H_∞ Control of a Building Structure with Time-Varying Delay

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(Received: 4 Aug 2013; Received revised form: 8 April 2014; Accepted: 6 Oct 2014)

Abstract: In this paper, a H_∞ controller for a building structure with time-varying delay is studied. A matrix inequality used for stability analysis is proposed, an H_∞ controller is designed based on the matrix inequality and by using the parameter-adjusting method. Three control cases are discussed detailly by transforming the problem into parameters optimization: (i) allowable time delay with known controller; (ii) controller design with known maximum time delay; (iii) the biggest allowable time delay to maintain system stability with unknown controller. Numerical simulations are also given to demonstrate the validity and feasibility of the proposed methods in this paper.

Key words: building structure, matrix inequality, H_∞ control, time-varying delay.

1. INTRODUCTION

In dynamic modeling and structural active control, errors may exist inevitably between the physical model and practical structures due to some uncertainties such as structural parameters and boundary conditions of the structures etc. Besides, signal noise and external disturbance may also degrade control efficiency in control implementation. So the designed controller is desired to have strong robustness for the uncertainties so as to eliminate the negative effect of the uncertainty factors on control performance. In modern control systems, robust control method is robust to the variances of structural parameters and external disturbance, so this method has received more and more researchers' attention and many studies have been done (Zhong 2006; Mahmoud 2000; Wu *et al*. 2010; Jia 2007).

Time delay exists inevitably in active control systems due to many reasons, such as online data acquisition from sensors at different locations of the structure, data processing and active control force calculation of the computer, control force signals transmission to the actuators to build up required control force. Various research results indicate that even a small time delay may cause actuators to apply energy to the control system when energy is actually not needed, which may cause degradation of control efficiency and even make the system unstable (Hu and Wang 2002; Cai *et al*. 2003; Chen 2009). So far some methods have been proposed to handle time delay problem in active control system, such as Taylor series expansion (Abdel-Rohman 1987), phase shift technique (Chung *et al*. 1988), state pre-estimation (Greery 1988) and two direct design methods for time-delay controller (Cai *et al*. 2003; Cai and Huang 2002). The first three methods work well with some small time delay problems, but can not deal with large time delay ones. Two direct design methods proposed by Cai *et al*. (2003) and Cai and Huang (2002) are to design time-delay controller directly from time-delay differential equation and no assumption is made in the entire design process, and they are suitable for both small and large time delays. Chen *et al*. (2009) and Chen (2009) verified these two methods by experiment using several flexible structures

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as research objects. Nowadays, time delay problem in robust H_∞ control has come to many researchers' attention. For example, Du and Zhang (2008) investigated an H_∞ controller design approach for vibration attenuation of seismic-excited building structures with uncertain time-invariant time delay in the control input. Zhang *et al*. (2008) studied the robust stability for a class of uncertain neutral systems with time-varying delay and nonlinear uncertainties, and by Lyapunov method, put forward a new delay-dependent stability criteria. Zhao *et al*. (2010) discussed the robust H_∞ state-feedback controller design for a class of semi-active seat suspension systems with norm-bounded parameter uncertainties, time-varying input delay and actuator saturation. The desired controller is derived by solving the LMIs and the corresponding closed-loop system is asymptotically stable with a guaranteed H_∞ performance. Chen *et al*. (2009) considered Takagi-Sugeno (T-S) fuzzy systems with both state and input time delays, robust H_∞ fuzzy controller is designed based on the Lyapunov-Krasoviskii functional method and numerical simulations are given to illustrate the effectiveness and feasibility of the proposed controller.

In this paper, we are interested in the problem of H_∞ control of a three-story building with time-varying delay. A matrix inequality for stability analysis is proposed by using the Lyapunov-Krasovskii functional and free-weighting matrix. An H_∞ controller is presented based on the matrix inequality. This paper is organized as follows. Section 2 briefly introduces motion equation of the structural system with time-varying delay. A matrix inequality and design of H_∞ controller is given in Section 3. Section 4 introduces three cases: (i) allowable time delay with known controller; (ii) controller design with known maximum time delay; (iii) the biggest allowable time delay to maintain system stability with unknown controller. Numerical simulations and comparisons of a three-story building structure using the proposed time-delay controller are carried out in Section 5. Finally, concluding remarks are given in Section 6.

2. MOTION EQUATION

Consider an n-story building, the structure undergoes an one-dimensional earthquake ground acceleration $w(t)$. $\tau(t)$ is the time-varying delay in control, the motion equation of the structural system is written as

$$\begin{aligned}&[\boldsymbol{M}]\{\ddot{\bar{\boldsymbol{Z}}}(t)\}+[\tilde{\boldsymbol{C}}]\{\dot{\bar{\boldsymbol{Z}}}(t)\}+[\tilde{\boldsymbol{K}}]\{\bar{\boldsymbol{Z}}(t)\}=\\&[\boldsymbol{H}]\{\boldsymbol{U}(t-\tau(t))\}-\{\boldsymbol{M}_0\}w(t)\end{aligned}\tag{1}$$

where, $\{\bar{\boldsymbol{Z}}(t)\} = \{\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_n\}^T$ is the interstory drift of each story unit of building structures; $[\boldsymbol{M}]$ is a lower triangular matrix whose elements are defined by $[\boldsymbol{M}](i, j) = m_i$ for $i = 1,\cdots, n$ and $j=1,\cdots, i$, where m_i represents the mass of the i-th floor; $[\tilde{\boldsymbol{K}}]$ is an upper triangular two band stiffness matrix whose elements are defined by $[\tilde{\boldsymbol{K}}](i, i) = k_i$ for $i = 1,\cdots, n$ and $[\tilde{\boldsymbol{K}}](i, i+1) = -k_{i+1}$ for $i = 1,\cdots, n-1$; $[\tilde{\boldsymbol{C}}]$ is an upper triangular two band damping matrix whose elements are defined by $[\tilde{\boldsymbol{C}}](i,i) = c_i$ for $i = 1,\cdots, n$ and $[\tilde{\boldsymbol{C}}](i, i+1) =-c_{i+1}$ for $i = 1, \cdots, n-1$; $\{\boldsymbol{M}_0\}$ is the vector whose elements are the mass of each story unit; $[\boldsymbol{H}]$ represents the location of active control force; $\{\boldsymbol{U}(t-\tau(t))\}$ is the active control force.

In the state space representation, Eqn 1 becomes

$$\{\dot{\boldsymbol{x}}(t)\}=[\boldsymbol{A}]\{\mathbf{x}(t)\}+[\boldsymbol{B}]\{\boldsymbol{U}(t-\tau(t))\}+\{\mathbf{B}_w\}w(t)\tag{2}$$

where

$$\{\boldsymbol{x}(t)\}=\begin{Bmatrix}\bar{\boldsymbol{Z}}(t)\\ \dot{\bar{\boldsymbol{Z}}}(t)\end{Bmatrix},\ [\boldsymbol{A}]=\begin{bmatrix}\boldsymbol{0} & \mathbf{I}\\ -\boldsymbol{M}^{-1}\tilde{\boldsymbol{K}} & -\boldsymbol{M}^{-1}\tilde{\boldsymbol{C}}\end{bmatrix},$$

$$[\boldsymbol{B}]=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{M}^{-1}\boldsymbol{H}\end{bmatrix},\ \{\boldsymbol{B}_w\}=\begin{Bmatrix}\boldsymbol{0}\\ -\boldsymbol{M}^{-1}\boldsymbol{M}_0\end{Bmatrix}.$$

3. MATRIX INEQUALITY AND DESIGN OF H_∞ CONTROLLER

The control system with output equation can be described as

$$\begin{cases}\{\dot{\boldsymbol{x}}(t)\}=[\boldsymbol{A}]\{\boldsymbol{x}(t)\}+[\boldsymbol{B}]\{\boldsymbol{U}(t-\tau(t))\}+\{\boldsymbol{B}_w\}w(t)\\ \{\mathbf{z}(t)\}=[\boldsymbol{C}]\{\boldsymbol{x}(t)\}+[\boldsymbol{D}_{12}]\{\boldsymbol{U}(t-\tau(t))\}+\{\boldsymbol{D}_{11}\}w(t)\end{cases}\tag{3}$$

Time-varying delay $\tau(t)$ satisfies

$$0\le\tau(t)\le\bar{\tau},\ \dot{\tau}(t)\le\mu,\ \forall t\ge 0\tag{4}$$

The memoryless controller $\{\boldsymbol{U}(t)\}$ is introduced into the control system

$$\{\boldsymbol{U}(t)\}=[\boldsymbol{K}]\{\boldsymbol{x}(t)\}\tag{5}$$

then the closed-loop control system is presented as

$$\begin{cases}\{\dot{\boldsymbol{x}}(t)\}=[\boldsymbol{A}]\{\boldsymbol{x}(t)\}+[\boldsymbol{B}_K]\{\mathbf{x}(t-\tau(t))\}+\{\boldsymbol{B}_w\}w(t)\\ \{\boldsymbol{z}(t)\}=[\boldsymbol{C}]\{\mathbf{x}(t)\}+[\boldsymbol{D}_K]\{\boldsymbol{x}(t-\tau(t))\}+\{\boldsymbol{D}_{11}\}w(t)\end{cases}\tag{6}$$

where $[\boldsymbol{B}][\boldsymbol{K}] = [\mathrm{B}_k]$, $[\boldsymbol{D}_{12}][\boldsymbol{K}] = [\mathrm{D}_k]$

