

Pilot Contamination Precoding in Multi-Cell Large Scale Antenna Systems

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Abstract—An LSAS entails a large number (tens or hundreds) of base station antennas serving a much smaller number of terminals, with large gains in spectral-efficiency and energy-efficiency compared with conventional MIMO technology. Until recently it was believed that in multi-cellular LSAS, even in the asymptotic regime, as the number of service antennas tends to infinity, the performance is limited by directed inter-cellular interference. The interference results from unavoidable re-use of reverse-link pilot sequences (pilot contamination) by terminals in different cells.

We devise a new concept that leads to the effective elimination of inter-cell interference in TDD LSAS systems. This is achieved by outer multi-cellular pre-coding, which we call pilot contamination pre-coding (PCP). The main idea of PCP is that each base station linearly combines messages aimed to terminals from different cells that re-use the same pilot sequence. Crucially, the combining coefficients depend only on the slow-fading coefficients between the terminals and the base stations. Each base station independently transmits its PCP-combined symbols using conventional linear pre-coding that is based on estimated fast-fading coefficients. Further we derive estimates for SINRs and a capacity lower bound for the case of LSASs with PCP and finite number of antennas M .

I. INTRODUCTION

The exponential increase in demand for high data rates, as well as the higher user density in cellular networks require new ways of mitigating interference allowing a larger number of users to share bandwidth. This, along with the Green Touch initiative to decrease the power consumption in communications networks, motivates the analysis of cellular systems with a very large number of base station (BTS) antennas. Such systems have been studied extensively.

In [5], [1] the authors derived estimates for SINR values in a non-cooperative cellular network in which the number of BTS antennas tends to infinity. It is shown in [5], [1], that not all interference vanishes, and therefore, SINR does not grow indefinitely. The reason is that the channel estimates made at the BTS contain not only the desired channel vector and additive white noise, but also components directed towards users from other cells who are assigned non-orthogonal pilot sequences.

A number of techniques were proposed for achieving higher SINRs. The numerical results obtained in [5], [1], [2] show that these techniques provide breakthrough data transmission rates for noncooperative cellular networks. Advanced network-MIMO TDD systems that allow some collaboration between

base stations were proposed recently in [3]. Unfortunately, in all these techniques SINR values remain finite and do not grow indefinitely with the number of base station antennas M .

In this paper we allow a limited collaboration between base stations and propose an outer multi-cellular pre-coding, which we call pilot contamination pre-coding. In the asymptotic regime, as M tends to infinity, this precoding allows us to construct interference free multi-cell wireless systems. We further proceed with analysis of LSASs with finite M .

The paper is organized as follows. First, in Section II, we describe our system model, assumptions, and TDD protocol. Then, in Section III, we explain the pilot contamination problem and show that the pilot contamination results in inter-cell interference that does not vanish as the number of base station antennas M is growing. In Section IV we propose the pilot contamination precoding and show that it allows achieving interference free TDD LSASs. Finally, in Section V we derive estimates for SINRs and a capacity lower bound for the case of LSASs with PCP and finite number of antennas M .

II. SYSTEM MODEL

We consider a cellular network composed of L hexagonal cells, each consisting of a central M -antenna base station and K single-antenna terminals that share the same bandwidth. We assume that Orthogonal Frequency-Division Multiplexing (OFDM) is used. Consequently, we consider a flat-fading channel model for each OFDM subcarrier. For a given subcarrier we denote by

$$g_{mikl} = h_{mikl} \sqrt{\beta_{ikl}} \quad (1)$$

the *channel (propagation) coefficient* between the m -th antenna of the i -th BTS and the k -th terminal of the l -th cell, Fig. 1. The first factor in (1) is the fast fading coefficient $h_{mikl} \sim \mathcal{CN}(0, 1)$. The second factor in (1) is slow fading coefficient (log normal and geometric decay). Coefficients β_{ikl} are constant with respect to frequency and BTS antenna index.

