Probabilistic Load Flow using Latin Hypercube Sampling with Dependence for Distribution Networks

Jun Cao, W. DU，H.F. Wang and L.Y. Xiao

***Abstract*-- The paper adopts the Latin Hypercube Sampling with Dependence (LHSD) method to solve the Probabilistic Load Flow (PLF) problem with correlated random variables for distribution networks. The proposed method is investigated using modified IEEE 34 distribution system with random loads, Wind power and Photovoltaics (PVs). Three different cases are studied and the comparison results shows that the LHSD method handles correlations efficiently and presents an accurate simulation result with a much smaller simulation size. The method has the potential to be applied in many power system probabilistic problems.**

***Index Terms*-- Latin Hypercube Sampling, correlation, probabilistic load flow, distribution networks**

I. INTRODUCTION

Nowadays, intensive efforts are made to utilize renewable energy such as wind power and photovoltaic (PV) system. These generators are mostly integrated into utility networks at distribution level which bring great uncertainties and randomness into the operation of power system due to their intermittence and variability. The probabilistic load flow (PLF) is adopted to analyze the performance of a distribution network under various possible uncertainties.

The PLF was first introduced by Borkowska [1]. Since then, lots of different methods have been proposed to solve it efficiently. The methods can be classified into two groups: analytical [1-5] and numerical [6, 7] methods. The analytical methods are based on the theory of convolution. But it is time consuming to obtain the PDF of even a single line when lots of random variables are considered. Various techniques are proposed to reduce the computational burden such as Fast Fourier Transform [2], Gram Charlier expansion series [3], Cornish-Fisher expansion series [9], etc. Another recent approach is point estimate method [23] that approximates the moments of interested variables. In the most literature, demands and generation sources are considered generally independent. Dependence is just included in few studies [6, 8, 9]. However, dependence between random variables should be considered especially in a distribution network due to the small geographical area that the network covers.

Most numerical methods evolve from Monte Carlo (MC) simulation. The best advantage of the MC approach is its simplicity. However, it needs heavy computation burden to converge. The variance reduction techniques such as control variates, antithetic variates, important sampling [15] are adopted to solve this problem. Recently, Latin Hypercube Sampling (LHS) has been introduced to solve the probabilistic problem in power systems. Reference [7] applies the LHS method to PLF problem. LHS is a variance reduction technique which is based on the stratified sampling. It was first developed by McKay *et al*. [10] in 1979 and further analyzed by Stein [11]. In this paper, the Latin Hypercube Sampling with Dependence (LHSD) is adopted to solve the PLF problem in distribution networks.

The paper is organized as follows: Section II explains the method of the standard LHS and the LHSD method in details. Section III presents the probabilistic model of load and distribution generators. The procedure of PLF analysis for radial distribution networks with correlated input variables is described in Section IV. In Section V, the IEEE 34-bus distribution network is used to demonstrate the effectiveness of the method. Finally, conclusions are given in Section VI.

II. LATIN HYPERCUBE SAMPLING

The basic procedure of LHS method is sampling and permutation, thereinto how to get a good permutation is very important. It is the key difference between the LHS with independence and dependence, while independence and dependence refer to non-correlated and correlated permutations respectively. In this section, the LHS with independence and dependence are described in detail.

A. Standard LHS with Independence

The standard LHS method is introduced to estimate the uncertainty of an output variable Y from a deterministic model $Y = g(X)$ with random input vector X which has K dimensions $X = (X^1, \ldots, X^K)$. To generate an independent Latin hypercube sample of size N in dimension K, let $U_i^{(j)}$ be an independent uniform random variable on [0, 1] for $i = 1,\ldots,N$ and $j =$

Jun Cao is with Department of Electrical Engineering, Queen's University Belfast, Belfast, UK (e-mail: jcao01@qub.ac.uk).

W. DU is with Department of Electrical Engineering, Queen's University Belfast, Belfast, UK (e-mail: ddwenjuan@gmail.com).

H.F.Wang is with Department of Electrical Engineering, Queen's University Belfast, UK (e-mail: hf.wang@qub.ac.uk).

L.Y. Xiao is with institute of Electrical Engineering, Chinese Academy of Sciences, Beijing, China.

