

Aristarchus observed that during a total eclipse of the sun the disc of the moon completely obscures the disc of the sun but just so—as shown in figure 9. Because the larger right triangle appearing in this figure is similar to the smaller one, the ratio of the distances of Sun and Moon  $SE / ME$  from Earth must equal the ratio of their radii  $R_S / R_M$ . Therefore, if the sun is, as Aristarchus believed, twenty times more distant than the moon, the sun must be twenty times larger than the moon. Since the sun is, in fact, about four hundred times more distant than the moon, the sun is actually about four hundred times larger than the moon.

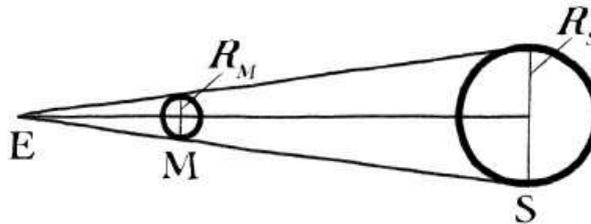


Figure 9

Aristarchus forged yet one more link in his chain of arguments. He noticed that the time required for the moon to pass into the earth's shadow during a lunar eclipse is close to the time the moon stays completely obscured by the earth's shadow. If so, the earth's radius must be twice that of the moon—assuming the earth's shadow from Earth to Moon is approximately cylindrical. And if the earth is two times larger than the moon and the sun is twenty times larger than the moon, then the sun must be ten times larger than the earth. Again, Aristarchus's argument is valid even if his data are not accurate. According to modern measurements the earth is about four times larger than the moon and the sun is some one hundred times larger than the earth.

Aristarchus's methods illustrate the intellectual trends of his time. Since he was a younger contemporary of Euclid (the latter flourished around 300 BCE), Aristarchus lived during a period in which the propositions of geometry had become widely known. Subsequently, astronomical knowledge was more frequently framed in geometrical language than before and, in this way, physical science began to distinguish itself from philosophical wisdom. At the same time, the center of Greek learning was migrating from Athens to the newly founded city of Alexandria near the mouth of the Nile. While Aristarchus may or may not have traveled from his native Samos to Alexandria, his life falls within the initial stages of a period of cultural ferment set in motion by the conquests and foundations of Alexander the Great.

## 7. Archimedes's Balance (250 BCE)

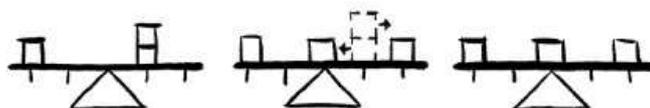


Figure 10

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Around 300 BCE, Euclid organized the mathematical knowledge of his time into *definitions, common notions, postulates, and the demonstrations of propositions*. Some of the definitions are familiar, for instance, “A line is breadthless length,” and some seem a little mysterious, “A straight line is a line that lies evenly with the points on itself.” The common notions are self-evident statements common to all kinds of reasoning such as “Things which are equal to the same thing are also equal to one other.” The postulates are a small group of unproven statements, assumed to be true, such as “All right angles are equal to one other,” and the propositions are statements whose truth Euclid demonstrates by valid argument from the postulates, the common notions, and previously demonstrated propositions. The result, contained within the thirteen books of Euclid’s *Elements*, is an extended deductive system that has, for 2,300 years, been a model of rigorous thinking. The outstanding lesson of Euclid’s system is that many truths can be demonstrated and not merely asserted.

Euclid’s *Elements* astonishes and charms its readers. It is said that Sir Thomas Hobbes, on first picking up Book I of the *Elements* and reading Proposition 47, the Pythagorean theorem, exclaimed, “By God, this is impossible.” Hobbes then read its demonstration and then the demonstrations of the propositions used to demonstrate Proposition 47 and so on until he had read a good part of Book I *in reverse order*—a method of reading Euclid I do not recommend. On the other hand, I do commend Edna St. Vincent Millay’s response to Euclid, a fourteen-line Shakespearean sonnet, “Euclid alone has looked on Beauty bare,” whose middle verses are as follows:

... let geese  
 Gabble and hiss, but heroes seek release  
 From dusty bondage into luminous air.  
 O blinding hour, O holy, terrible day,  
 When first the shaft into his vision shone  
 Of light anatomized! Euclid alone  
 Has looked on Beauty bare. ...