The channel coefficients of the i -th base station form the *channel vector* $\mathbf{g}_{ikl} = (g_{1ikl}, g_{2ikl}, \dots, g_{Mikl}) \in \mathbb{C}^{1 \times M}$, and the corresponding fast fading coefficients form *fast fading vector* $\mathbf{h}_{ikl} = (h_{1ikl}, h_{2ikl}, \dots, h_{Mikl}) \in \mathbb{C}^{1 \times M}$. We assume that fading coefficients are i.i.d. and therefore $\mathbf{h}_{ikl} \sim \mathcal{CN}(0, I_M)$.

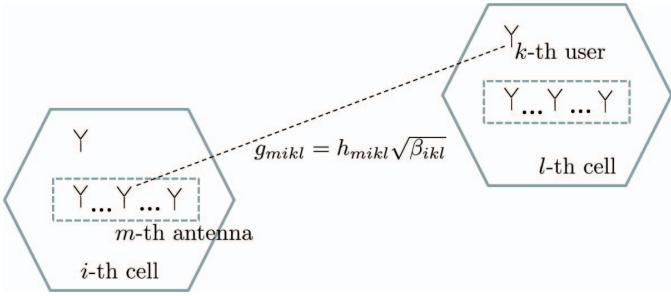


Fig. 1. The channel coefficient g_{mikl} between the m -th antenna of the i -th cell and the k -th terminal in the l -th cell

We further assume that the fast fading channels of different terminals are statistically independent.

We assume a time block fading model. Thus fast fading vectors \mathbf{h}_{ikl} stay constant during coherence intervals of T OFDM symbols. These vectors are assumed to be independent in different coherence blocks.

We assume reciprocity between uplink and downlink channels, i.e. that β_{ikl} and \mathbf{h}_{mikl} are the same for these channels.

Let us consider a Time-Division Duplexing (TDD) scheme, Fig. 2. Each coherence interval is organized in four phases:

- first, each terminal sends uplink data to its BTS;
- second, all terminal synchronously send pilot sequences of length τ ;
- base stations use these pilots to estimate the corresponding channel vectors and use the obtained estimates for maximum ratio combining processing of the uplink data
- base stations transmit downlink data to their terminals using the channel estimates as beamforming vectors;

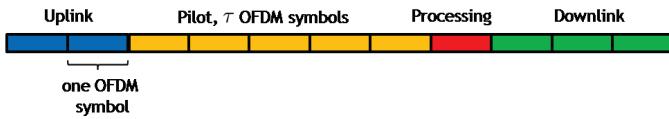


Fig. 2. Coherence interval of $T = 11$ OFDM symbols

Below we describe details of this protocol for downlink transmission. We assume that K terminals located in the same cell use orthogonal pilots. The maximum number of orthogonal τ -tuples is τ , therefore $K \leq \tau$. The coherence intervals of fast moving terminals are short, typically $T \in [4, 30]$. So τ should be small. Therefore users in the neighboring cells unavoidably use nonorthogonal pilots, or reuse the same pilots. We assume that in all cells the same set ψ_1, \dots, ψ_τ of orthogonal pilots ($\psi_i^\dagger \psi_j = \delta_{ij}$) is used. While not optimal, this choice completely eliminates intra-cell interference as $M \rightarrow \infty$, as we detail below.

The k -th terminals in all cells synchronously send ψ_k . The i -th base station receives the $M \times \tau$ matrix

$$\mathbf{Y}_i = \sum_{l=1}^L \sum_{k=1}^K \sqrt{\rho_r \tau} \mathbf{g}_{ikl}^\top \psi_k + \mathbf{z}_i, \quad (2)$$

where $\mathbf{z}_i \in \mathbb{C}^{M \times K}$ is the additive noise. Without loss of generality, we assume that the entries of \mathbf{z}_i are i.i.d. $\mathcal{CN}(0, 1)$ random variables and that all gains are scaled accordingly.

