite element analysis*. New York: John Wiley & Sons, 1991.
121. Ribeiro P. First-order shear deformation, p-version, finite element for laminated plate non-linear vibrations. *AIAA J* 2005; 43: 1371–1379.