3.1. Matrix Inequality

Theorem 1. (Wu 2008) Given the upper bound $\bar{\tau}$ of time-varying delay $\tau(t)$, if there exist real matrices

$[\boldsymbol{P}] > 0$, $[\boldsymbol{R}] > 0$ and $[\boldsymbol{Q}] > 0$, and the two free-weighting matrices $[\boldsymbol{N}_1]$ and $[\boldsymbol{N}_2]$, satisfying the matrix inequality

$$[\Xi] = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \boldsymbol{PB}_w & \bar{\tau}\boldsymbol{A}^T & \bar{\tau}\boldsymbol{N}_1 & \boldsymbol{C}^T \\ * & \Xi_{22} & \boldsymbol{0} & \bar{\tau}\boldsymbol{B}_K^T & \bar{\tau}\boldsymbol{N}_2 & \boldsymbol{D}_K^T \\ * & * & -\gamma^2\boldsymbol{I} & \bar{\tau}\boldsymbol{B}_w^T & \boldsymbol{0} & \boldsymbol{D}_{11}^T \\ * & * & * & -\bar{\tau}\boldsymbol{R}^{-1} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & -\bar{\tau}\boldsymbol{R} & \boldsymbol{0} \\ * & * & * & * & * & -\boldsymbol{I} \end{bmatrix} < 0 \quad (7)$$

where $[\boldsymbol{B}_k] = [\boldsymbol{B}][\boldsymbol{K}]$, $[\boldsymbol{D}_k] = [\boldsymbol{D}_{12}][\boldsymbol{K}]$; $[\Xi_{11}] = [\boldsymbol{P}][\boldsymbol{A}] + [\mathrm{A}]^T[\boldsymbol{P}] + \{\boldsymbol{Q}\} + [\boldsymbol{N}_1] + [\boldsymbol{N}_1]^{\mathrm{T}}$, $[\Xi_{12}] = [\boldsymbol{P}][\boldsymbol{B}_k] - [\boldsymbol{N}_1] + [\boldsymbol{N}_2]^T$, $[\Xi_{22}] = -(1-\mu)[\boldsymbol{Q}]-[\boldsymbol{N}_2] + [\boldsymbol{N}_2]^T$. Then the closed-loop control system Eqn 6 should be asymptotically stable with H_∞ performance index γ for any time-varying delay $\tau(t)$, satisfied Eqn 4.

3.2. Design of H_∞ Controller

In last section, a matrix inequality is presented. Eqn 7 is a nonlinear matrix inequality due to the introduction of free-weighting matrices $[\boldsymbol{N}_1]$ and $[\boldsymbol{N}_2]$ which can not be solved directly by using the LMI toolbox in MATLAB software. In this section, the parameter adjustment method given by Wu (2008) is used to handle Eqn 7 by introducing two adjustable parameters λ and ρ. When λ and ρ are given and $\bar{\tau}$ is known, Eqn 7 will becomes a linear matrix inequality (LMI), so the LMI Toolbox can be used to obtain H_∞ controller.

Theorem 2. Given $\bar{\tau} > 0$, $\lambda \in R$ and $0 \neq \rho \in \mathrm{R}$, if there exist real matrices $[\bar{\boldsymbol{P}}] > 0$, $[\bar{\boldsymbol{R}}] > 0$, $[\bar{\boldsymbol{Q}}] > 0$, and $[\boldsymbol{Y}]$ satisfying the matrix inequality

$$\begin{bmatrix} \omega_{11} & \omega_{12} & \boldsymbol{B}_w & \omega_{13} & \boldsymbol{0} & \omega_{14} & \bar{\mathbf{P}} \\ * & \omega_{22} & \boldsymbol{0} & \bar{\tau}\rho\boldsymbol{Y}^T\boldsymbol{B}^T & \bar{\tau}\bar{\boldsymbol{R}} & \rho\boldsymbol{Y}^T\boldsymbol{D}_{12}^T & \boldsymbol{0} \\ * & * & -\gamma^2\boldsymbol{I} & \bar{\tau}\boldsymbol{B}_w^T & \boldsymbol{0} & \boldsymbol{D}_{11}^T & \boldsymbol{0} \\ * & * & * & -\bar{\tau}\bar{\boldsymbol{R}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & -\bar{\tau}\bar{\boldsymbol{R}} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & -\boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & * & * & -\bar{\boldsymbol{Q}} \end{bmatrix} < 0 \quad (8)$$

where $[\omega_{11}] = [\bar{\boldsymbol{P}}][\boldsymbol{A}]^T + [\boldsymbol{A}][\bar{\boldsymbol{P}}] + \lambda[\boldsymbol{B}][\boldsymbol{Y}] + \lambda[\boldsymbol{Y}]^T[\boldsymbol{B}]^T - (1-\mu)\lambda^2[\bar{\boldsymbol{Q}}]$, $[\omega_{12}] = [\bar{\boldsymbol{P}}]-\lambda[\bar{\boldsymbol{Q}}] + \rho[\boldsymbol{B}][\boldsymbol{Y}] - (1-\mu)\lambda\rho[\bar{\boldsymbol{Q}}]$, $[\omega_{22}] = -(1-\mu)\rho^2[\bar{\boldsymbol{Q}}] - 2\rho[\bar{\boldsymbol{Q}}]$, $[\omega_{13}] = \bar{\tau}[\bar{\boldsymbol{P}}][\boldsymbol{A}]^T + \lambda[\boldsymbol{Y}]^T[\boldsymbol{B}]^T$, $[\omega_{14}] = [\bar{\boldsymbol{P}}][\boldsymbol{C}]^T + \lambda[\boldsymbol{Y}]^T[\boldsymbol{D}_{12}]$.

Then the closed-loop control system is asymptotically stable with H_∞ performance index γ for any time-varying delay $\tau(t)$ satisfied Eqn 4, and the H_∞ control feedback gain is $[\boldsymbol{K}] = [\boldsymbol{Y}][\bar{\mathrm{Q}}]^{-1}$.

4. THREE CONTROL CASES

4.1. Allowable Time Delay with Known Controller

Time delay exists inevitably in active control systems, even a small time delay may cause actuators to apply energy to the control system when energy is actually not needed, which may cause degradation of control efficiency and even make the system unstable (Hu and Wang 2002; Cai *et al*. 2003; Chen 2009). Different active control methods adopt different feedback gain $[\boldsymbol{K}]$. We hope that the controller designed should have strong robustness to the variation of structural intrinsic parameters and external disturbance. In this section, we consider the following problem: in what range of time delay the controller $[\boldsymbol{K}]$ designed in the case of no time delay is applicable? The genetic algorithm will be used in this section to calculate the maximum time delay for the case when $[\boldsymbol{K}]$ is known.

In section 3, it is indicated that Eqn 8 is equivalent to Eqn 7, that is, if Eqn 8 holds, the closed-loop control system is asymptotically stable for any time-varying delay $\tau(t)$ satisfied Eqn 4. In this section, the maximum time delay can be calculated by using the following method. Assume that the feedback gain $[\boldsymbol{K}]$ has been obtained using a certain active control method. Substituting a known feedback gain $[\boldsymbol{K}]$ and a small delay $\tau(t)$ into Eqn 8 and then using the feasp function of LMI Toolbox to verify if Eqn 8 is satisfied. If Eqn 8 holds, increasing $\tau(t)$, repeat the above process until Eqn 8 does not hold. In this way, the maximum time delay can be determined.

Eqn 8 is still nonlinear after substituting a known feedback gain $[\boldsymbol{K}]$ into matrix inequality Eqn 8 because parameters λ and ρ are not determined. Here we use the genetic algorithm to optimize λ and ρ. The objective function is the maximum time delay $\bar{\tau}$ and it can be described as follows

$$\max_{\lambda,\rho} \bar{\tau} \quad \text{subject to LMI Eqn 8} \qquad (9)$$

When $[\boldsymbol{K}]$ is known, the genetic algorithm randomly generates initial λ and ρ which changes thereafter within the evolution procedure according to objective Eqn 9. The detailed technique can be described in the following steps:

Step 1: use the binary string to encode λ and ρ;

Step 2: randomly generate an initial population of N_p chromosomes, $N_p = 20$ is taken in this paper;

Step 3: substituting λ and ρ into Eqn 8 to solve the objective function $\bar{\tau}$. Decode the initial population produced in Step 2 into real values for λ_j and ρ_j, j=1, 2 ... N_p. For every λ_j and ρ_j, solving the LMI Eqn 8 to obtain the objective function $\bar{\tau}$, and then associate every λ_j and

ρ_j with a suitable fitness value according to rank-based fitness assignment approach, and then go to Step 4;

Step 4: use stochastic universal sampling to choose the offspring;

Step 5: perform one-point crossover with probability p_c to produce new offspring, $p_c = 0.7$ is taken in this paper;

Step 6: do bit mutation in the population of chromosomes with a small mutation probability p_m, in this paper $p_m = 0.02$ is taken;

Step 7: retain the best chromosomes in the population with fitness-based reinsertion method;

Step 8: the evolution process will repeat for N_g generations or being ended when the search process converges with a given accuracy. Or else go to Step 3. In this paper, $N_g = 100$ is taken.

