Proceedings of the 26th Chinese Control Conference
July 26-31, 2007, Zhangjiajie, Hunan, China

An MPC Approach to Networked Control Design*

Wu Jing[1], Zhang Liqian[2], Chen Tongwen[1]

1. Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4
E-mail: jingwu@ece.ualberta.ca

2. Department of Chemical and Materials Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2G6

Abstract: This paper investigates the problem of model predictive control for a class of networked control systems. Both sensor-to-controller and controller-to-actuator delays are considered and described by Markovian chains. The resulting closed-loop systems are written as jump linear systems with two modes. The control scheme is characterized as a constrained delay-dependent optimization problem of the worst-case quadratic cost over an infinite horizon at each sampling instant. A linear matrix inequality approach for the controller synthesis is developed. It is shown that the proposed state feedback model predictive controller guarantees the stochastic stability of the closed-loop system.

Key Words: Model predictive control (MPC), Networked control systems (NCSs), Linear matrix inequalities (LMIs), Jump linear systems, Stochastic stability

1 INTRODUCTION

With the development of complex industrial systems, communication networks play a more and more important role, by which tremendous amount of information is sensed, processed and transmitted. Such control systems, where control loops are closed through real-time communication networks, are referred to as networked control systems (NCSs). Examples include high-speed paper production, power generation plants, petrochemical processing facilities and so on. The presence of the network brings new functionalities that were not available in the past, such as low cost, reduced system wiring, simple system diagnosis and maintenance, and increased system agility. However, the data exchanged between NCS components are exposed to stochastic or deterministic delays [13, 14], losses [4], and asynchronization [10], which may degrade performance and even cause instability of the feedback control loop. To solve these problems, various methods and many results have been developed. Network-induced delay, as one of the main issues, has been the focus of attention [3, 6, 7, 9]. In [9], the stability analysis and control design of NCSs were studied when the network-induced delay at each sampling instant is random and less than one sampling time. The results in [9] have been extended to the case with longer delays in [3]. The stability of NCSs was also formulated, respectively, by a hybrid system approach with deterministic delays in [14], by a switched system approach with constant controller gain in [7], and by a jump linear system approach with random delays in [6].

Model predictive control has received much attention in the past decades due to its extensive applications in the control of industrial processes such as distillation and oil fractionation, pulp and paper processing, and so on [11]. Its essence is as follows: at every sampling instant, solutions to an optimization problem over a fixed number of future time instants are obtained; only the first optimal control move is implemented as the current control law; at the next sampling time, the measurement is used to update the state estimate and the same procedure is repeated. This feature renders the MPC approach very appropriate to incorporate I/O constraints into the on-line optimization as well as compensate time delays, which increases the possibility of its application in the synthesis and analysis of NCSs [2, 8, 12]. In [2], an MPC strategy for multivariable plants was presented. The sensor-to-controller delays were described by stochastic and deterministic quantities, respectively, but controller-to-actuator delays were assumed to be known and fixed. In [8, 12], modified MPC methods were introduced to compensate the delayed or missing control signals. In [12] both current and future control increment signals were used to update control signals of the plant; an adaptive predictive controller with variable horizon was designed, but no stability was considered. In [8], future control move was chosen from the received control sequences to compensate the delayed control signals; the stability of the system was discussed with the consideration of fixed delay in the sensor-to-controller side and fixed or random delay in the controller-to-actuator side. The goal of this study is to design a predictive control strategy for an NCS such that at each sampling instant the infinite horizon quadratic objective is minimized while guaranteeing the stochastic stability of the closed-loop system. The actuator will implement most recently received signal directly to the plant and only the first control move will be used. The networked communication delays are assumed to be random and bounded, without loss of generality, which are described by Markovian chains. Delay-dependent conditions for the existence of such controllers are given and an LMI approach is developed. A numerical example [5] is given to show the feasibility and efficiency of the proposed method.