1,…,*K*. An independent uniform random LHS realization set is generated as [15]:

$$V_i^{(j)} = \frac{U_i^{(j)}}{N} + \frac{\pi_i^{(j)} - 1}{N} \quad , \quad i = 1,...,N, \quad j = 1,...,K \quad (1)$$

where $\pi_i^{(j)}$ is an independent random permutation of {1,…, *N*}. From (1), it is obvious that $(\pi_i^{(j)} - 1)/N < V_i^{(j)} < \pi_i^{(j)}/N$ which means that a random value is sampled from each non-overlapping equal probability interval. The sample values thus represent the actual distribution better than MC method. Then, the *j*th sample of X_i can be determined by $X_i^{(j)} = F_i^{-1}(V_i^{(j)})$ where F_i^{-1} is the inverse cumulative probability function (cdf).

However, unexpected correlations between different input random variables are uncontrollable which affect the quality and accuracy of the result. Various methods have been proposed in LHS to generate an independent random permutation $\pi_i^{(j)}$ with minimum correlation such as genetic algorithm [17], columnwise-pairwise algorithm [18], single-switch optimization [19], Cholesky decomposition [20], ranked Gram-Schmidt orthogonalization [21], etc. Here, Cholesky decomposition method is chosen because it can get good performance with less computational burden [7].

B. LHS with Dependence

Correlation should be included in Probabilistic Load Flow evaluation especially when analyzing distribution networks. A method proposed by Stein [11] is used for including correlation among the input random variables. The general idea of the LHSD is to generate a Latin hypercube sample, albeit with the following modification: Instead of choosing an independent random permutation in each dimension, a particular permutation will be correlated according to the rank correlation of the target correlation matrix. The method is summarized as follows:

1) Generate a sample ***S*** of standard normal variables using the standard Latin Hypercube Sampling;
2) Calculate ***T***, the correlation matrix of ***S***;
3) Decompose the target correlation matrix Σ to get the lower triangular matrix ***A*** using the Cholesky factorization $\Sigma = AA^T$ and also ***Q*** the lower triangular matrix of ***T***, $T = QQ^T$; P^T and Q^T are transposed matrixs of ***A*** and ***Q***, respectively.
4) Build a new correlation matrix ***P*** by using a matrix transformation: $P = S(AQ^{-1})^T$. One can prove that the correlation matrix of ***P*** (denoted by Σ') is identical to the target correlation matrix Σ :
$\Sigma' = (AQ^{-1})T(AQ^{-1})^T = (AQ^{-1})QQ^T(AQ^{-1})^T = AA^T = \Sigma$
(Explain: In mathematics, if $X \sim N(a,V)$ and *C* is constant matrix, then $Y = CX \sim N(Ca, CVC^T)$)
5) Obtain the matrix ***RC*** which contains the same rank correla tion corresponding to the correlation matrix ***P***;
6) Generate the Latin hypercube sample ***X*** with correlation matrix ***RC*** by

$$X_i^{(j)} = F_i^{-1}\left(\frac{RC_i^{(j)} - U_i^{(j)}}{N}\right)$$

For the sake of clarity, the pseudocode of generating correlated samples using LHSD method is described as follows:

```
//N : sample number, K : number of input random variables
Read Σ    // Σ is the target correlation matrix
for j = 1 to K do
    Generate U^(j) and π^(j) // U^(j) an independent uniform
//distribution on [0, 1], π^(j) is independent random
permutation
    V^(j) = U^(j)/N + (π^(j) − 1)/N ,   j = 1,..., K
    S^(j) ← Φ^(−1)(V^(j)) ,   j = 1,…, K
end for   // Φ^(−1) is the inverse cdf of norm distribution
    Calculat T      //T is the correlation matrix of S
    Compute Q such that QQ^T = T    //Cholesky factorisation
    Compute A such that AA^T = Σ    //Cholesky factorisation
    P ← S(AQ^(−1))^T
    RC ← Ranking(P)
    X = F^(−1)((RC − U)/N)
```

Note that: sometimes, the measured target correlation matrix ***R*** is non-positive semi-definite. The Modified Cholesky Factorization is adopted to decompose the non-positive semi-definite matrix ***R***. It is a numerically stable algorithm that produces a positive definite matrix differing from the original matrix only in its diagonal elements. A detailed description of the method can be found in [22].