Finally, the best chromosome is decoded into real λ and ρ, substituting $[\boldsymbol{K}]$, λ and ρ into Eqn 8 to obtain the maximum time delay $\bar{\tau}$.

4.2. Controller Design with Known Maximum Time Delay

In section 4.1, we assume that feedback gain $[\boldsymbol{K}]$ can be obtained using a certain active control method and parameters λ and ρ can be determined by the genetic algorithm, then the maximum time delay $\bar{\tau}$ can be calculated by Eqn 8. However, this method can not guarantee that γ is minimum. In other words, $[\boldsymbol{K}]$ can only guarantee that the closed-loop control system is asymptotically stable for the time-varying delay $\bar{\tau}$ satisfied Eqn 4, but can not guarantee that the control effectiveness is the best. Here we design the best $[\boldsymbol{K}]$ using the following treating process: (i) a feedback gain $[\boldsymbol{K}]$ is first designed using the classical LQR method; (ii) substituting $[\boldsymbol{K}]$ into Eqn 8 to get a maximum time delay $\bar{\tau}$; and (iii) under this $\bar{\tau}$, the best $[\boldsymbol{K}']$ can be obtained using the method in Section 4.1 and $[\boldsymbol{K}']$ can achieve better control effectiveness than $[\boldsymbol{K}]$ which will demonstrate in the next Section.

The objective function is the minimum γ and it can be described as follows

$$\min_{\lambda,\rho} \gamma \quad \text{subject to LMI Eqn 8} \tag{10}$$

The genetic algorithm is also adopted to calculate parameters λ and ρ which is similar to Section 4.1 and omitted herein.

4.3. The Biggest Allowable Time Delay for System Stability with Unknown Controller

In section 4.1, the maximum time delay $\bar{\tau}$ corresponding to a known $[\boldsymbol{K}]$ is discussed. In this section, we will discuss the problem of calculation of maximum time delay when feedback gain $[\boldsymbol{K}]$ is unknown and, for this case, how to determine the feedback gain $[\boldsymbol{K}]$.

Herein we directly use the optimization problem given by Eqn 8 to determine the maximum time delay, where the objective function is the same as Eqn 9, and the optimization variables are λ and ρ. The genetic algorithm is also the same as that used in Sections 4.1 and 4.2. After the maximum time delay $\bar{\tau}$ is obtained, the corresponding feedback gain $[\bar{\boldsymbol{K}}]$ can be solved using the method in Section 4.2. Since the feedback gain is not specified in advance in the solution of $\bar{\tau}$, this $\bar{\tau}$ is the biggest one of all the maximum time delays.

5. NUMERICAL SIMULATIONS

A three-story building is considered in this section as the structural model, as shown in Figure 1. The Tianjin earthquake with a maximum ground acceleration of 0.4 g, as shown in Figure 2, is used as the external excitation and the earthquake episode is 10s. Two active brace systems (ABS) are installed in the first and second floors, denoted as 'Actuator I'and 'Actuator II', respectively. The mass, damping and elastic stiffness of each story unit are given as 1000 kg, 1.407 kN·s/m, and

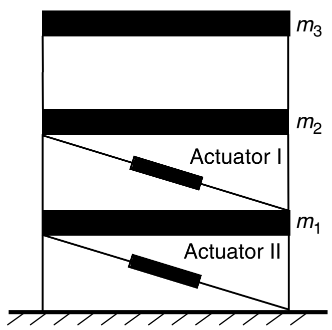

Figure 1. Structural model of a three-story building

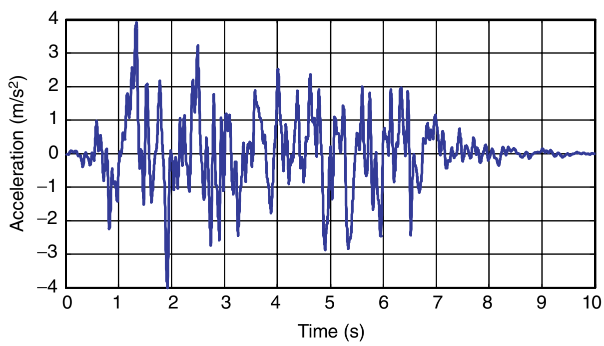

Figure 2. Time history of the Tianjin earthquake

980 kN/m, respectively. The data sampling period and computation time step are both taken by 10^{-3}s. The initial value of vector $\{\bar{\boldsymbol{Z}}\}$ is zero.

5.1. Allowable Time Delay with Known Controller

In this section, the method mentioned in section 4.1 is used to calculate the maximum time delay when the feedback gain is known. The feedback gain can be determined by using the LQR control method firstly. Du and Zhang (2008) ever studied H_∞ controller for building structures with time delay in control by linear matrix inequality, and the maximum time delay was investigated using the genetic algorithm method. The optimization problem adopted in (Du and Zhang 2008) is the same as Eqn 8, but the linear matrix inequality used in (Du and Zhang 2008) is different from Eqn 8. In this paper, the result using the method given by Du and Zhang (2008) will be compared with that using the proposed method in Section 4.1.

Firstly, we consider the case with time-invariant delay, that is, $\dot{\tau}(t)$ is chosen to be 0 in Eqn 4. The weighting matrices when using the LQR method are chosen as $[\hat{\boldsymbol{Q}}]$ = diag $\{10^4, 10^3, 10^2, 1,1,1\}$ and $[\hat{\boldsymbol{R}}]$ = diag $\{1.906 \times 10^{-7}, 1.906\times10^{-7}\}$, respectively. Using the method in Section 4.1, the maximum time delay corresponding to the feedback gain determined by the LQR method is $\bar{\tau} = 0.0382$s, that is, the feedback gain $[\boldsymbol{K}]$ calculated when using $[\hat{\boldsymbol{Q}}]$ = diag $[10^4, 10^3, 10^2, 1,1,1\}$ and $[\hat{\boldsymbol{R}}]$ diag $\{1.906\times10^{-7}, 1.906\times10^{-7})$ in the LQR method is available for vibration control of the building structure when the real time delay in the control system is within $0 \le \bar{\tau} \le 0.0382$s. Using the method given by Du and Zhang (2008), the result is $\bar{\tau} = 0.0205$s, namely the stability range of time delay is $0 \le \bar{\tau} \le 0.0205$s. Figure 3 shows the results against time of the interstory drift and the absolute acceleration of each story unit when time delay in the control system is chosen as $\tau_1 = \tau_2 = 0.029$s, denoted by the solid line. As observed from Figure 3, the interstory drift and the absolute acceleration of each story unit are evidently reduced. It is also observed from Figure 3 that, Although the real time delay $\tau_1 = \tau_2 = 0.029$s is larger than $\bar{\tau} = 0.0205$s, the control system is still stable which indicates that the given method in (Du and Zhang 2008) is somewhat conservative. Table 1 shows the maximum interstory drift x_i and the maximum absolute acceleration a_i of

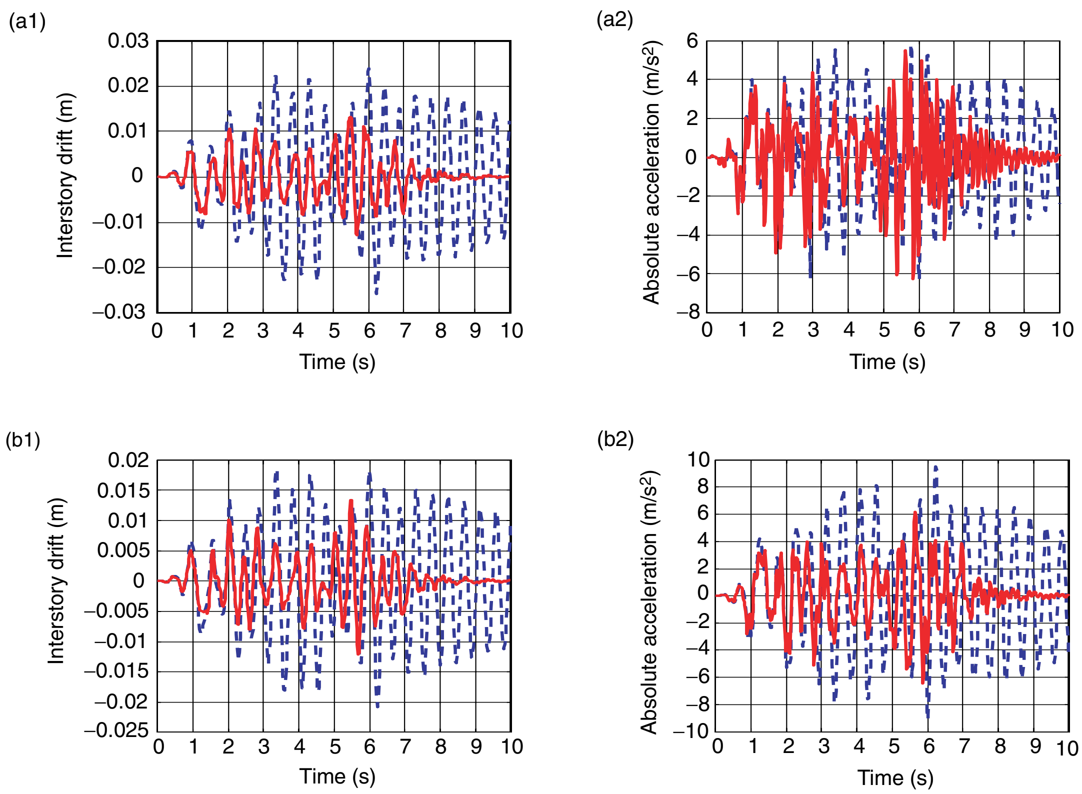

Figure 3. (***Continued***)