2 PROBLEM FORMULATION

Consider the networked control setup in Fig. 1, where the plant is a linear time-invariant discrete-time system, $\tau_k \geqslant 0$ is the random time delay from the sensor to the controller, $d_k \geqslant 0$ is the random time delay from the controller to the actuator, and the controller F_k is to be designed by the MPC method. $v(k)$ is the output of the controller and satisfies

* This research was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

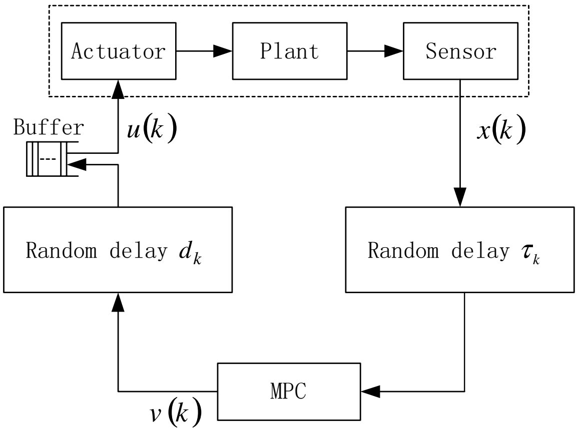

Fig. 1 The setup of the networked control system

$$v(k) = F_k x(k - \tau_k) \tag{1}$$

$u(k)$ is the control input of the plant, which equals to

$$u(k) = v(k - d_k) = F_{k-d_k} x(k - d_k - \tau_{k-d_k}) \tag{2}$$

Let $\mathcal{F}_k = \sigma\{x_0, \tau_0, d_0, \cdots, x_k, \tau_k, d_k\}$ be the σ-algebra generated by $\{(x_l, \tau_l, d_l), 0 \leqslant l \leqslant k\}$. The quadratic objective we consider is

$$J_\infty(k) = \sum_{m=0}^{\infty} \mathcal{E}\left\{x(k+m|k)^{\mathrm{T}} Q x(k+m|k) + u(k+m|k)^{\mathrm{T}} R u(k+m|k) | \mathcal{F}_k\right\} \tag{3}$$

where $Q > 0$, $R \geqslant 0$ are symmetric weighting matrices, $x(k+m|k)$ is the state predicted at time $k+m$ based on the measurements of time k, $x(k|k)$ is the state measured at time k, $u(k+m|k)$ is the corresponding predicted control input at time $k+m$, and $u(k|k)$ is the control move to be implemented at time k. It is assumed that both τ_k and d_k are bounded, that is,

$$\underline{\tau} \leqslant \tau_k \leqslant \overline{\tau}, \ \ \underline{d} \leqslant d_k \leqslant \overline{d}$$

Without loss of generality, we assume that $\underline{\tau} = 0$ and $\underline{d} = 0$. Since current time delays are usually correlated with the previous time delays, it is reasonable to model two random delays τ_k and d_k as two independent homogeneous Markov chains that take values in $\mathcal{M} = \{0, 1, \cdots, \overline{\tau}\}$ and $\mathcal{N} = \{0, 1, \cdots, \overline{d}\}$, and their transition probability matrices are $\Lambda = [\lambda_{ij}]$ and $\Pi = [\pi_{rs}]$ respectively [9, 13]. That is, τ_k and d_k jump from mode i to j and from mode r to s, respectively, with probabilities λ_{ij} and π_{rs}, which are defined by

$$\begin{aligned} \lambda_{ij} &= \Pr(\tau_{k+1} = j \,|\tau_k = i) \\ \pi_{rs} &= \Pr(d_{k+1} = s \,|d_k = r) \end{aligned} \tag{4}$$

where $\lambda_{ij}, \pi_{rs} \geqslant 0$ and

$$\sum_{j=0}^{\overline{\tau}} \lambda_{ij} = 1, \ \ \sum_{s=0}^{\overline{d}} \pi_{rs} = 1 \tag{5}$$

for all $i, j \in \mathcal{M}$ and $r, s \in \mathcal{N}$. Suppose the model of the plant is a linear time-invariant discrete-time model as follows

$$x(k+1) = Ax(k) + Bu(k) \tag{6}$$

and that the exact measurement of the system state is available at each sampling time k, i.e.,

$$x(k|k) = x(k) \tag{7}$$

The controller design scheme can be stated as following: At each sampling time k,

1. measure the state $x(k)$;

2. compute the state-feedback gain F_k in

$$u(k+m|k) = F_{k-d_{k+m|k}} \times x(k+m-\tau_{k+m-d_{k+m|k}} - d_{k+m|k}|k) \tag{8}$$

such that the performance objective in (3) is minimized;