III. Probabilistic Model of Load and Distribution Generators

A. Probabilistic load model

The load is assumed to be a normal distribution. So the probabilistic density function (pdf) of load P_L is given by the following equation:

$$f(P_L) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(P_L - \mu_p)^2}{2\sigma_P^2}\right) \quad (2)$$

where μ_P is the mean value, σ_P is the standard deviation.

All the loads are correlated by specified correlation coefficient and follow the same pdf.

B. Probabilistic Wind Power model [25]

The wind speed *V* is assumed to be Weibull distribution with a scale parameter *c* and a shape parameter *k*. Its pdf takes the following form:

$$f(\upsilon) = \frac{k}{c}\left(\frac{V}{c}\right)^k \exp\left[-\left(\frac{V}{c}\right)^k\right] \quad (3)$$

The parameters of Weibull distribution can be estimated by the following equations:

$$\hat{k} = \left(\frac{\sigma}{\mu}\right)^{-1.086} \quad (4)$$

$$\hat{c} = \frac{\mu}{\Gamma(1+\frac{1}{k})} \quad (5)$$

where μ is mean value of wind speed, σ is the standard deviation. Γ is the gamma function.

The relation between the input wind speed and the output wind power is dependent on several factors, such as the type of the wind turbine and the efficiencies of the generator. For a generic wind turbine, a simplified model is adopted to characterize the relation between the wind speed and the output of wind power

$$P_w = \begin{cases} 0; & (V < \upsilon_{in} \text{ or } V > \upsilon_{out}) \\ P_r; & (\upsilon_r < V < \upsilon_{out}) \\ k_1 V + k_2; & (\upsilon_{in} < V < \upsilon_r) \end{cases} \quad (6)$$

where

$k_1 = \dfrac{P_r}{\upsilon_r - \upsilon_{in}}$;

$k_2 = -k_1 \upsilon_{in}$;

P_r is the rated wind power;

$\upsilon_{in}, \upsilon_r, \upsilon_{out}$ are the cut-in, rated and cut-out wind speeds.

C. Probabilistic PV generator model

The solar irradiance is assumed to be a Beta distribution [25] and the pdf of solar irradiance with two positive shape parameters, typically denoted by α and β is:

$$f(r) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{r}{r_{\max}}\right)^{\alpha-1}\left(1-\frac{r}{r_{\max}}\right)^{\beta-1} \quad (7)$$

where r and r_{max} are actual and maximum solar irradiance respectively. The unit is W/m^2.

The parameters of Beta distribution can be estimated by the following equations:

$$\hat{\alpha} = \bar{x}\left(\frac{\bar{x}(1-\bar{x})}{\sigma} - 1\right) \quad (8)$$

$$\hat{\beta} = \left(1-\bar{x}\right)\left(\frac{\bar{x}(1-\bar{x})}{\sigma} - 1\right) \quad (9)$$

Since the PV system is generally equipped with Maximum Power Point Tracker (MPPT) and the relationship between the input solar irradiance r and total output active power P_r is

$$P_r = rA\eta \quad (10)$$

where A is total area of the modules, η is efficiency.

The pdf of total output active power P_r can be obtained from (7) and (10)

$$f(P_r) = \frac{1}{rA}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{P_r}{P_{r_{\max}}}\right)^{\alpha-1}\left(1-\frac{P_r}{P_{r_{\max}}}\right)^{\beta-1} \quad (11)$$

where $P_{r_{\max}} = r_{\max} A\eta$ is the maximum power available from the module.

IV. PLF ANALYSIS WITH CORRELATED INPUT VARIABLES

The iterative backward/forward sweep method [13, 14] is adopted to solve the load flow problem in radial distribution systems. The procedure of this method can be divided into two main steps including backward and forward sweep. The objective of backward sweep is to calculate each branch current or power flow based on the KCL and update each node voltage. The forward sweep is used to find each bus voltage based on the KVL and update each branch current or power flow. Its convergence doesn't affected by the high ratio of *R/X* in distribution system. Reference [14] provides a detailed description of the algorithm.