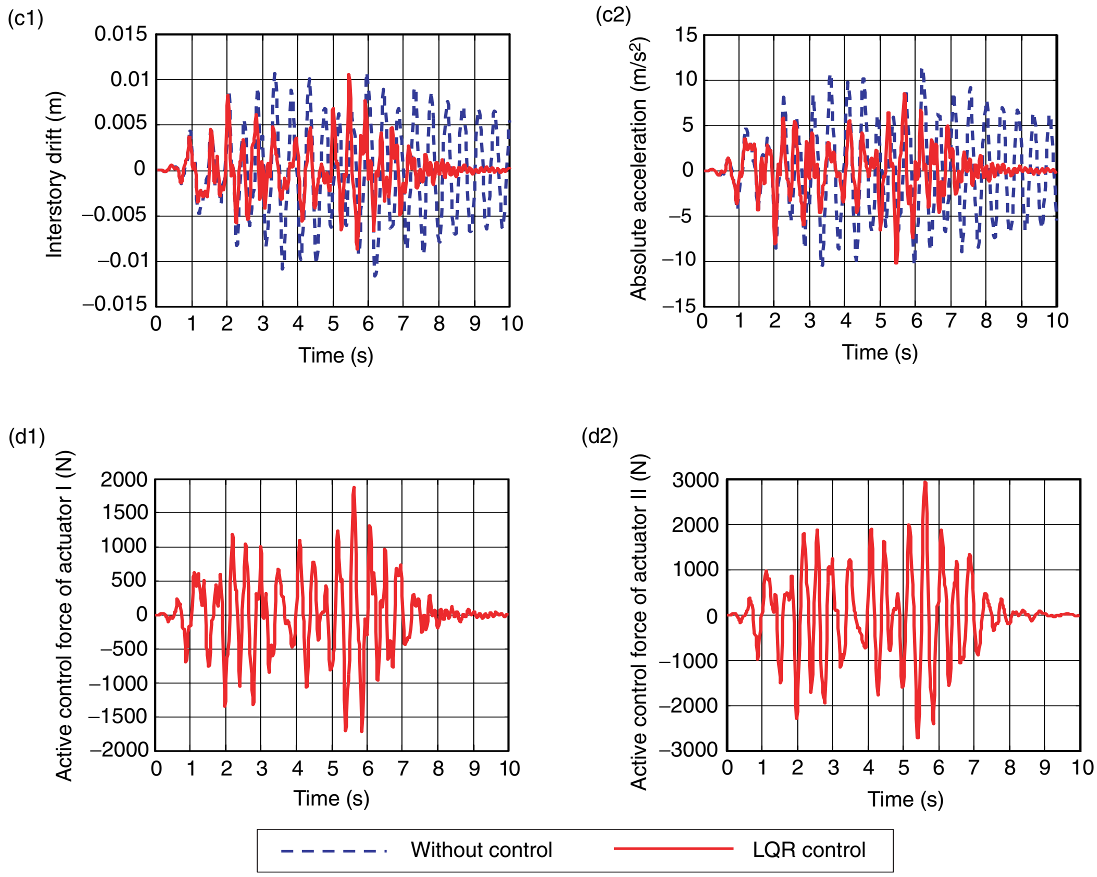

Figure 3. Time histories of system response when using LQR control and time delay is $\tau_1 = \tau_2 = 0.029$s: (a) the first story unit; (b) the second story unit; (c) the third story unit; and (d) active control forces of Actuator I and II

Table 1. Maximum interstory drift x_i and maximum absolute acceleration a_i of each story unit when using LQR control and time delay is $\tau_1 = \tau_2 = 0.04$s (x_i: m, a_i: m/s²)

Story	No control		LQR control U_{1max}=2151.42 N U_{2max}=3241.44 N	
	x_i	a_i	x_i	a_i
(1)	(2)	(3)	(4)	(5)
1	0.0258	6.29	0.0146	8.09
2	0.0209	9.49	0.0146	7.98
3	0.0117	11.44	0.0122	11.94

each story unit when time delay is chosen as $\tau_1 = \tau_2 = 0.04$s It is observed from Table 1 that the control effectiveness becomes worse when the time delay is beyond the maximum time delay, since the maximum response quantities of some story units with control are larger than those without control.

The case with time-varying delay is considered herein. The weighting matrices when using the LQR method are also chosen as $[\hat{Q}]$ = diag $\{10^4, 10^3, 10^2, 1,1,1\}$ and $[\hat{R}]$ = diag$\{1.906 \times 10^{-7}, 1.906 \times 10^{-7}\}$, respectively. In Eqn 4, $\dot{\tau}(t) \leq 0.3$, that is, $\mu = 0.3$. Using the method in Section 4.1, the maximum time delay corresponding to the feedback gain determined by the LQR method is $\overline{\tau} = 0.0501$s. Figure 4 shows the results against time of the interstory drift and the absolute acceleration of each story unit when time-varying delay in the control system is chosen as $\tau_1(t) = \tau_2(t) = |0.03\sin(10t)|$, denoted by the solid line. As observed from Figure 3, the LQR control method may achieve good control effectiveness due to the time-varying delay in the control system satisfies Eqn 4. Table 2 shows the maximum interstory drift x_i and the maximum absolute acceleration a_i of each story unit when time-varying delay is chosen as $\tau_1(t) = \tau_2(t) = |0.0587\sin(11t)|$. It is observed from Table 2 that the control effectiveness becomes worse when the time-varying delay is not satisfied $0 \leq \tau(t) \geq \overline{\tau}$ in Eqn 4, the

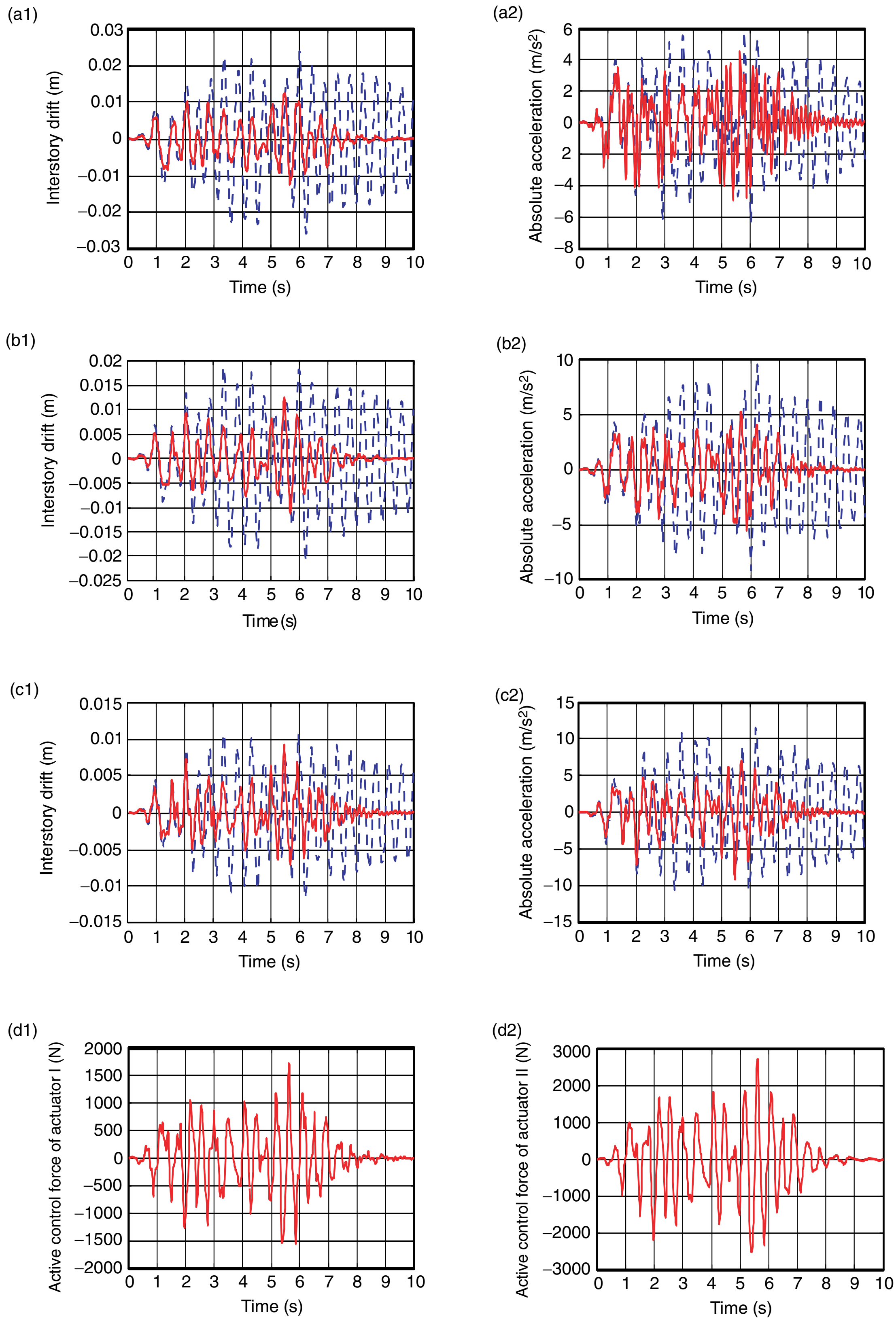

Figure 4. (*Continued*)