The i -th base station estimates the vectors \mathbf{g}_{iki} for terminals located in the same cell as $\hat{\mathbf{g}}_{iki}^\top = \mathbf{Y}_i \cdot \psi_k^\dagger / \sqrt{\tau \rho_r}, k = 1, \dots, K$, which results in

$$\hat{\mathbf{g}}_{iki} = \mathbf{g}_{iki} + \sum_{l=1, l \neq i}^L \mathbf{g}_{ikl} + \mathbf{z}'_i, \quad (3)$$

where $\mathbf{z}'_i = \frac{\mathbf{z}_i \psi_k^\dagger}{\sqrt{\tau \rho_r}} \sim \mathcal{CN}(0, \frac{1}{\tau \rho_r} I_M)$. The estimate (3) is contaminated, as it involves the channel vectors of users from different cells that use pilot ψ_k .

The base station uses the estimates $\hat{\mathbf{g}}_{iki}$ for computing precoding beamforming vectors. For $M \rightarrow \infty$ it is convenient to use conjugate beam-forming defined by

$$\mathbf{w}_{ki} = \hat{\mathbf{g}}_{iki}^\dagger. \quad (4)$$

With this precoding the base station transmits with the power $\rho_f \|\hat{\mathbf{g}}_{iki}\|^2$, which varies from one coherent interval to another. For transmitting with a constant power the base station should use normalized beamforming vector:

$$\mathbf{w}_{ki} = \frac{\hat{\mathbf{g}}_{iki}^\dagger}{\|\hat{\mathbf{g}}_{iki}\|} = \frac{\hat{\mathbf{g}}_{iki}^\dagger}{\alpha_{ki} \sqrt{M}}. \quad (5)$$

The scalar

$$\alpha_{ki} = \frac{\|\hat{\mathbf{g}}_{iki}\|}{\sqrt{M}}$$

is a normalization factor.

After computing vectors \mathbf{w}_{ki} BTS transmits from its M antennas the vector

$$\mathbf{v}_i = \mathbf{w}_{1i} q_{1i} + \dots + \mathbf{w}_{Ki} q_{Ki},$$

where q_{ki} is the signal intended to the k -th terminal in the i -th cell.

The k -th terminal of the i -th cell receives the signal

$$x_{ki} = \sum_{l=1}^L \sum_{k'=1}^K \sqrt{\rho_f} \mathbf{g}_{lki} \mathbf{w}_{k'l} q_{k'l} + z_{ki}, z_{ki} \in \mathcal{CN}(0, 1). \quad (6)$$

III. PILOT CONTAMINATION PROBLEM IN LSAS

The analysis of the asymptotic behavior of the SINR was conducted in [5] for the precoding defined in (4) and in [1] for precoding (5). Below we summarize the obtained results.

We assume precoding (5). The following well known Lemma will be useful for us.

Lemma 1. Let $\mathbf{x}, \mathbf{y} \in \mathbb{C}^{M \times 1}$ be two independent vectors with distribution $\mathcal{CN}(\mathbf{0}, c \mathbf{I})$. Then

$$\lim_{M \rightarrow \infty} \frac{\mathbf{x}^\dagger \mathbf{y}}{M} \stackrel{a.s.}{=} 0 \text{ and } \lim_{M \rightarrow \infty} \frac{\mathbf{x}^\dagger \mathbf{x}}{M} \stackrel{a.s.}{=} c. \quad (7)$$

Using the fact that the channel vectors of different terminals are independent, and applying the above lemma, we can derive the asymptotic behavior of α_{ki}^2 :

$$\lim_{M \rightarrow \infty} \alpha_{ki}^2 \stackrel{a.s.}{=} \sum_{l=1}^L \beta_{ikl} + \frac{1}{\tau \rho_r}. \quad (8)$$

Denote the terms in the double sum of (6) by $Q_{k'l}$ and let $S_{k'l} = |Q_{k'l}|^2$. Again using Lemma 1, we obtain

$$\lim_{M \rightarrow \infty} \frac{S_{k'l}}{M} \stackrel{\text{a.s.}}{=} \begin{cases} \frac{\rho_f \beta_{lki}^2}{\alpha_{kl}^2}, & k' = k, \\ 0, & k' \neq k. \end{cases} \quad (9)$$