The procedure of the PLF analysis for radial distribution networks with correlated input variables via the LHSD method is described as follows:

Step 1) Read the data for radial distribution networks, the sample size *N*, the correlation matrix and the data obtain from HOMER [26] software are imported into programme;

Step 2) The parameters of Weibull and Beta distribution are estimated by (4), (5) and (8), (9), based on the data from Step 1) ,respectively;

Step 3) Generate a correlated sampling matrix using the LHSD method described in Section II; the correlated sampling matrix used for MC method is generated by Gaussian Copula method [24] described as below;

Step 4) Solve a deterministic power flow problem for the sam ple vector obtained in Step 3) using backward/forward sweep method;

Step 5) Repeat Step 4 until a sufficient number *N* is performed.

Step 6) Compute and plot means, standard deviations and any other statistical information of interest.

The pseudocode of Gaussian Copula method is

//N : sample number, K : number of input random variables
Read Σ // Σ is the target correlation matrix
Compute $\boldsymbol{A}$ such that $AA^T = \Sigma$ //Cholesky factorisation
for i = 1 to N **do**
  for j = 1 to K **do**
    generate $L_i^j \sim N(0,1)$ //independent of L_i^j
  end for
  $(Z_i^1,\cdots,Z_i^K)^T \leftarrow A\cdot(L_i^1,\cdots,L_i^K)^T$ //vector of correlated
  //standard norm samples
  $(U_i^1,\cdots,U_i^K)^T \leftarrow [\Phi(Z_i^1),\cdots,\Phi(Z_i^K)]^T$
  //Φ is the cumulative density function of norm distribution
  //$(U_i^1,\cdots,U_i^K)^T$ are the correlated uniform distribution
  $(X_i^1,\cdots X_i^K)^T \leftarrow F^{-1}[(U_i^1,\cdots,U_i^K)^T]$
  //F_i^{-1} is the inverse cumulative probability function
end for

V. CASE STUDY

The method described in the preceding sections is applied to an modified IEEE 34 Distribution Network. The programs are developed with Matlab R2009b on a PC with Intel Core 2 2.66-GHz CPU and 4 GB RAM.

The network diagram is shown by Fig.1 and the data are presented in [12]. HOMER software is used to generate the wind speed and solar irradiance data for one year. The estimated parameters of wind speed are k=1.8785 and c=3.3795 using (4) and (5), respectively. The comparison between actual wind speed data and Best-fit Weibull distribution is shown by Fig. 2. The type of the wind turbines is Enercon E33 with rated power 330kW. The cut-in, rated and cut-out wind speed are 3m/s, 13m/s and 25m/s. The estimated parameters of solar irradiance are $\alpha = 0.2249$ and $\beta = 1.1245$ using (8) and (9). The comparison between actual solar irradiance data and Best-fit Beta distribution is shown by Fig.3.