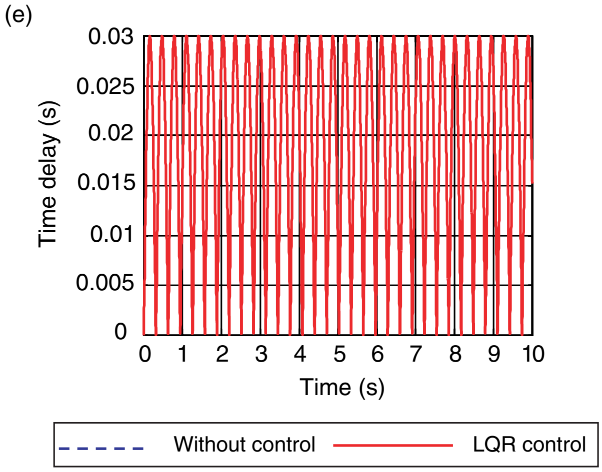

Figure 4. Time histories of system response when using LQR control and time-varying delay is τ_1 (t) = τ_2 (t) = |0.03sin(10t)| (a) the first story unit; (b) the second story unit; (c) the third story unit; (d) active control forces of Actuator I and II; and (e) time-varying delay

Table 2. Maximum interstory drift x_i and maximum absolute acceleration a_i of each story unit when using LQR control and time-varying delay is $\tau_1(t) = \tau_2(t) = |0.0587\sin(5.11t)|$ (x_i: m, a_i: m/s^2)

Story	No control		LQR control U_{1max}=2151.42N U_{2max}=3241.44N	
	x_i	a_i	x_i	a_i
(1)	(2)	(3)	(4)	(5)
1	0.0258	6.29	0.0136	7.67
2	0.0209	9.49	0.0147	8.20
3	0.0117	11.44	0.0118	11.53

maximum response quantities of some story units with control are larger than those without control.

5.2. Controller Design with Known Maximum Time Delay

In last section, the control feedback gain [K] is firstly designed using the LQR method, and the corresponding maximum time delay $\bar{\tau}$ can be determined by using the method proposed in Section 4.1. In this section, the maximum time delay $\bar{\tau}$ is substituting into Eqn 8 and the best control feedback gain [$\bar{K}$] may be obtained according to the method mentioned in Section 4.2.

Firstly, we consider the case with time-invariant delay, that is, $\dot{\tau}(t)$ is chosen to be 0 in Eqn 4 and $\bar{\tau}$ = 0.0382s. By using the method in Section 4.2, the best controller can be determined as follows

$$[\bar{\mathbf{K}}] = 10^3 \times \begin{bmatrix} 2.2561 & 0.0354 & -3.2227 & -3.6988 & 0.5267 & 0.5247 \\ 3.7796 & 1.8580 & -0.4097 & -9.4487 & -9.4519 & -5.2287 \end{bmatrix} \tag{11}$$

Figure 5 shows the results against time of the interstory drift and the absolute acceleration of each story unit when feedback gain [$\bar{K}$] is chosen as Eqn 11 and time delay in the control system as τ_1 = τ_2 = 0.029s, denoted by the solid line. As observed from Figure 3, the interstory drift and the absolute acceleration of each story unit are evidently reduced. Columns 4–5 and 6–7 of Table 3 show, when time delay is $\tau_1 = \tau_2$ = 0.029s, the maximum interstory drift x_i and the maximum absolute acceleration a_i of each story unit using feedback gain [K] and [$\bar{K}$], respectively. It is observed that the maximum response quantities using [$\bar{K}$] is smaller than that using [K] which indicate better control effectiveness can be obtained by using [$\bar{K}$]. Columns 8–9 and 10–11 of Table 3 show, when time delay is τ_1 = τ_2 =, 0.04s, the maximum response quantities using feedback gain [K] and [$\bar{K}$], respectively. It is observed from Table 3 that the control effectiveness becomes worse using both [K] and [$\bar{K}$] when the time delay is beyond the maximum time delay, since the maximum response quantities of some story units with control are larger than those without control.

The case with time-varying delay is considered herein. In Eqn 4, $\dot{\tau}(t) \le 0.3$, that is, μ = 0.3 and $\bar{\tau}$ = 0.0501s. By using the method in Section 4.2, the best controller can be determined as follows

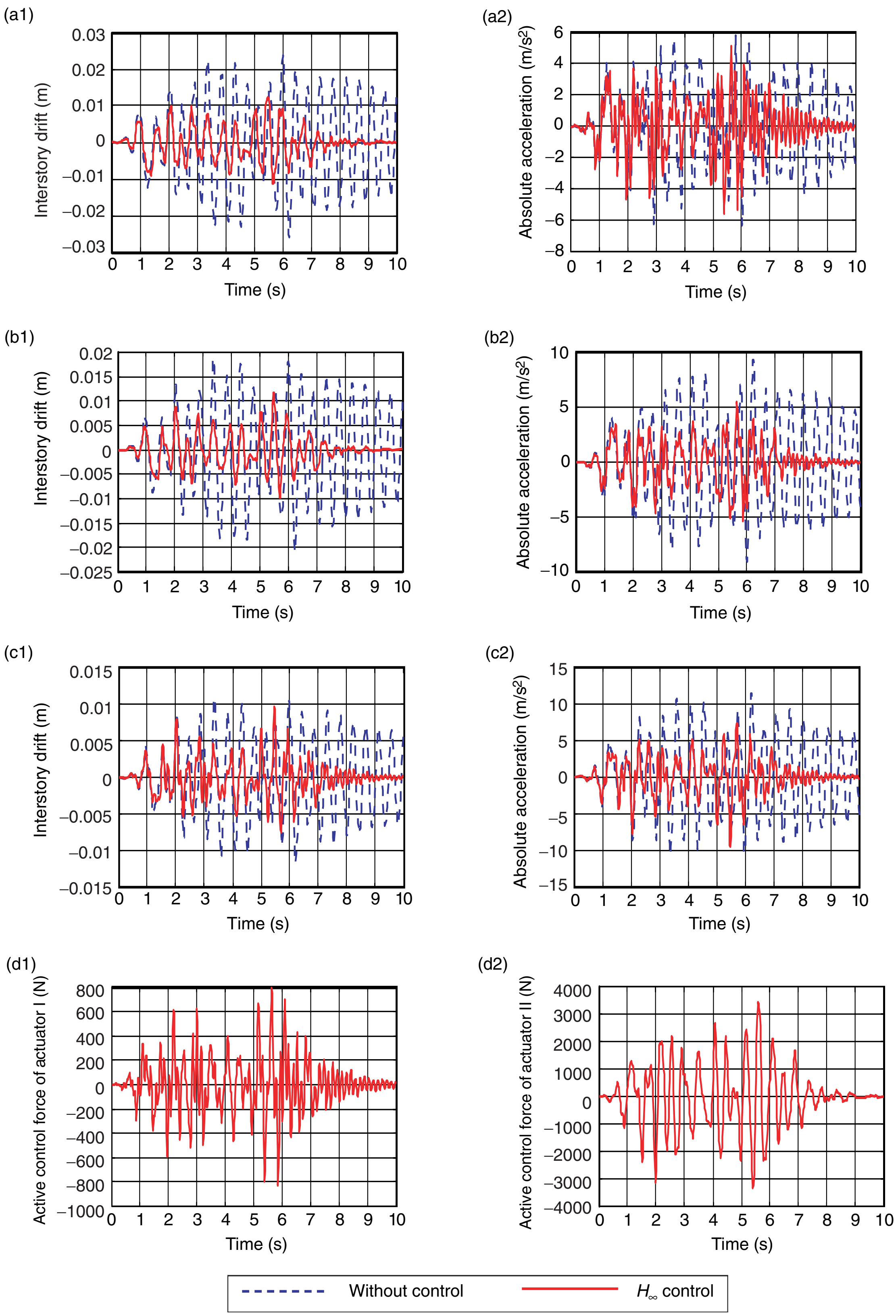

Figure 5. Time histories of system response when using H_∞ control and time delay is $\tau_1 = \tau_2 = 0.029$s: (a) the first story unit; (b) the second story unit; (c) the third story unit; and (d) active control forces of Actuator I and II

Table 3. Maximum interstory drift x_i and maximum absolute acceleration a_i of each story unit when using LQR control and H_∞ control with time-invariant delay (x_i: m, a_i: m/s^2)