3. implement the first control move $u(k|k)$, that is,

$$u(k) = u(k|k) = F_{k-d_k} x(k - \tau_{k-d_k} - d_k) \tag{9}$$

By the control input given in (9), the resulting closed-loop system can be written as

$$x(k+1) = Ax(k) + BF_{k-d_k} x(k - \tau_{k-d_k} - d_k) \tag{10}$$

With the modelings of τ_{k-d_k} and d_k as two Markov chains, it can be seen that the system in (10) is a Markovian jump linear delay system with two modes, and the time delays are mode-dependent. Furthermore, it can be seen from the 2nd step and the plant model in (6) that the predicted state $x(k+m|k)$ satisfies the following difference equation

$$x(k+m+1|k) = Ax(k+m|k) + BF_{k-d_{k+m|k}} \times x(k+m-\tau_{k+m-d_{k+m|k}|k} - d_{k+m|k}|k) \tag{11}$$

The key to solving the MPC problem is to find a way to solve the optimization problem in step 2 at each sampling time k. In the following, we will give the sufficient conditions for the $\gamma-$suboptimal problem

$$J_\infty(k) < \gamma \tag{12}$$

for a given $\gamma > 0$. Consider the quadratic function which is given by

$$V\left(x(k+m|k), k\right) = \sum_{t=1}^{4} V_t$$

where

$$\begin{aligned} V_1 &= x^{\mathrm{T}}(k+m|k) P\left(\tau_{k+m-d_{k+m|k}|k}, d_{k+m|k}\right) \\ &\quad \times x(k+m|k) \\ V_2 &= \sum_{\theta=\theta_0}^{0} \sum_{l=k+m+\theta-1}^{k+m-1} y^{\mathrm{T}}(l|k) W y(l|k) \\ V_3 &= \sum_{l=k+m-\tau_{k+m-d_{k+m|k}|k}-d_{k+m|k}}^{k+m-1} x^{\mathrm{T}}(l|k) S x(l|k) \\ V_4 &= (1-\underline{\lambda\pi}) \sum_{\theta=-\overline{\tau}-\overline{d}+1}^{-1} \sum_{l=k+m+\theta}^{k+m-1} \left[x^{\mathrm{T}}(l|k) S x(l|k)\right. \\ &\quad \left. + y^{\mathrm{T}}(l|k) W y(l|k)(l-k-m-\theta+1)\right] \\ \theta_0 &= -\tau_{k+m-d_{k+m|k}|k} - d_{k+m|k} + 1 \end{aligned}$$

and $y(k+m|k) = x(k+m+1|k) - x(k+m|k)$. At the sampling time k, suppose that the following inequality holds for all $x(k+m|k)$ and $u(k+m|k)$, $m \geqslant 0$ satisfying (11):

$$\begin{aligned} & \mathcal{E}\{V(x(k+m+1|k),k) - V(x(k+m|k),k)|\mathcal{F}_k\} \\ \leqslant & -\mathcal{E}\left\{x(k+m|k)^{\mathrm{T}} Q x(k+m|k)\right. \\ & \left. + u(k+m|k)^{\mathrm{T}} R u(k+m|k)|\mathcal{F}_k\right\} \end{aligned} \tag{13}$$

For the control performance $J_\infty(k)$ to be finite, we must have $\mathcal{E}\{V(x(\infty|k),k)\} = 0$. Thus, from (13), we obtain

$$-V(x(k|k),k) \leqslant -J_\infty(k) \tag{14}$$

Definition 1 *[1] The system in (10) is stochastically stable if for all finite $x(k) = \varphi$ defined on $[-\overline{\tau}-\overline{d}, 0]$ and initial mode τ_0, d_0, there exists a finite number $\tilde{\Xi}(\varphi, \tau_0, d_0) > 0$ such that*

$$\lim_{N\to\infty} \mathcal{E}\left\{ \sum_{k=0}^{N} \|x(k)\|^2 \middle| \varphi, \tau_0, d_0 \right\} < \tilde{\Xi}(\varphi, \tau_0, d_0) \tag{15}$$

holds, where $\mathcal{E}(\cdot)$ is the statistical expectation operator.

Theorem 1 *Consider the stochastic system in (10) and let $x(k|k), x(k-1|k), \ldots, x(k-\overline{\tau}-\overline{d}|k)$ be the measured state x at time instant k, $k-1, \cdots, k-\overline{\tau}-\overline{d}$ respectively. Then there exists a state feedback controller in (9) such that both (12) and (13) hold if there exist matrices $P(i,r) > 0$, $P_1(i,r)$, $P_2(i,r)$, $Y(i,r)$ $W > 0$, $S > 0$, $Z(i,r)$ and a scalar $\gamma > 0$ such that the following optimization problem holds:*