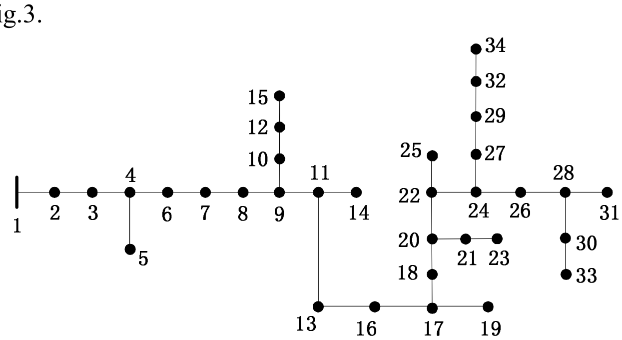


Fig. 1. IEEE 34 Distribution Network

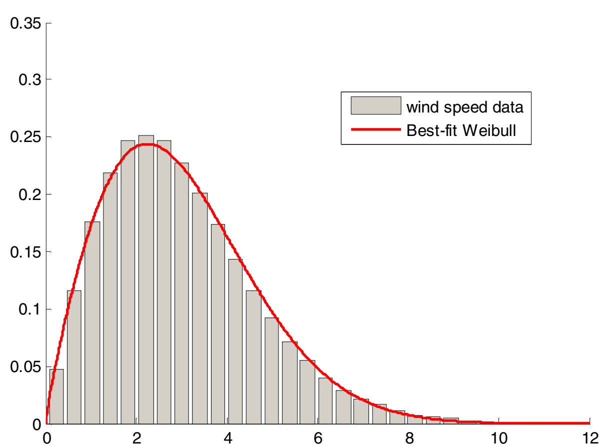


Fig. 2 Comparison between wind speed data and Best-fit Weibull distribution

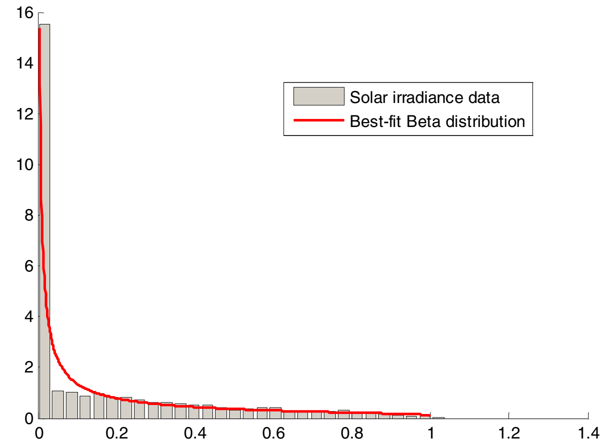


Fig. 3 Comparison between solar irradiance data and Best-fit Beta distribution

The probabilistic results of 50 000 times of Monte Carlo (MC) simulation are assumed to be accurate and used to determine the effectiveness of the LHSD method. The error indices are used to indicate the distribution accuracy of output random variables as follows:

$$\varepsilon_{\mu} = \left| \frac{\mu_{MC} - \mu_{LHSD}}{\mu_{MC}} \right| \times 100\%$$

$$\varepsilon_{\sigma} = \left| \frac{\sigma_{MC} - \sigma_{LHSD}}{\sigma_{MC}} \right| \times 100\%$$

where μ_{MC} , σ_{MC} are the results of 50 000 times of MC simulation and μ_{LHSD} , σ_{LHSD} are the results of LHSD method, respectively.

Three different cases are identified and studied:

Case 1. Three wind turbines are located at Bus 29, 31 and 34. All the input random variables are independent;

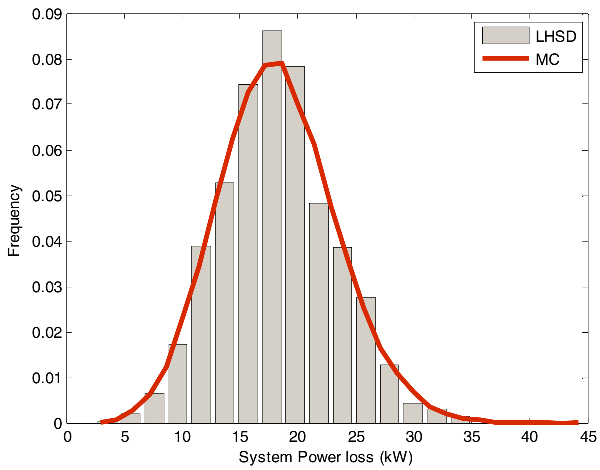
Case 2. All the correlation coefficient of loads is 0.8. All wind powers are correlated with a correlation coefficient of 0.8. The wind powers and loads are assumed to be uncorrelated for simplicity.

Case 3. Based on Case 2, the Photovoltaic (PVs) are added in Bus 29 and 31. The correlation coefficient between wind powers and PV is -0.7 due to the complement between the wind power and PV.

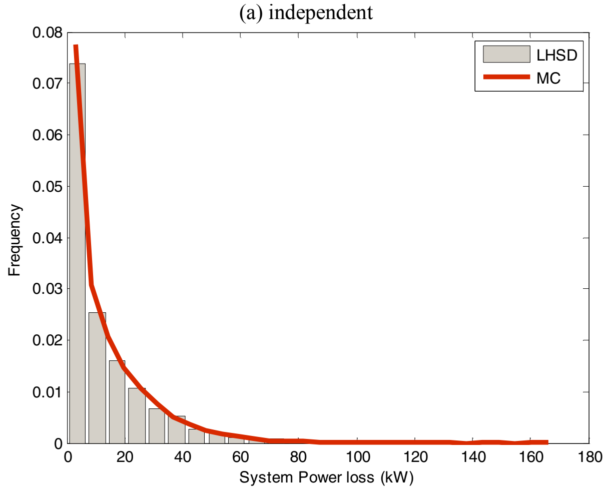
In order to evaluate and illustrate the effectiveness of the LHSD method, the error comparisons between MC and LHSD method are shown in Table I. It is observed that the LHSD method can get very accuracy results with much smaller sample size and has a much smaller error value compared with MC simulation.