Archimedes (287–212 BCE), certainly the most original mathematician and physicist of antiquity, also fell under Euclid’s spell. We know this because he followed Euclid in organizing what he had discovered about the equilibrium of heavy bodies into a system of postulates, propositions, and demonstrations.

Figure 10 illustrates Propositions 6 and 7 of Archimedes’s *On the Equilibrium of Planes* that together compose his law of the balance: *two objects balance at distances inversely proportional to their weights*—a law illustrated each time two children of unequal weight balance themselves on a teeter-totter. The dark line in the diagram is the balance beam, the triangle is the beam’s support or pivot, short, light lines mark the beam at equally spaced intervals on either side of the pivot, and the blocks stand for units of weight. In the left panel a weight of two units is one unit to the right of the pivot and a weight of one unit is two units to the left of the pivot, so that each weight is at a distance from the pivot inversely proportional to its magnitude.

Figure 10 illustrates a demonstration of a particular case of the law of the balance. The demonstration requires only two premises, both quite reasonable. One of these is Archimedes’s Postulate 1, *equal weights at equal distances (from the pivot) balance*, and the other is a previously demonstrated proposition, Proposition 4, *the center of*

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gravity of two equal weights taken together is in the middle of a line joining their centers. The phrase *center of gravity* refers to the location at which a pair of identical weights can be replaced by a single weight equal in magnitude to the total weight of the pair. Thus, Proposition 4 justifies the transition, shown in the center panel that makes the stability of the left and right panels equivalent. Of course, according to Postulate 1, the weights in the right panel balance. Therefore, the sequence of panels from left to right (or from right to left) demonstrates that a weight of two units located one unit to the right of the pivot balances a weight of one unit located two units to the left of the pivot.

Archimedes's demonstration is more general than ours. Nevertheless, our demonstration exploits the rule justified by his Proposition 4: *a weight at a particular location can be replaced by two weights, each equal to half the original weight, placed equal distances on either side of their original location*. By applying this rule several times one can show that the two balance beams illustrated in figure 11 are physically equivalent. Give it a try. And remember: You are allowed to place blocks on top of the pivot.

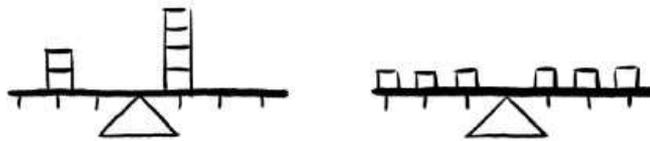


Figure 11

Archimedes may have sojourned for a while in Alexandria, and if so, he may have known the somewhat younger Eratosthenes (276–194 BCE) who plays a role in a later essay. Even so Archimedes lived the greater part of his life in his native Syracuse, a Greek city on the island of Sicily. Greek colonists had inhabited Sicily and the southeastern coast of the Italian mainland since the eighth century BCE. During Archimedes's lifetime, the Romans extended their dominion over the Italian peninsula and Sicily and engaged the North African city of Carthage in a life-and-death struggle. Syracuse, and therefore Archimedes, stood directly in the path of this Roman expansion.