Story	No control		LQR control $\tau_1 = \tau_2 = 0.029$s $U_{1max} = 1871.64$ N $U_{2max} = 2933.77$ N		H_∞ control $\tau_1 = \tau_2 = 0.029$s $U_{1max} = 832.40$ N $U_{2max} = 3425.51$ N		LQR control $\tau_1 = \tau_2 = 0.04$s $U_{1max} = 2151.42$N $U_{2max} = 3241.44$N		H_∞ control $\tau_1 = \tau_2 = 0.04$s $U_{1max} = 893.55$N $U_{2max} = 3762.17$N	
	x_i	a_i	x_i	a_i	x_i	a_i	x_i	a_i	x_i	a_i
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	0.0258	6.29	0.0132	6.27	0.0123	5.60	0.0146	8.09	0.0128	6.33
2	0.0209	9.49	0.0134	6.42	0.0118	5.51	0.0146	7.98	0.0123	6.51
3	0.0117	11.44	0.0106	10.35	0.0096	9.46	0.0122	11.94	0.0106	10.40

$$[\bar{K}] = 10^3 \times \begin{bmatrix} 1.5366 & 0.1629 & -2.7250 & -2.9610 & 0.6661 & 0.6646 \\ 2.4709 & 1.9190 & 1.8659 & -9.5040 & -9.5050 & -5.8789 \end{bmatrix} \tag{12}$$

Figure 6 shows the results against time of the interstory drift and the absolute acceleration of each story unit when feedback gain $[\bar{K}]$ is chosen as Eqn 12 and time-varying delay in the control system as $\tau_1(t) = \tau_2(t) = |0.03\sin(10t)|$, denoted by the solid line. As observed from Figure 6, the interstory drift and the absolute acceleration of each story unit are evidently reduced. Columns 4–5 and 6–7 of Table 4 show, when time-varying delay is $\tau_1(t) = \tau_2(t) = |0.03\sin(10t)|$, the maximum interstory drift x_i and the maximum absolute acceleration a_i of each story unit using feedback gain $[K]$ and $[\bar{K}]$, respectively. It is observed that, except the maximum absolute acceleration of the first story unit, the maximum response quantities using $[\bar{K}]$ is smaller than that using $[K]$ which indicate better control effectiveness can be obtained by using $[\bar{K}]$. Columns 8–9 and 10–11 of Table 4 show, when time delay is $\tau_1(t) = \tau_2(t) = |0.0587\sin(5.11t)|$, the maximum response quantities using feedback gain $[K]$ and $[\bar{K}]$, respectively. It is observed from Table 4 that the

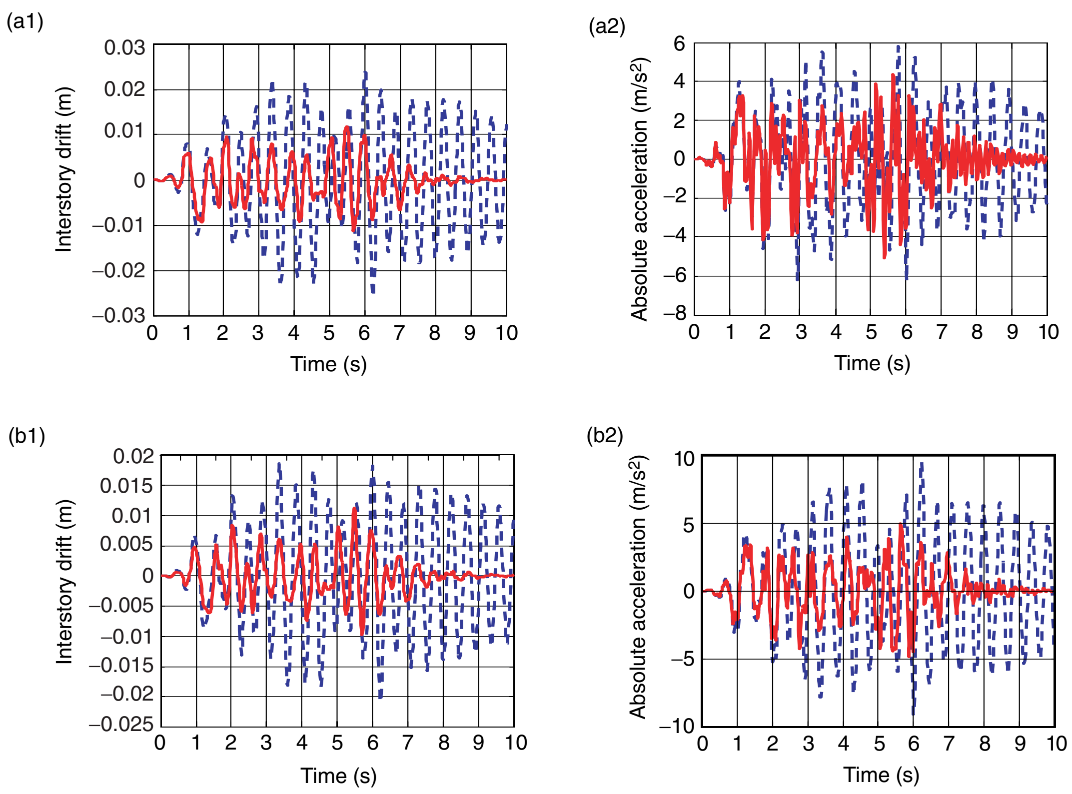

Figure 6. (*Continued*)

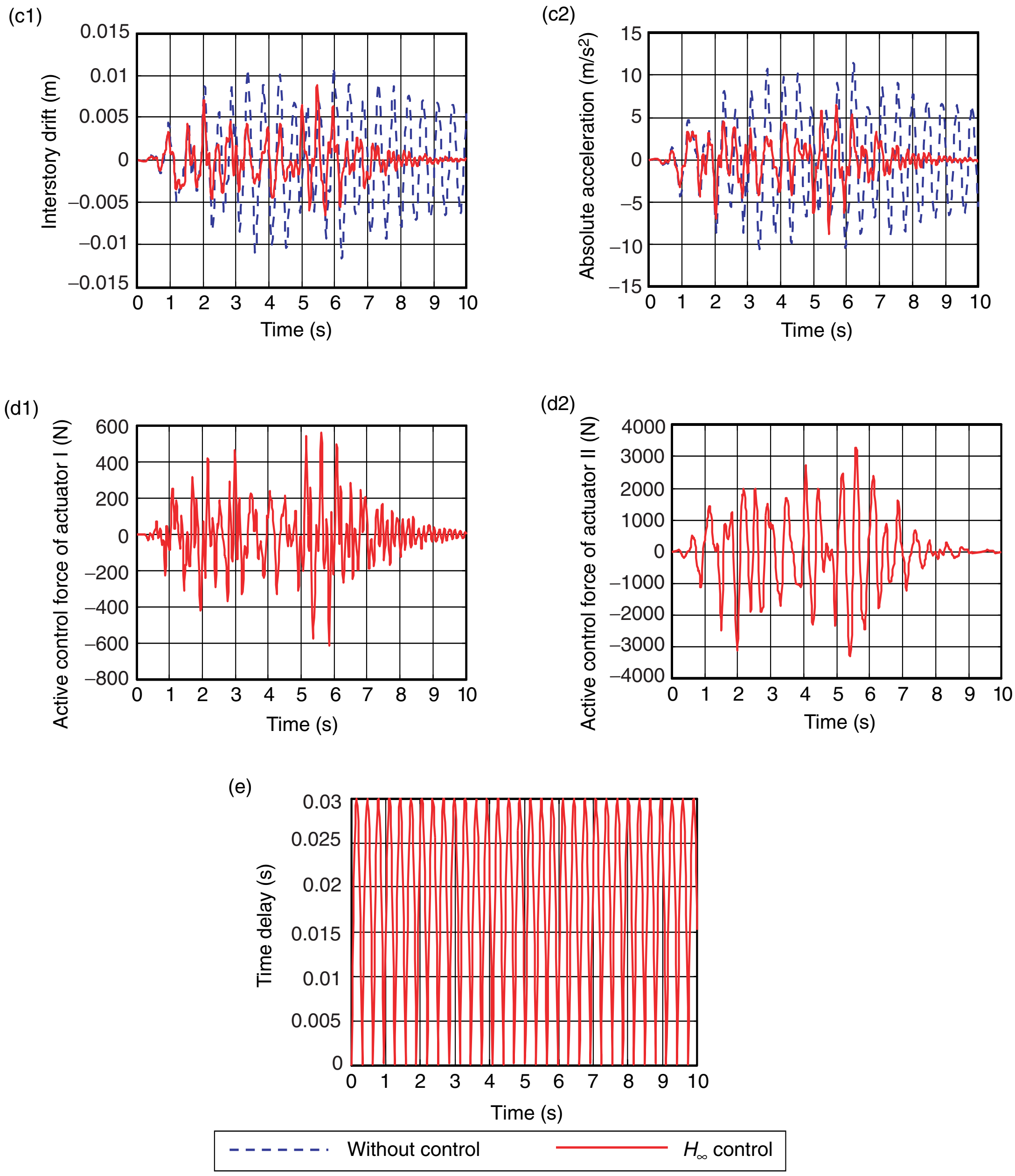

Figure 6. Time histories of system response when using H_∞ control and time-varying delay is $\tau_1(t) = \tau_2(t) = |0.03\sin(10t)|$: (a) the first story unit; (b) the second story unit; (c) the third story unit; (d) active control forces of Actuator I and II; and (e) time-varying delay