In (6) S_{ki} is the signal power and all other terms $S_{k'l}$, contribute to interference. From (9) we see that only the terminals that use the same pilot ψ_k create interference that does not vanish as $M \rightarrow \infty$. The reason for this is that the beamforming vectors of these terminals contain components directed towards the k -th terminal in the i -th cell generating directed interference that does not vanish as $M \rightarrow \infty$. The variance of the additive noise, however, is unitary regardless of the number of BTS antennas, thus rendering the effect of the noise null in the asymptotic region.

Equations (8) and (9) define the asymptotic behavior of the numerator and denominator of the SINR, which are formed of independent variables. Therefore, we have the following theorem (analysis for uplink is similar).

Theorem 1. [5] [1] *The downlink SINR of the k -th terminal in the i -th cell for precoding (4) is*

$$\varsigma_{ik}^D = \frac{\beta_{lki}^2}{\sum_{l=1, l \neq i}^L \beta_{lki}^2}, \quad (10)$$

and for precoding (5) the SINR is

$$\varsigma_{ik}^D = \frac{\beta_{lki}^2 / \alpha_{ki}^2}{\sum_{l=1, l \neq i}^L \beta_{lki}^2 / \alpha_{kl}^2}, \quad (11)$$

with $\alpha_{kl}^2 = \sum_{j=1}^L \beta_{lkj} + \frac{1}{\tau \rho_r}$.

For uplink transmission the SINR is

$$\varsigma_{ik}^U = \frac{\beta_{lki}^2}{\sum_{l=1, l \neq i}^L \beta_{lki}^2}, \quad (12)$$

IV. PILOT CONTAMINATION PRECODING AND INTERFERENCE FREE LSAS

The SINR values from Theorem 1 can be improved by different techniques (for instance, we can allow different transmit powers $\rho_{f,l}$; their optimization gives a significant gain in SINRs [1]). In all these techniques, however, the SINRs do not grow with M . One may try to use network MIMO approach for canceling the inter-cell interference. Network MIMO, however, is based on the assumption that the base stations send to each other estimates $\hat{\mathbf{g}}_{ikl}$, which is not feasible in LSAS systems.

One possible conclusion of these arguments can be that in both noncooperative LSASs and LSASs with cooperation SINRs do not grow with M beyond certain limits. Below we show that this conclusion is wrong. We demonstrate that a limited collaboration between base stations allows us to radically resolve the pilot contamination problem and to construct interference free LSASs.

We make the following assumption on collaboration between base stations.

- First, we assume that all signals q_{ki} are accessible to all base stations across the entire network.
- Second, we assume that the slow fading coefficients β_{ikl} can be accurately estimated and made available to all base stations or alternatively to a network hub.

Remark We assume that q_{ki} and β_{ikl} are accessible across the entire network only to obtain a simple theoretical model. In a real system it will be enough if q_{ki} and β_{ikl} are available only to a limited number of cell neighboring the i -th cell.

We would like to point out that the slow fading coefficients β_{ikl} are easy to estimate since they are constant over the M antennas, frequency, and over many time slots. In a typical system with 20MHz bandwidth and the useful symbol duration of $2/3 \times 100$ microseconds there are about 1400 tones per OFDM symbol. We assign to each terminal a dedicated OFDM tone. The terminals transmit pilots of duration 1 (for instance $\psi_{kl} = 1$ for all k and l), simultaneously in the assigned tones. The i -th base station receives in the appropriate tone: $\mathbf{y}_i = \sqrt{\beta_{ikl}} \mathbf{h}_{ikl}^\top \psi_{kl}$, and computes the estimate as $\hat{\beta}_{ikl} = \frac{1}{M} \mathbf{y}_i^\top \mathbf{y}_i$. It is easy to see that

$$\lim_{M \rightarrow \infty} \frac{1}{M} \mathbf{y}_i^\top \mathbf{y}_i \stackrel{\text{a.s.}}{=} \beta_{ikl}.$$

So we need only one OFDM symbol for getting these estimates. Terminals that are located in remote cells can reuse the same OFDM tone. If this separation is not enough the terminals may use orthogonal pilots ψ_{kl} of length n . In this case the same estimation procedure can be applied and only every $(1400 \times n)$ -th terminal will be reusing the same pilot.