## 8. Archimedes's Principle (250 BCE)

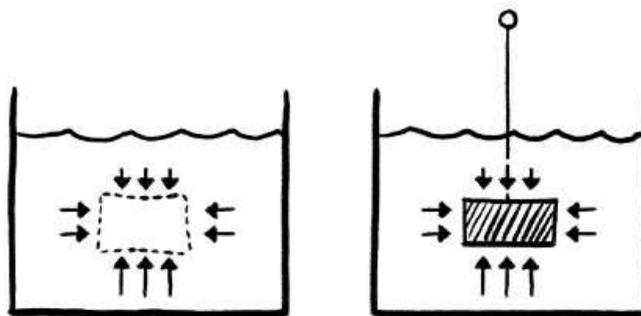


Figure 12

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The story goes that when the solution to a particularly challenging problem came to Archimedes in his bath, he leapt from the tub shouting “Eureka! Eureka!” (I have found it! I have found it!) But what had Archimedes found? According to Vitruvius (ca. 75–15 BCE), the Roman military engineer who told the story almost two centuries after the event, Archimedes had discovered a method for determining whether a crown that had been made for King Hieron of Syracuse was of pure gold, as per instructions, or mixed with silver. The story is a good one—almost too good to be true—for the method Archimedes discovered concerned bodies submerged in a fluid just as Archimedes’s body was submerged in his bathwater. However, Vitruvius does not tell us the details of Archimedes’s method.

Figure 12 illustrates the physics behind what physics teachers call *Archimedes’s principle*—the content of which Archimedes outlined in Propositions 3–7 of Book I of his text *On the Equilibrium of Floating Bodies*. Archimedes’s principle is simply stated and elegantly proved and could have been used to determine the composition of King Hieron’s crown.

The left panel shows a container filled with water, or any other fluid, at rest. The dashes outline a region of the fluid while the arrows represent the direction and magnitude of the pressure exerted by the fluid *outside* the outlined region on the fluid *inside* the outlined region. (The longer the arrows, the larger the pressure.) Note that, as one might expect, the magnitude of this pressure increases with depth. Therefore, the upward push, on the fluid in the outlined region, is larger than the downward push. In fact, the net upward push must be just enough to support the weight of the fluid in the outlined region in order to keep the fluid at rest.

In the right panel of figure 12 the fluid that was in the outlined region has been replaced with an identically shaped object. A string attached to the object keeps it from sinking. Since the fluid outside the object in the right panel is identical to the fluid outside the region outlined in the left panel, the net force the outside fluid exerts is also identical. Thus, *the fluid outside an object exerts a net upward force on the object equal in magnitude to the weight of the fluid the object displaces*. This is Archimedes’s principle. The argument with which we have reached Archimedes’s principle applies to floating as well as to fully submerged objects.

Once we understand Archimedes’s principle we can use it to solve problems. Here is one problem physics teachers sometimes assign their students. A cargo ship containing iron ore is in a watertight lock as illustrated in figure 13. The captain orders his crew to dump the iron ore into the bottom of the lock. Does the water level in the lock rise, fall, or stay the same when the ore is dumped? The answer (the water level falls) requires the creative use of Archimedes’s principle.

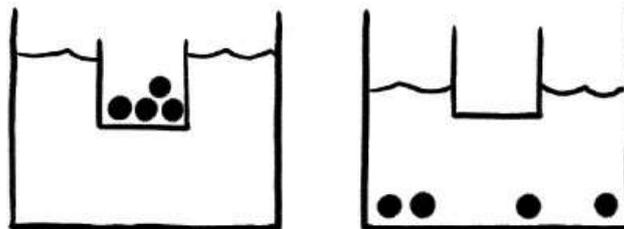


Figure 13

Archimedes also proved that *the surface of a fluid at rest is part of the surface of a sphere whose center is at the center of the earth*. Stop a moment and consider this claim. The surface of every glass of water, every cup of coffee, and every farmer's pond is curved, concave downward, with its center of curvature at the center of the earth! Of course, Archimedes's claim ignores the distorting effect of the water's surface tension. But, still, the claim is amazing. The bulk of the earth pulls on a fluid in such a way as to shape it into a section of a sphere. Figure 14 illustrates this claim and provides the seed of Archimedes's proof, which, however, I do not spell out.