TABLE I
ERROR COMPARISONS BETWEEN MC AND LHSD METHOD

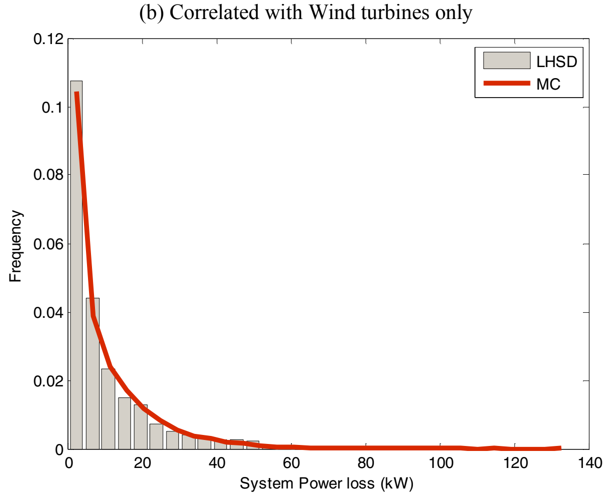
Sample Size N	500		1000		1500	
	MC	LHSD	MC	LHSD	MC	LHSD
ε_{μ}	1.6%	0.03%	0.08%	0.005%	0.06%	0.005%
ε_{σ}	2.31%	1.29%	4.81%	0.57%	3.95%	0.41%



(a) independent

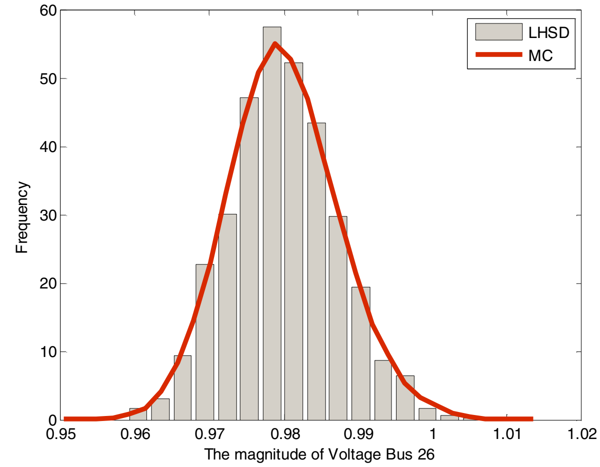


(b) Correlated with Wind turbines only

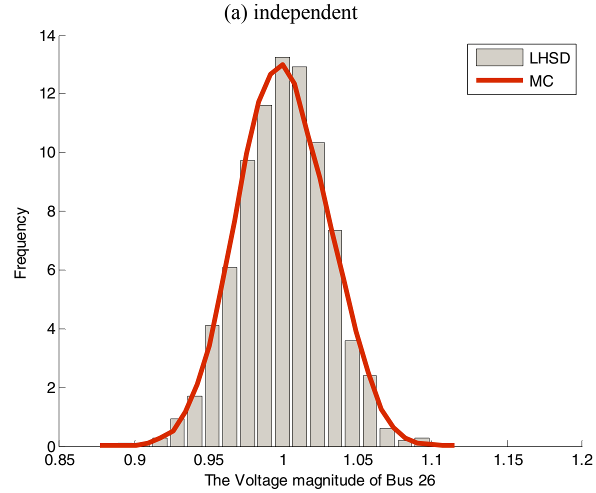


(c) Correlated with Wind turbines and PV hybrid system

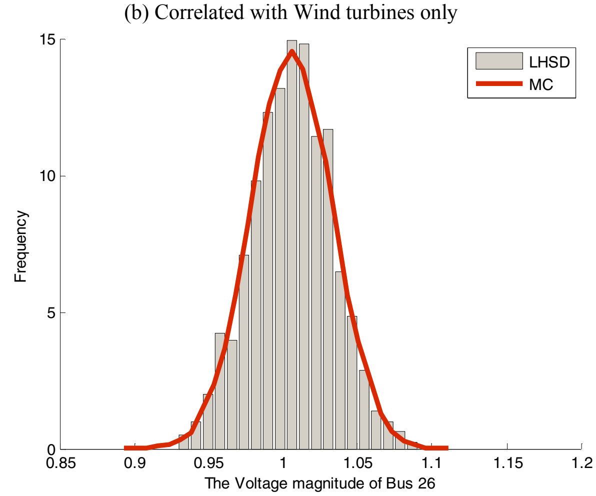
Fig. 2 Comparison between total system loss by LHSD with $N = 1000$ and MC with $N = 50\ 000$