$$\min \gamma \tag{16}$$

subject to

$$\begin{aligned} & x(k|k)^{\mathrm{T}} P(\tau_{k-r}, r) x(k|k) + \sum_{l=k-\tau_{k-r}-r}^{k-1} x^{\mathrm{T}}(l|k) S x(l|k) \\ & + \sum_{\theta=-\tau_{k-r}-r+1}^{0} \sum_{l=k+\theta-1}^{k-1} y(l|k)^{\mathrm{T}} W y(l|k) \\ & + (1-\underline{\lambda\pi}) \sum_{\theta=-\overline{\tau}-\overline{d}+1}^{-1} \sum_{l=k+\theta}^{k-1} \left[x^{\mathrm{T}}(l|k) S x(l|k)\right. \\ & \left. + y^{\mathrm{T}}(l|k) W y(l|k)(l-k-\theta+1)\right] \leqslant \gamma \end{aligned} \tag{17}$$

and

$$\begin{bmatrix} Z(i,r) & Y(i,r) \\ * & W \end{bmatrix} \geqslant 0 \tag{18}$$

$$\Theta(i,r) = \begin{bmatrix} \Psi + \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix} & \Theta_{12}(i,r) \\ * & \Theta_{22}(i,r) \end{bmatrix} < 0 \tag{19}$$

for any $i \in \mathcal{M}$ and $r \in \mathcal{N}$, where $\underline{\lambda} = \min_i \lambda_{ii}$, $\underline{\pi} = \min_r \pi_{rr}$, $\overline{P}(i,r) = \sum_{j=0}^{\overline{\tau}} \sum_{s=0}^{\overline{d}} \lambda_{ij}\pi_{rs} P(j,s)$ and

$$\begin{aligned} \Theta_{12}(i,r) &= G^{\mathrm{T}}(i,r) \begin{bmatrix} 0 \\ BF_{k-r} \end{bmatrix} - Y(i,r) \\ \Theta_{22}(i,r) &= -S + F_{k-r}^{\mathrm{T}} B^{\mathrm{T}} R B F_{k-r} \\ \rho &= [1 + (1-\underline{\lambda\pi})(\overline{\tau}+\overline{d})] \\ \mu(i,r) &= \sum_{j=0}^{\overline{\tau}} \sum_{s=0}^{\overline{d}} \lambda_{ij}\pi_{rs}(j+s) + \\ &\quad (1-\underline{\lambda\pi}) \frac{(\overline{\tau}+\overline{d}-1)(\overline{\tau}+\overline{d})}{2} \end{aligned}$$

$$\begin{aligned} \Psi = & \begin{bmatrix} \overline{P}(i,r) - P(i,r) + \rho S & 0 \\ 0 & \overline{P}(i,r) + \mu(i,r) W \end{bmatrix} \\ & + G^{\mathrm{T}}(i,r) \begin{bmatrix} 0 & I \\ A-I & -I \end{bmatrix} + \begin{bmatrix} Y(i,r) & 0 \end{bmatrix} \\ & + \begin{bmatrix} 0 & I \\ A-I & -I \end{bmatrix}^{\mathrm{T}} G(i,r) + \begin{bmatrix} Y^{\mathrm{T}}(i,r) \\ 0 \end{bmatrix} \\ & + (i+r) Z(i,r) \end{aligned}$$

Proof Omitted here for the length limitation of the paper.

From (17)∼(19), it can be observed that when $r > 0$, F_{k-r} is known, and thus, (17)∼(19) are LMIs. However, when $r = 0$, since F_k is unknown, inequality (19) is not linear with unknown variables. In the following theorem, we will give an equivalent LMI condition for (19) for any $i \in \mathcal{M}$ and $r \in \mathcal{N}$.

Theorem 2 *The matrix inequalities in (18)-(19) are equivalent to the following LMIs:*

$$\begin{bmatrix} \tilde{Z}_1(i,r) & \tilde{Z}_2(i,r) & 0 \\ * & \tilde{Z}_3(i,r) & \delta(i,r)\widetilde{W} \\ * & * & \widetilde{W} \end{bmatrix} \geqslant 0 \tag{20}$$

$$\begin{bmatrix} \Gamma_{11} & * \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} < 0 \tag{21}$$

for any $i \in \mathcal{M}$ and $r \in \mathcal{N}$, where

$$\Gamma_{11} = \begin{bmatrix} \Theta_{11} & * & * \\ \Theta_{12}^{\mathrm{T}} & \Theta_{22} & * \\ 0 & [BU - \delta(i,r)\widetilde{S}]^{\mathrm{T}} & -\widetilde{S} \end{bmatrix}$$