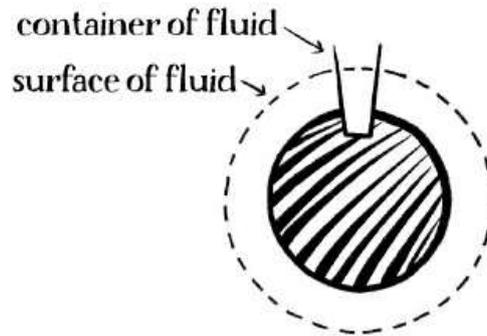


Figure 14

Although Archimedes was primarily a mathematician and physicist, he also invented devices that exploit physical principles: the so-called Archimedean screw that could pump water from a lower to a higher level and the compound pulley that could, in principle, allow one person to, very slowly, lift a massive ship. Some of Archimedes's inventions were weapons of war, for instance, burning mirrors and catapults, which he devised in order to defend his native Syracuse from the Roman army that besieged it in 212 BCE.

But the Romans prevailed and Archimedes died as he had lived—absorbed in a problem of mathematical science. The Roman commander of the besieging army, Marcellus, had given orders that the famous Archimedes, then seventy-five years old, be spared. But when confronted by an armed Roman soldier Archimedes, who had been studying some figures drawn in the sand, brusquely demanded, “Stand back from my diagrams!” Those were his last words. Evidently, this was no way to address an armed Roman soldier.

## 9. The Size of the Earth (225 BCE)

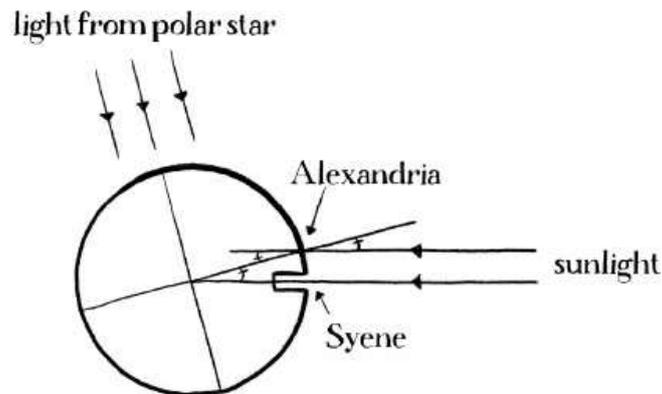


Figure 15

Eratosthenes (276–194 BCE) was born in the North African town of Cyrene (in modern Libya) and educated in Athens, but lived the greater part of his life in Alexandria where around 244 BCE he became the head of its great library. The wealth of this library's holdings can be inferred from the task given Callimachus, a contemporary of Eratosthenes—to catalog the library's books—and the result of his effort: 120 volumes of bibliography. Thus, we can understand Callimachus's famous complaint, "A great [or large] book is a great evil." Nevertheless, the library and its great books made Eratosthenes's career possible.

Eratosthenes's writings include the text *Geographica*, now lost but often cited in antiquity. This book gathered together what was then known about *geography*—a word he was the first to use in its modern sense. Eratosthenes's compilation of the geographical wisdom of the past into a single, expansive treatise had many imitators during the centuries of Roman domination that followed his death. Pliny's *Natural History*, for instance, included all that was known of the natural world. However, even the best Roman scholars were more concerned with the utility and entertainment value of the learning inherited from the Greeks than with creatively understanding or extending that learning.

But Eratosthenes was an Alexandrian Greek who not only preserved but also built upon the wisdom of the past. For instance, he was the first to add lines of longitude to a map of the known world. On a globe these lines are great circles that pass through both poles. The particular line of longitude or meridian that connects Alexandria and Syene (modern Aswan) plays a role in Eratosthenes's determination of the circumference of the earth.

The diagram illustrates Eratosthenes's method. Eratosthenes noticed that when rays of sunlight reach the bottom of a deep well at Syene, rays of sunlight at Alexandria make an angle equal to  $1/50$  of a circle with a straight vertical pole or *gnomon*. Eratosthenes also knew that, so distant was the sun from the earth, the different rays of sunlight striking the earth are essentially parallel, and that, according to Euclid, a line falling upon two parallel lines makes the alternate interior angles equal—as illustrated in figure 15. Therefore, according to the geometry of the diagram and Eratosthenes's measurement, an angle equal to  $1/50$  of a circle (about 7 degrees) with vertex at the center of the earth subtends or encompasses the meridian connecting Syene and Alexandria along the surface of the earth. Consequently, the distance between Syene and

Alexandria is  $1/50$  the circumference of the earth. It only remained for Eratosthenes to determine this distance and multiply by 50.