Now we re-examine equation (6). Instead of treating the terms Q_{kl} as interference and estimating their powers, we look at the terms themselves:

$$\begin{aligned} \frac{1}{\sqrt{M}} Q_{kl} &= \frac{\sqrt{\rho_f}}{\sqrt{M}} \hat{\mathbf{g}}_{lki} \mathbf{w}_{kl} q_{kl} \\ &= \frac{\sqrt{\rho_f \beta_{lki}}}{M \alpha_{kl}} \mathbf{h}_{lki} \left(\sum_{l_1=1}^L \sqrt{\beta_{lkl_1}} \mathbf{h}_{lkl_1}^\top + \mathbf{z}_l'^\top \right) q_{kl} \end{aligned} \quad (13)$$

Applying Lemma 1 we obtain

$$\lim_{M \rightarrow \infty} \frac{1}{\sqrt{M}} Q_{kl} \stackrel{\text{a.s.}}{=} \frac{\sqrt{\rho_f} \beta_{lki}}{\alpha_{kl}} q_{kl}.$$

From the above expression and (9) we get

$$\frac{1}{\sqrt{M}} x_{ki} = \sqrt{\rho_f} \left(\frac{\beta_{1ki}}{\alpha_{k1}} q_{k1} + \frac{\beta_{2ki}}{\alpha_{k2}} q_{k2} + \dots + \frac{\beta_{Lki}}{\alpha_{kL}} q_{kL} \right). \quad (14)$$

Let us denote

$$\mathbf{x}_k = \frac{1}{\sqrt{M}} \begin{pmatrix} x_{k1} \\ \vdots \\ x_{kL} \end{pmatrix} \quad \text{and} \quad \mathbf{q}_k = \begin{pmatrix} q_{k1} \\ \vdots \\ q_{kL} \end{pmatrix}$$

and

$$\mathbf{B}_k = \begin{pmatrix} \beta_{1ki} / \alpha_{k1} & \dots & \beta_{Lki} / \alpha_{kL} \\ \vdots & & \vdots \\ \beta_{1kL} / \alpha_{k1} & \dots & \beta_{LkL} / \alpha_{kL} \end{pmatrix}.$$

Then from (28) we have

$$\frac{1}{\sqrt{M}} \mathbf{x}_k = \sqrt{\rho_f} \mathbf{B}_k \mathbf{q}_k. \quad (15)$$

Now we can describe the pilot contamination precoding.

Downlink PCP

The i -th base station estimates the slow fading coefficients β_{ikl} , $k = 1, \dots, K; l = 1, \dots, L$, and transmits them to the network hub (alternatively to all other base stations). The network hub computes the PCP precoding matrices

$$\mathbf{A}_k = \mathbf{B}_k^{-1}, k = 1, \dots, K \quad (16)$$

and transmits the l -th row \mathbf{a}_{kl} of \mathbf{A}_k to the l -th base station. The l -th base station computes the signals

$$\mathbf{q}'_{kl} = \mathbf{a}_{kl} \mathbf{q}_k. \quad (17)$$

and transmits from its M antennas the vector

$$\mathbf{v}_l = \mathbf{w}_{1l} q'_{1l} + \mathbf{w}_{2l} q'_{2l} + \dots + \mathbf{w}_{Kl} q'_{Kl}.$$

The end of Downlink PCP

From (15), (17), and (16) we obtain

$$\frac{1}{\sqrt{M}} \mathbf{x}_k = \mathbf{B}_k \begin{pmatrix} q'_{k1} \\ \vdots \\ q'_{KL} \end{pmatrix} = \mathbf{B}_k \mathbf{A}_k \begin{pmatrix} q_{k1} \\ \vdots \\ q_{KL} \end{pmatrix} = \begin{pmatrix} q_{k1} \\ \vdots \\ q_{KL} \end{pmatrix}.$$

From this equation it follows that both the additive noise and interference vanishes and we have SINR:

$$\lim_{M \rightarrow \infty} \zeta_{kl}^D \rightarrow \infty. \quad (18)$$

Thus, under our network assumptions, we constructed a noise free and interference free multi-cell LSAS.