Table 4. Maximum interstory drift x_i and maximum absolute acceleration a_i of each story unit when using LQR control and H_∞ control with time-varying delay (x_i: m, a_i: m/s^2)

Story	No control		LQR control $\tau_1(t) = \tau_2(t) = \|0.03\sin(10t)\|$ U_{1max} = 1727.43N U_{2max} = 2737.49N		H_∞ control $\tau_1(t) = \tau_2(t) = \|0.03\sin(10t)\|$ U_{1max} = 609.72N U_{2max} = 3293.16N		LQR control $\tau_1(t) = \tau_2(t) = \|0.0587\sin(5.11t)\|$ U_{1max} = 2047.09N U_{2max} = 3011.26N		H_∞ control $\tau_1(t) = \tau_2(t) = \|0.0587\sin(5.11t)\|$ U_{1max} = 736.36N U_{2max} = 3511.11N	
	x_i	a_i	x_i	a_i	x_i	a_i	x_i	a_i	x_i	a_i
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	0.0258	6.29	0.0125	4.96	0.0118	5.09	0.0136	7.67	0.0133	6.68
2	0.0209	9.49	0.0123	5.54	0.0112	4.95	0.0147	8.20	0.0126	7.17
3	0.0117	11.44	0.0093	9.10	0.0089	8.72	0.0118	11.53	0.0103	10.10

control effectiveness becomes worse using both $[\boldsymbol{K}]$ and $[\bar{\boldsymbol{K}}]$ when the time-varying delay is not satisfied $0 \le \tau(t) \le \bar{\tau}$, in Eqn 4, the maximum response quantities of some story units with control are larger than those without control.

5.3. The Biggest Allowable Time Delay for System Stability with Unknown Controller

In this section, the controller is unknown in advance. The biggest time delay for stability is solved at first using the optimization algorithm and then H_∞ controller is determined using the method in Section 4.2.

Firstly, we consider the case with time-invariant delay, that is, $\dot{\tau}(t)$ is chosen to be 0 in Eqn 4. Using the objective function Eqn 9, the adjusting parameters are $\lambda = 0.0758$ and $\rho = 4.2231$, so the biggest time delay is $\bar{\tau} = 0.0744$s and the corresponding feedback gain $[\tilde{\boldsymbol{K}}]$can be determined as follows

$$[\tilde{\boldsymbol{K}}] = 10^3 \times \begin{bmatrix} -0.3383 & -0.3232 & -0.1772 & -4.6678 & -2.7456 & -0.8111 \\ 0.1986 & 0.1888 & 0.1036 & -10.0343 & -8.5945 & -5.1584 \end{bmatrix} \tag{13}$$

Figure 7 shows the results against time of the interstory drift and the absolute acceleration of each

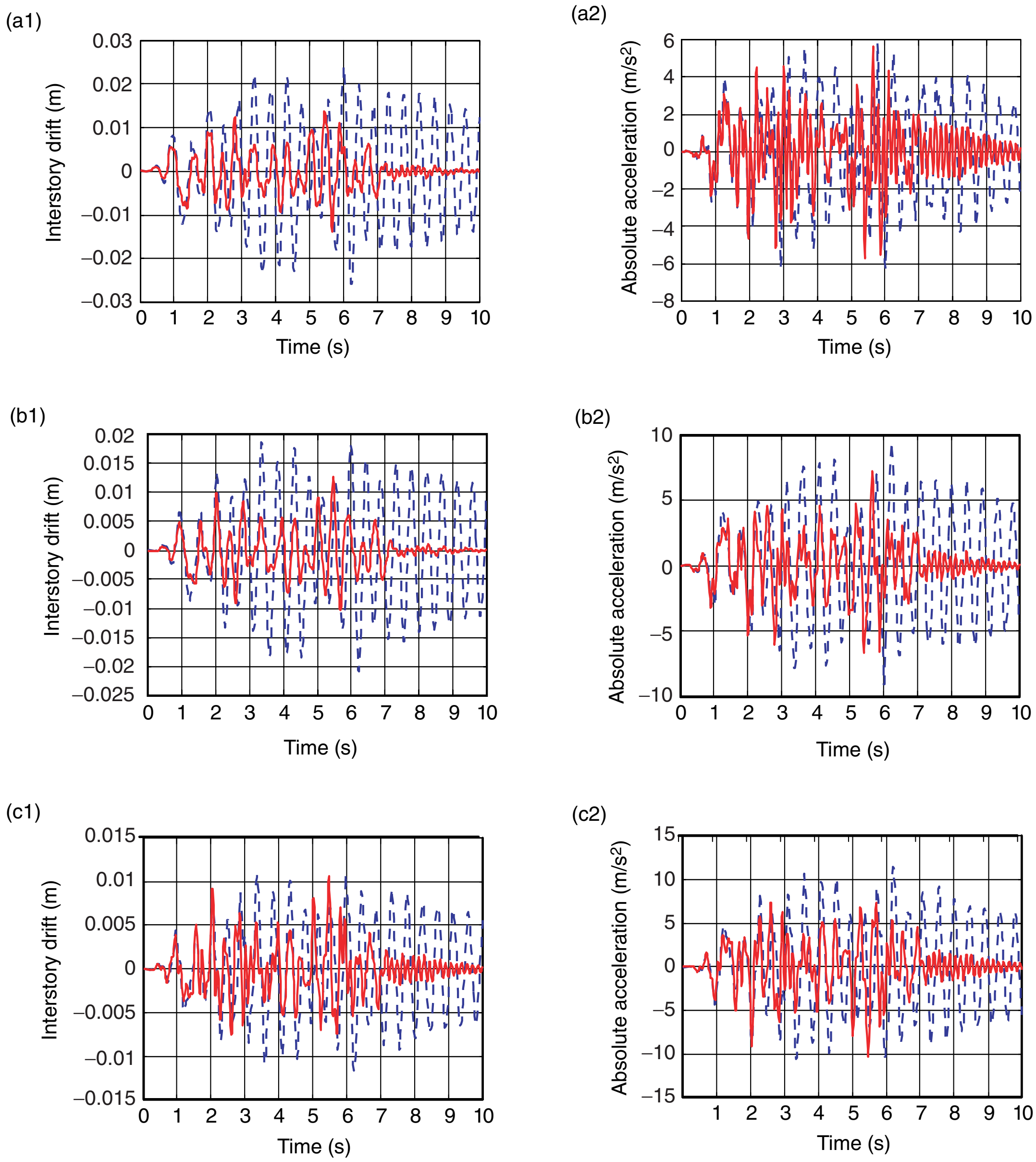

Figure 7. (*Continued*)

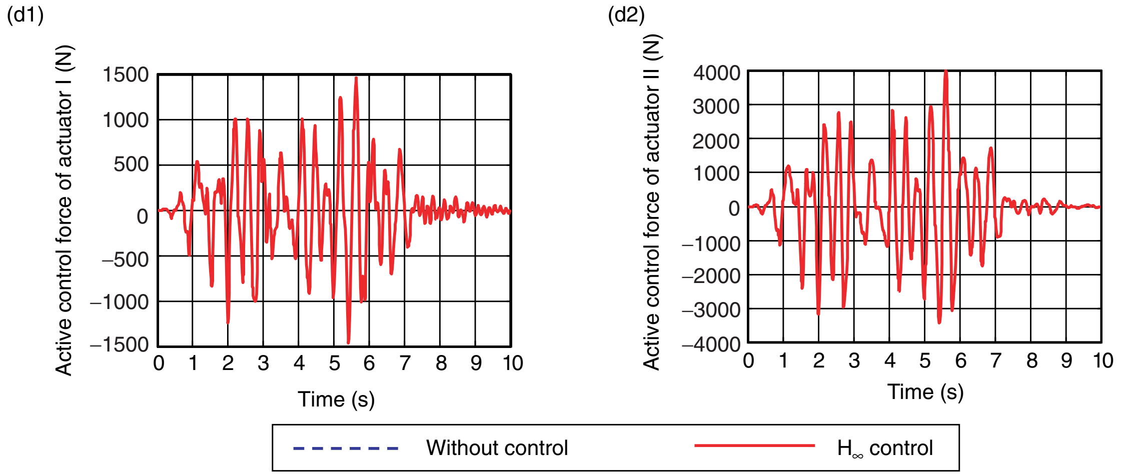

Figure 7. Time histories of system response when using H_∞ control and time delay is $\tau_1 = \tau_2 = 0.05$s : (a) the first story unit; (b) the second story unit; (c) the third story unit; and (d) active control forces of Actuator I and II

Table 5. Maximum interstory drift x_i and maximum absolute acceleration a_i of each story unit when using H_∞ control and time delay is $\tau_1 = \tau_2 = 0.078$s (x_i: m, a_i: m/s^2)

Story	No control		H_∞ control U_{1max} = 2165.96N U_{2max} = 6298.36N	
	x_i	a_i	x_i	a_i
(1)	(2)	(3)	(4)	(5)
1	0.0258	6.29	0.0205	8.04
2	0.0209	9.49	0.0169	10.06
3	0.0117	11.44	0.0135	13.28

story unit when time delay in the control system is chosen as $\tau_1 = \tau_2 = 0.05$s, denoted by the solid line. As observed from Figure 7, the interstory drift and the absolute acceleration of each story unit are evidently reduced. Table 5 shows the maximum interstory drift x_i and the maximum absolute acceleration a_i of each story unit when time delay is chosen as $\tau_1 = \tau_2 = 0.078$s. It is observed from Table 5 that the control effectiveness becomes worse when the time delay is beyond the maximum time delay, since the maximum response quantities of some story units with control are larger than those without control.