As it happens, Syene is located near the first waterfall upstream from the Nile's mouth at Alexandria. Between Syene and Alexandria, the Nile flooded often, and, consequently, was measured frequently by the cadre of Egyptian *geometers*, literally "land measurers," whose job was to preserve the identity of property along the Nile. Eratosthenes had access to the land measurers' records and from them inferred that the distance from Syene to Alexandria was about 5,000 stadia. Therefore, according to Eratosthenes's method, the circumference of the earth is about 250,000 stadia.

But how long is a single *stade*? Ancient documents provide at least two different answers to this question. The Egyptian stade is 158 meters and the more commonly used Greek stade is 185 meters. The first produces a circumference within one percent of the modern value, 40,000 kilometers, while the second produces one 17 percent too large. But comparing Eratosthenes's value to that determined by modern methods teaches us little. It is more important to understand that Eratosthenes's method is sound and based on measurements rather than on speculations.

Eratosthenes also understood that his measurements were uncertain. We know this because Eratosthenes attempted to quantify their uncertainty. For instance, he determined that on the longest day of the year, sunlight reaches the bottom of any well at Syene within a circle with radius of about 300 stadia. This effect alone limits the accuracy of Eratosthenes's determination of the circumference of the earth to plus or minus 6 percent.

That Eratosthenes assumed the earth is spherical is unremarkable. For by his time it had long been known that (1) as we travel north the southern constellations sink toward the horizon and the Pole Star rises higher in the nighttime sky, and (2) during a lunar eclipse the shadow of the earth on the moon is a section of a circle. No observant person could argue with these facts. Only a desire to make sense of them was needed. We know from his determination of the size of the earth that Eratosthenes had that desire.

## Notes

11. "The Sun puts the shine in the Moon." Nahm, *Selections from Early Greek Philosophy* (1964), 143.

13. "You cannot step twice into the same river ..." Nahm, *Selections from Early Greek Philosophy* (1964), 70.

13. "As when a young girl, playing with a clepsydra ..." Curd, *A Presocratics Reader* (2011), 97.

17. "one of the greatest philosophers and scientists of all times." Sarton, *Introduction to the History of Science* (1927), 127.

17. "carried on immense botanical, zoological, and anatomical investigations ..." Sarton, *Introduction to the History of Science* (1927), 127.

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23. “a line is breadthless length,’ and some seem a little mysterious, ‘A straight line is a line that lies evenly with the points on itself.” Euclid, *The Elements* (1956), 153.
23. “Things which are equal to the same thing are also equal to one other.” Euclid, *The Elements* (1956), 155.
24. “ ... let geese / Gabble and hiss, but heroes seek release ... ” From “Euclid Alone Has Looked on Beauty Bare” by Edna St. Vincent Millay, in Salter, Ferguson, and Stallworth, eds., *Norton Anthology of Poetry* (2005), 1383.