Remark In this algorithm vectors \mathbf{w}_{kl} are computed solely on the basis of $\hat{\mathbf{g}}_{kl}$. An exchange of M dimensional vectors is not required (opposite to the network MIMO).

Let us consider now **the uplink transmission**. Let the k -th user in the l -th cell transmit s_{kl} . The i -th base station receives

$$\mathbf{y}_i = \sum_{l=1}^L \sum_{k=1}^K \sqrt{\rho_r} \mathbf{g}_{ikl}^\top s_{kl} + \mathbf{z}_i.$$

To decode the signal s_{ki} the base station applies Maximum Ratio Combining by multiplying the received signal \mathbf{y}_i by the scaled channel estimate from (3) and obtains

$$\begin{aligned} \hat{s}_{ki} &= \frac{1}{\sqrt{M}} \hat{\mathbf{g}}_{iki}^* \mathbf{y}_i \\ &= \frac{1}{\sqrt{M}} \sum_{l_1=1}^L \mathbf{g}_{ikl_1}^* \sum_{l=1}^L \sum_{k=1}^K \sqrt{\rho_r} \mathbf{g}_{ikl}^\top s_{kl} \\ &+ \sum_{l_1=1}^L \mathbf{g}_{ikl_1}^* \mathbf{z}_i + \mathbf{z}'_i \sum_{l=1}^L \sum_{k=1}^K \mathbf{g}_{ikl}^\top s_{kl} + \mathbf{z}'_i \mathbf{z}_i. \end{aligned} \quad (19)$$

Here $*$ denotes complex conjugation of all entries of the corresponding vector. Applying Lemma 1, we obtain

$$\hat{s}_{ki} \stackrel{\text{a.s.}}{=} \sqrt{\rho_r} (\beta_{ik1} s_{k1} + \beta_{ik2} s_{k2} + \dots + \beta_{ikL} s_{kL}).$$

Let

$$\mathbf{B}_k^U = \begin{pmatrix} \beta_{1k1} & \dots & \beta_{1kL} \\ \vdots & & \vdots \\ \beta_{Lk1} & \dots & \beta_{LkL} \end{pmatrix}.$$

Then

$$(\hat{s}_{k1}, \dots, \hat{s}_{kL})^\top = \sqrt{\rho_r} \mathbf{B}_k^U (s_{k1}, \dots, s_{kL})^\top.$$

Uplink PCP

The base stations estimate their slow fading coefficients β_{ikl} and transmit them to the network hub. The hub computes

$$\mathbf{A}_k = (\sqrt{\rho_r} \mathbf{B}_k^U)^{-1}, k = 1, \dots, K,$$

and passes the l -th row \mathbf{a}_{kl} of \mathbf{A}_k to the l -th base station.

The i -th base station computes the estimates \hat{s}_{ki} , $k = 1, \dots, K$, according to (19) and passes them to all other base stations. The i -th base stations reconstruct the symbol s_{ki} as

$$s_{ki} = \mathbf{a}_{ki} \cdot (\hat{s}_{k1}, \dots, \hat{s}_{kL})^\top.$$

The end of uplink PCP

Similar to the downlink case we have $\lim_{M \rightarrow \infty} \zeta_{kl}^U = \infty$.

V. FINITE M ANALYSIS

For a finite M we make small changes in the TDD protocol.