(a) independent



(b) Correlated with Wind turbines only



(c) Correlated with Wind turbines and PV hybrid system

Fig. 3 Comparison between the magnitude of Voltage Bus 26 by LHSD with $N = 1000$ and MC with $N = 50\ 000$

The three different cases comparisons between total system loss and the magnitude of Voltage Bus 26 by LHSD method with $N = 1000$ and MC with $N = 50\ 000$ are plotted in Fig. 2 and 3, respectively. It is clear that the distributions of output random variables are fitted very well.

The results of case 1 and case 2 show that the mean values of output variables remain basically unchanged as the correlation among the input variables increase; however, the standard deviations of these variables grow significantly when the correlation is considered. The results prove that the correlation between the input random variables should be considered, or else the results are over-optimistic.

The comparison between case 2 and case 3 indicates that wind and PV hybrid system can decrease the probability of voltage limit violation and has a smaller system loss due to the complement between the wind power and PV.

Table II is the computation time comparisons between MC and the LHSD method. It shows that the calculation speed of LHSD is comparable with MC simulation.

TABLE II
COMPUTATION TIME (SECONDS) COMPARISONS BETWEEN MC AND LHSD METHOD

Sample Size N	500	1000	1500
MC	2.0247s	3.9602s	5.7440s
LHSD	2.5525s	4.5976s	7.4660s

VI. CONCLUSION

The paper adopts the LHSD method to solve the PLF problem with correlated random variables for distribution networks. The results obtained by applying the proposed method allow deriving the relevant conclusions below:

1) The LHSD method handles correlations efficiently and presents an accurate simulation result with a much smaller simulation size. The computation time of LHSD method is comparable with MC simulation.

2) The expected values of output variables remain basically unchanged as the correlation among the input variables increase; however, the standard deviations of these variables grow significantly as the correlation increases. The results indicate the importance of correlation between the input random variables.

3) Wind and PV hybrid system show better voltage limit violation and smaller system loss due to the complement between the wind power and PV system.

VII. ACKNOWLEDGMENT

The authors would like to acknowledge the support of the EPSRC UK-China joint research consortium (EP/F061242/1), the science bridge award (EP/G042594/1), Jiangsu Power Company, China, the Chinese Scholarship Council and the Queen's University of Belfast, UK. Prof. Haifeng Wang is a member of the international innovation team of superconducting technology for electrical engineering at the Institute of Electrical Engineering, Beijing, China, sponsored by the Chinese Academy of Sciences, China.

VIII. REFERENCES

[1] B. Borkowska, "Probabilistic Load Flow," *IEEE Trans. Power App. Syst.*, vol. PAS-93, pp. 752-759, Apr. 1974.

[2] R. N. Allan, A. M. Leite da Silva and R. C. Burchett, "Evaluation Methods and Accuracy in Probabilistic Load Flow Solutions," *IEEE Trans. Power App. Syst.,* vol. PAS-100, pp. 2539-2546, May. 1981.

[3] Pei Zhang and S. T. Lee, "Probabilistic load flow computation using the method of combined cumulants and Gram-Charlier expansion," *IEEE Trans. Power Systems,* vol. 19, pp. 676-682, Feb. 2004.

[4] Z. Hu and Xifan Wang, "A probabilistic load flow method considering branch outages," *IEEE Trans. Power Systems,* vol. 21, pp. 507-514, May. 2006.

[5] N. D. Hatziargyriou, T. S. Karakatsanis and M. Papadopoulos, "Probabilistic load flow in distribution systems containing dispersed wind power generation," *IEEE Trans. Power Systems,* vol. 8, pp. 159-165, Feb. 1993.

[6] A. M. Leite da Silva, V. L. Arienti and R. N. Allan, "Probabilistic Load Flow Considering Dependence Between Input Nodal Powers," *IEEE Trans. Power App. Syst.,* vol. PAS-103, pp. 1524-1530, June. 1984.