## Middle Ages

### 10. Philoponus on Free Fall (550 CE)

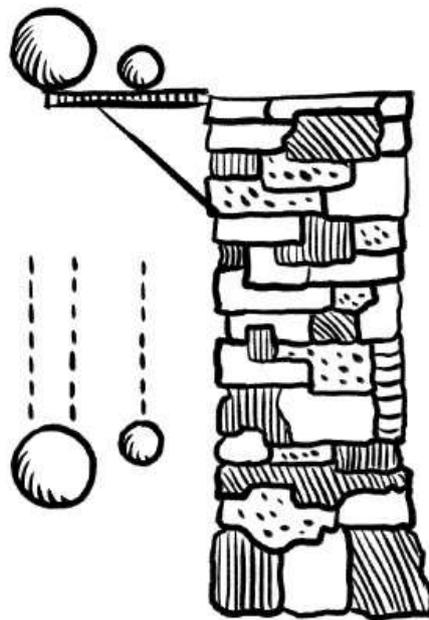


Figure 16

John Philoponus (490–570 CE), whose surname means *lover of toil*, was a Greek Christian who lived and worked as a philosopher, theologian, and scientist in the century immediately following the invasion of Roman Italy by Germanic tribes in 476 CE. While he flourished more than a century after Theodosius (347–395 CE) had established Catholic Christianity as the official religion of the empire in 380 CE, Philoponus was taught by and worked with pagan philosophers associated with the library in Alexandria. Philoponus wrote extensive commentaries on Aristotle and in several treatises argued against the Aristotelian doctrine of the eternity of the world. He believed that the heavens have the same properties as the earth and, as well a Christian might, that the heavens are not divine.

Philoponus's analysis of motion critiqued Aristotle's. Aristotle had argued that continuous motion requires either an internal or an external mover in continuous contact with the object moved. Accordingly, the natural downward motion of a heavy object is caused by the object's inner nature and opposed by the air through which it falls. In contrast, the horizontal motion of a projectile is unnatural and requires external

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movers: at first a mover that initiates the horizontal motion and then the continued push of the air. Philoponus, quite reasonably, doubted that the air could at the same time resist a projectile's natural downward motion and cause its unnatural horizontal motion.

Aristotle also claimed that the time required for an object to fall from rest from a given height is in inverse proportion to its weight. Thus, the heavier an object, the more quickly it should fall. But, according to Philoponus:

This view of Aristotle's is completely erroneous, and our view may be corroborated by actual observations more effectively than by any sort of verbal argument. For if you let fall from the same height two weights, one many times as heavy as the other, you will see that the ratio of the times required for the motion does not depend [solely] on the ratio of the weights, but that the difference in time is very small. And so if the difference in the weights is not considerable, that is, if one is, let us say, double the other there will be no difference, or else an imperceptible difference, in time.

Figure 16 illustrates the situation Philoponus describes. When two objects, one several times heavier than the other, are simultaneously released, the heavier object, according to Philoponus's observation, reaches the ground only slightly ahead of the lighter object—certainly not, as according to Aristotle, several times more quickly.

However, Aristotle's view is not without foundation. Given the difficulty of measuring small time intervals, Aristotle may well have simulated descent in air with descent in water by, for instance, simultaneously dropping heavy and light stones in a pool of clear water. If so, Aristotle would have observed that heavier objects do, indeed, as illustrated in figure 17, fall significantly faster than lighter ones.

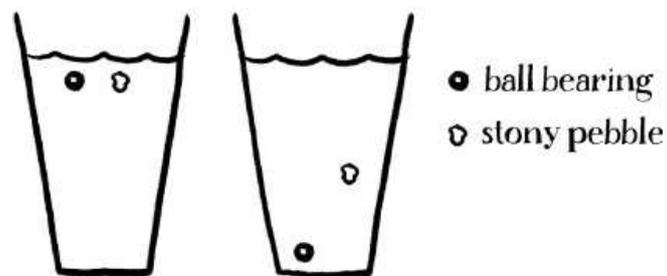


Figure 17

We now know that *free fall*, that is, fall through a near vacuum or through a relatively short distance in air, is not comparable to fall through water or oil. In a vacuum, all massive objects fall at exactly the same rate just as, for instance, a hammer and feather dropped together on the surface of our relatively airless moon do. However, in a sufficiently viscous fluid, two similarly shaped objects fall at terminal speeds that are, as Aristotle expected, proportional to the object's weight. To observe such descent all one needs is a tall glass of water and two objects of approximately the same shape and size but with very different masses—perhaps a stony pebble and a ball bearing.

The barbarian invasions that led to the fall of the Western Empire and the subsequent breakdown of Roman institutions disrupted communication between the Latin West and the Greek East. As a result, Philoponus's books and commentaries as well as many