A. TDD Protocol and Downlink PCP

The i -th base station receives at the m -th antenna the m -th row \mathbf{y}_{mi} of the matrix \mathbf{Y}_i defined in (2). It further multiples \mathbf{y}_{mi} by ψ_k^\dagger and obtains *the processed pilot signal*

$$y_{mik} = \mathbf{y}_{mi} \psi_k^\dagger = \sqrt{\rho_r \tau} \sum_{l=1}^L g_{mikl} + z_{mik}, z_{mik} \in \mathcal{CN}(0, 1).$$

From y_{mik} we get MMSE linear estimates:

$$\hat{g}_{mikl} = \frac{\sqrt{\rho_r \tau} \beta_{ikl} y_{mik}}{1 + \rho_r \tau \sum_{s=1}^L \beta_{iks}}. \quad (20)$$

The estimation error $\tilde{g}_{mikl} = \hat{g}_{mikl} - g_{mikl}$ is uncorrelated with the estimate \hat{g}_{mikl} . It is easy to see that

$$E \{ |\tilde{g}_{mikl}|^2 \} = \beta_{ikl} - \frac{\rho_r \tau \beta_{ikl}^2}{1 + \rho_r \tau \sum_{s=1}^L \beta_{iks}}, \quad (21)$$

and

$$E \{ \hat{g}_{mikl} \hat{g}_{miki}^* \} = \frac{\rho_r \tau \beta_{ikl} \beta_{iki}}{1 + \rho_r \tau \sum_{s=1}^L \beta_{iks}}. \quad (22)$$

Under the conjugate beam-forming precoding (4) the m -th antenna of the i -th base station transmits

$$v_{mi} = \frac{1}{\sqrt{\gamma}} \sum_{k=1}^K y_{mik}^* q_{ki}, \quad (23)$$

where γ is a normalization coefficient chosen so that to satisfy the average power constraint per cell:

$$E \left\{ \frac{1}{L} \sum_{l=1}^L \sum_{m=1}^M |v_{ml}|^2 \right\} = 1. \quad (24)$$

The k -th terminal in the l -th cell receives the signal

$$x_{kl} = \sqrt{\rho_f} \sum_{j=1}^L \sum_{m=1}^M g_{mjk} v_{mj} + z_{kl}, \quad (25)$$

If we use downlink PCP we have

$$v_{mj} = \frac{1}{\sqrt{\gamma}} \sum_{k=1}^K y_{mjk}^* q'_{kj} = \frac{1}{\sqrt{\gamma}} \sum_{k=1}^K \sum_{l=1}^L y_{mjk}^* a_{kjl} q_{kl}, \quad (26)$$

where a_{kjl} , $l = 1, \dots, L$, form the j -th row of \mathbf{A}_k . To satisfy the power-constraint (24) we choose:

$$\gamma = \frac{M}{L} \sum_{k=1}^K \sum_{i=1}^L \sum_{l=1}^L \left(1 + \rho_r \tau \sum_{s=1}^L \beta_{iks} \right) |a_{kil}|^2. \quad (27)$$

B. SINR estimates and Capacity Lower-Bound for PCP

Substituting (26) into (25), we obtained

$$x_{kl} = \sqrt{\frac{\rho_f}{\gamma}} \sum_{m=1}^M \sum_{j=1}^L \sum_{n=1}^K \sum_{u=1}^L g_{mjk} y_{mjn}^* a_{nju} q_{nu} + z_{kl}.$$

After some manipulations we can write this expression in the following form

$$x_{kl} = s_{kl} q_{kl} + w_{kl}^{(1)} + w_{kl}^{(2)} + w_{kl}^{(3)} + w_{kl}^{(4)} + w_{kl}^{(5)}, \quad (28)$$

where

$$s_{kl} = \sqrt{\frac{\rho_f}{\gamma}} \sum_{m=1}^M \sum_{j=1}^L \mathbb{E} \{ \hat{g}_{mjk} y_{mjk}^* \} a_{kjl}, \quad (29)$$