The case with time-varying delay is considered herein. In Eqn 4, $\dot{\tau}(t) \leq 0.3$, that is, $\mu = 0.3$. Using the objective function Eqn 9, the adjusting parameters are $\lambda = 0.0829$ and $\rho = 2.7528$, so the biggest time delay is $\bar{\tau} = 0.0863$s and the corresponding feedback gain $[\tilde{\boldsymbol{K}}]$ can be determined as follows

$$[\tilde{\boldsymbol{K}}] = 10^3 \times \begin{bmatrix} 0.0488 & -0.0042 & -0.0017 & -4.4759 & -2.2087 & -0.5053 \\ -0.0379 & -0.0059 & -0.0033 & -9.4956 & -8.3742 & -5.0379 \end{bmatrix}$$

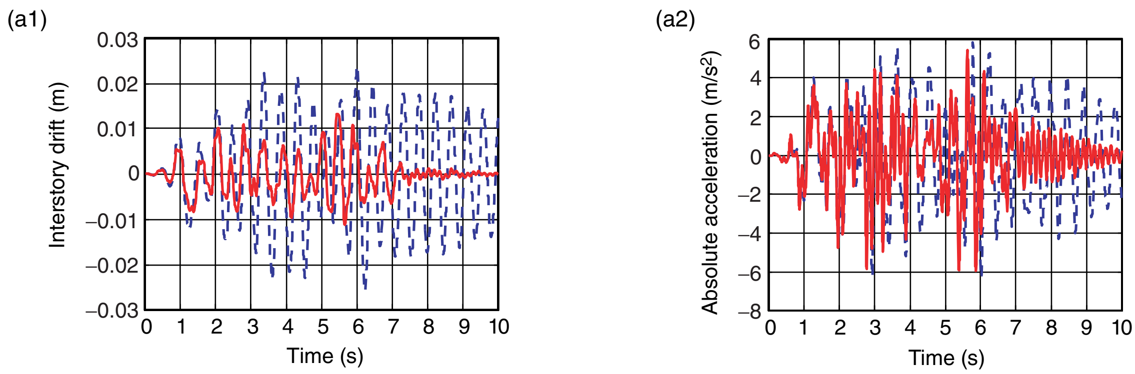

Figure 8. (*Continued*)

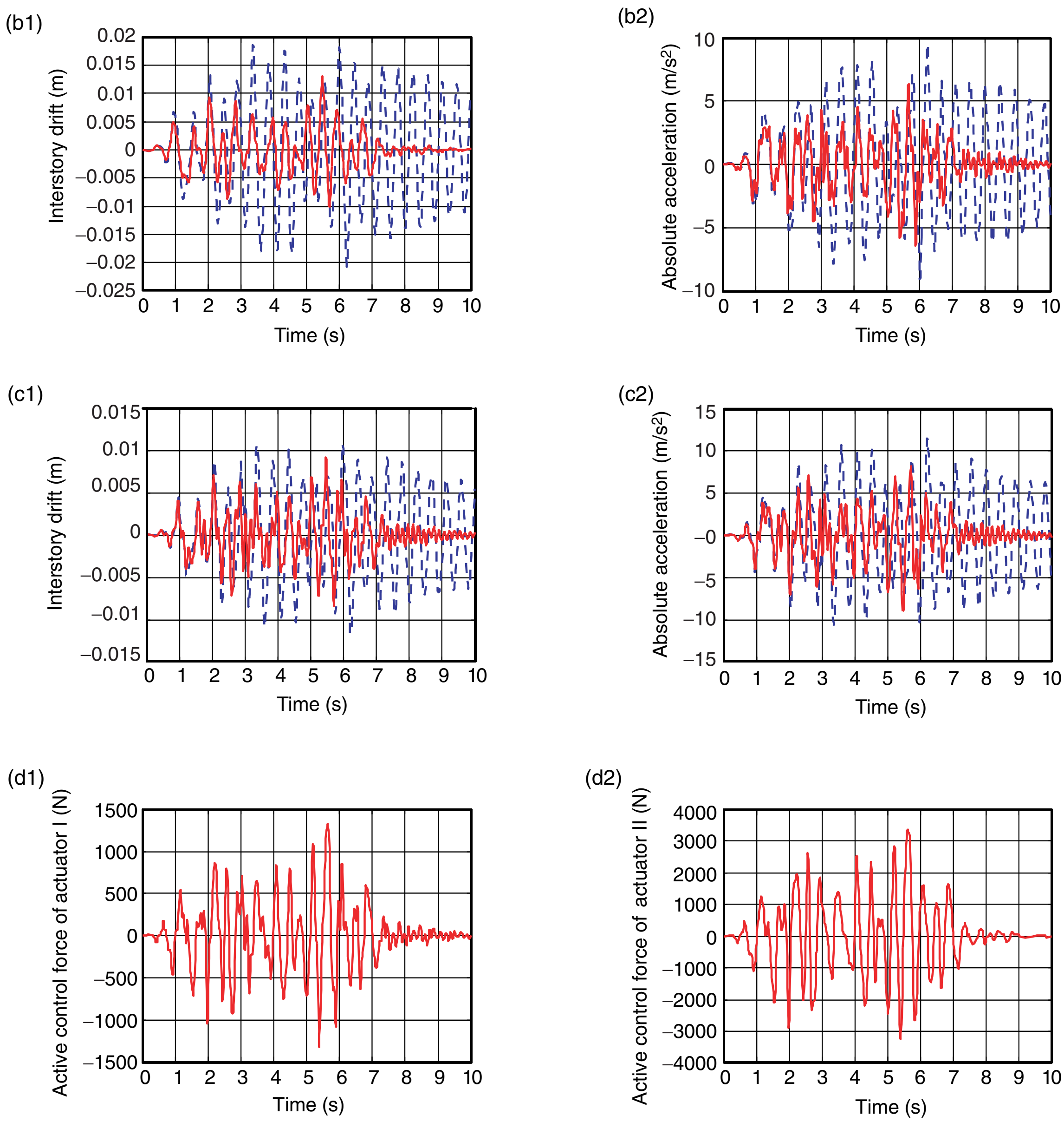

Figure 8. Time histories of system response when using H_∞ control and time-varying delay is τ_1 (t) = τ_2 (t) = $|0.065\sin(4.61t)|$: (a) the first story unit; (b) the second story unit; (c) the third story unit; (d) active control forces of Actuator I and II; and (e) time-varying dela

(14)

Figure 8 shows the results against time of the interstory drift and the absolute acceleration of each story unit when time-varying delay in the control system is chosen as $\tau_1(t) = \tau_2(t) = |0.065\sin(4.61(t)|$, denoted by the solid line. As observed from Figure 8, the LQR control method may achieve good control effectiveness due to the time-varying delay in the control system satisfies Eqn 4. Table 6 shows the maximum interstory drift x_i and the maximum absolute acceleration a_i of each story unit when time-varying delay is chosen as $\tau_1(t) = \tau_2(t) = |0.09$

Table 6. Maximum interstory drift x_i and maximum absolute acceleration a_i of each story unit when using H_∞ control and time-varying delay is $\tau_1(t) = \tau_2(t) = |0.09\sin(3.33t)|$ (x_i : m, a_i : m/s²)

Story	No control		H_∞ control U_{1max} = 1610.99N U_{2max} = 4675.90N	
	x_i	a_i	x_i	a_i
(1)	(2)	(3)	(4)	(5)
1	0.0258	6.29	0.0175	6.43
2	0.0209	9.49	0.0153	8.01
3	0.0117	11.44	0.0121	11.84

$\sin(3.33t)\,|$. It is observed from Table 6 that the control effectiveness becomes worse when the time-varying delay is not satisfied $0 \le \tau(t) \le \bar{\tau}$ in Eqn 4, the maximum response quantities of some story units with control are larger than those without control.

6. CONCLUDING REMARKS

By using the H_∞ control method, this paper studies the active control of a building structure with external disturbance and time-varying delay. A matrix inequality is proposed for stability analysis of the system and a H_∞ controller is designed based on this matrix inequality. Simulation results indicate that the proposed time-delay controller can effectively reduce the responses of the building structure and has strong robustness for the variances of time-varying delay.

ACKNOWLEDGEMENTS

This work is supported by the Natural Science Foundation of China (11132001, 11272202 and 11472171), the Key Scientific Project of Shanghai Municipal Education Commission (14ZZ021), and the Natural Science Foundation of Shanghai (14ZR1421000).

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