[7] H. Yu, C. Y. Chung, K. P. Wong, H. W. Lee and J. H. Zhang, "Probabilistic Load Flow Evaluation With Hybrid Latin Hypercube Sampling and Cholesky Decomposition," *IEEE Trans. Power Systems,* vol. 24, pp. 661-667, May. 2009.

[8] J. M. Morales, L. Baringo, A. J. Conejo and R. Minguez, "Probabilistic power flow with correlated wind sources," *IET Gemer. Transm. Distrib.,* vol. 4, pp. 641-651, 2010.

[9] J. Usaola, "Probabilistic load flow with correlated wind power injections," *Electr. Power Syst. Res.,* vol. 80, pp. 528-536, May. 2010.

[10] M. D. McKay, R. J. Beckman and W. J. Conover, "A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code," *Technometrics,* vol. 21, pp. 239-245, May, 1979.

[11] M. Stein, "Large Sample Properties of Simulations Using Latin Hypercube Sampling," *Technometrics,* vol. 29, pp. 143-151, May, 1987.

[12] R. M. Ciric, A. Padilha-Feltrin and I. F. E. D. Denis, "Observing the performance of distribution systems with embedded generators," *European Transactions on Electrical Power,* vol. 14, pp. 347-359, 2004.

[13] G. W. Chang, S. Y. Chu and H. L. Wang, "An Improved Backward/Forward Sweep Load Flow Algorithm for Radial Distribution Systems," *IEEE Trans. on Power Syst.,* vol. 22, pp. 882-884, May, 2007.

[14] E. Bompard, E. Carpaneto, G. Chicco and R. Napoli, "Convergence of the backward/forward sweep method for the load-flow analysis of radial distribution systems," *Electrical Power & Energy Systems,* vol. 22, pp. 521-530, Oct., 2000.

[15] P. Glasserman, *Monte Carlo Methods in Financial Engineering*, New York: Springer, 2003, pp. 236-238.

[16] P.Jirutitijaroen and C.Singh, "Comparison of simulation methods for power system reliablity indexs and their distributions," *IEEE Trans. Power Systems,* vol. 23, pp. 486-493, May. 2008.

[17] M. Liefvendahl and R.Stocki, "A study on algorithms for optimization of Latin hypercubes," *J. Statist. Plan. Infer.*, vol. 136, pp. 3231-3247, 2006.

[18] K. Q. Ye, W. Li, and A. Sudjianto, "Algorithmic construction of optimal symmetric Latin hypercube designs," *J. Statist. Plan. Infer.*, vol. 90, pp. 145-159, 1998.

[19] D. E. Huntington and C. S. Lyrintzis, "Improvements to and limitations of Latin hypercube sampling," *Prob. Eng. Mech.*, vol. 13, no. 4, pp. 245-253,1998.

[20] R. L. Iman and W. J. Conover, "A distribution-free approach to inducing rank correlation among input variables," *Commun. Statist.-Sim. Comput.*, pp. 311-334, Jan. 1982.

[21] A. B. Owen, "Controlling correlations in Latin hypercube samples," *J. Amer. Statist. Assoc.*, vol. 89, no. 428, pp. 1517-1522, Dec. 1994.

[22] Philip E. Gill, Walter Murray and Margaret H.Wright, *Practical Optimization*, London: Academic Press, 1981, pp. 108-111.

[23] C. L. Su, "Probabilistic Load-Flow Computation Using Point Estimate Method," *IEEE Trans. Power Systems,* vol. 20, pp. 1843-1851, Nov. 2005.

[24] G. Papaefthymiou and D. Kurowicka, "Using Copulas for Modeling Stochastic Dependence in Power System Uncertainty Analysis," *IEEE Trans. Power Systems,* vol. 24, pp. 40-49, Feb. 2009.

[25] S. H. Karaki, R.B. Chedid and R. Ramadan, "Probabilistic Performance Assessment of Autonomous Solar-Wind Energy Conversion Systems," *IEEE Trans. Energy Convers*., vol. 14, pp. 766-772, Sep. 1999.

[26] NERL. Hybrid Optimization Model for Electric Renewable (HOMER). [Online]. Available: http://www.nrel.gov/international/tools/HOMER/ h omer.html.