and

$$w_{kl}^{(1)} = z_{kl}, \quad (30)$$

$$w_{kl}^{(2)} = -\sqrt{\frac{\rho_f}{\gamma}} \sum_{m=1}^M \sum_{j=1}^L \sum_{n=1}^K \tilde{g}_{mjk} y_{mjn}^* q'_{nj}, \quad (31)$$

$$w_{kl}^{(3)} = \sqrt{\frac{\rho_f}{\gamma}} \sum_{m=1}^M \sum_{j=1}^L \sum_{n \neq k} \hat{g}_{mjk} y_{mjn}^* q'_{nj}, \quad (32)$$

$$w_{kl}^{(4)} = \sqrt{\frac{\rho_f}{\gamma}} \sum_{m=1}^M \sum_{j=1}^L (\hat{g}_{mjk} y_{mjk}^* - \mathbb{E} \{ \hat{g}_{mjk} y_{mjk}^* \}) q'_{kj}, \quad (33)$$

$$w_{kl}^{(5)} = \sqrt{\frac{\rho_f}{\gamma}} \sum_{m=1}^M \sum_{j=1}^L \sum_{u=1, u \neq l}^L \mathbb{E} \{ \hat{g}_{mjk} y_{mjk}^* \} a_{kju} q_{ku}. \quad (34)$$

The terminal can compute s_{kl} in advance and use it for estimation its SINR value. Thus s_{kl}^2 is the signal power. All other terms in (28) form the effective noise. The quantity (30) corresponds to channel estimation error, (31) to non-orthogonal channels, (32) to beam-forming gain uncertainty, and (33) to pilot-contamination. All these noise components are mutually uncorrelated.

Estimating (29) and combining (30)-(33), after calculations, we obtain for given \mathbf{A}_k and β_{jkl} :

Theorem 2.

$$\varsigma_{kl}^D = \frac{M \rho_f \rho_r \tau \left| \sum_{j=1}^L \beta_{jkl} a_{kjl} \right|^2}{T_1 + M \cdot T_2}, \quad (34)$$

where

$$T_1 = \sum_{j=1}^L \sum_{n=1}^K \left(\frac{1}{L} + \rho_f \beta_{jkl} \right) \left(1 + \rho_r \tau \sum_{s=1}^L \beta_{jns} \right) \sum_{u=1}^L |a_{nju}|^2$$

and

$$T_2 = \rho_f \rho_r \tau \sum_{u \neq l} \left| \sum_{j=1}^L \beta_{jkl} a_{kju} \right|^2.$$

It is proven in [4] that assuming the worst case Gaussian noise in the effective channel we obtain a capacity bound:

$$C_{kl} > \log_2 (1 + \varsigma_{kl}^D). \quad (35)$$

The expression (34) allows one to simulate SINRs in a fast way without generating M -dimensional vectors.

Theorem 2 shows that reduction ρ_f by the factor of 2 can be compensated by doubling M . Thus LSAsSs can be very useful for building energy efficient wireless systems.

If PCP is not used, that is $a_{kjl} = \delta_{jl}$, we have:

Theorem 3. If PCP is not used then

$$\varsigma_{kl}^D = \frac{M \rho_f \rho_r \tau \beta_{lkl}^2}{R_1 + M \cdot R_2},$$

where

$$R_1 = \sum_{j=1}^L \sum_{n=1}^K \left(\frac{1}{L} + \rho_f \beta_{jkl} \right) \left(1 + \rho_r \tau \sum_{s=1}^L \beta_{jns} \right)$$

and $R_2 = \rho_f \rho_r \tau \sum_{u \neq l} \beta_{ulk}^2$.

From this we have $\lim_{M \rightarrow \infty} \varsigma_{kl}^D = \frac{\beta_{lkl}^2}{\sum_{u \neq l} \beta_{ulk}^2}$ which coincides with (12). If we use PCP with $\mathbf{A}_k = \mathbf{B}_k^{-1}$ then $T_2 = 0$ and

$$\lim_{M \rightarrow \infty} \varsigma_{kl}^D = \infty$$

as it is supposed to be according to (18).

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