$$\Gamma_{21} = \begin{bmatrix} \Gamma_{211} & TX_2(i,r) & 0 \\ X_1(i,r) & X_2(i,r) & 0 \\ 0 & 0 & BU \\ X(i,r) & 0 & 0 \\ X(i,r) & 0 & 0 \end{bmatrix}$$

$$\Gamma_{22} = \mathrm{diag}\begin{bmatrix} \Lambda & -\frac{1}{\mu(i,r)}\widetilde{W} & -\widetilde{R} & -\widetilde{Q} & -\frac{1}{\rho}\widetilde{S} \end{bmatrix}$$

$$\Gamma_{211} = T^{\mathrm{T}}[X(i,r) + X_1(i,r)]$$

$$\Theta_{11} = -X(i,r) + (i+r)\tilde{Z}_1(i,r)$$

$$\Theta_{12} = -X_1(i,r) + (i+r)\tilde{Z}_2(i,r)$$

$$+X(i,r)(A - I + \delta(i,r)I)^{\mathrm{T}}$$

$$\Theta_{22} = -X_2(i,r) - X_2^{\mathrm{T}}(i,r) + (i+r)\tilde{Z}_3(i,r)$$

$$U = F_{k-r}\widetilde{S}, \quad \widetilde{S} = S^{-1}, \quad \widetilde{W} = W^{-1}$$

$$\widetilde{R} = R^{-1}, \quad \widetilde{Q} = Q^{-1}$$

$$T = \begin{bmatrix} \sqrt{\lambda_{i0}\pi_{r0}}I & \cdots & \sqrt{\lambda_{i\overline{\tau}}\pi_{r\overline{d}}}I \end{bmatrix}$$

$$\Lambda = \mathrm{diag}\begin{bmatrix} -X(0,0) & \cdots & -X(\overline{\tau},\overline{d}) \end{bmatrix}$$

$$Z(i,r) = \begin{bmatrix} Z_1(i,r) & Z_2(i,r) \\ Z_2^{\mathrm{T}}(i,r) & Z_3(i,r) \end{bmatrix}$$

$$\tilde{Z}(i,r) = \tilde{G}^{\mathrm{T}}(i,r)Z(i,r)G(i,r) = \begin{bmatrix} \tilde{Z}_1(i,r) & \tilde{Z}_2(i,r) \\ \tilde{Z}_2^{\mathrm{T}}(i,r) & \tilde{Z}_3(i,r) \end{bmatrix}$$

Moreover, if (17), (20) and (21) are feasible, then

$$F_k = U\widetilde{S}^{-1}$$

3 FEASIBILITY ANALYSIS

From now on, the variables in Theorem 1 will be denoted as $P^k(i,r) > 0$, $P_1^k(i,r)$, $P_2^k(i,r)$, $Y^k(i,r)$, $W^k > 0$, $S^k > 0$, $Z^k(i,r)$ to show that they are computed at time k. For simplicity, throughout the rest of this paper, denote $\tau_{k|k} = \tau_k$, $\tau_{k+1|k+1} = \tau_{k+1}$, $d_{k|k} = d_k$, and $d_{k+1|k+1} = d_{k+1}$.

Theorem 3 *If the matrix inequalities in (17)∼(19) are feasible at time k, then they are also feasible for all time $t > k$.*

Proof Let us assume that the optimization problem in Theorem 1 is feasible at the sampling time k. The only LMI in the problem that depends explicitly on the measured state $x(k|k) = x(k)$ of the system is the constraint in (17). Inequalities (18) and (19) can be easily proved by setting the decision variables at time $k+1$ equal to be the optimal values computed at time k since the parameters are independent of k. Thus, to prove this theorem, we need only to prove that inequality (17) is feasible for all the future measured states $x(k+m|k+m) = x(k+m)$, $m \geqslant 1$. We will show that the theorem holds at the sampling time $k+1$ first.
Since Q and R in (13) are positive definite matrices, we have

$$\mathcal{E}\{V(x(k+m+1|k),k) - V(x(k+m|k),k)|\mathcal{F}_k\} < 0$$

Iteratively applying this inequality for $m = 0, 1, \cdots$, we obtain

$$\mathcal{E}\{V(x(k+m+1|k),k)|\mathcal{F}_k\} < V(x(k|k),k) \quad (22)$$

if $x(k+m|k) \neq 0$. Since $V(x(k|k),k) \leqslant \gamma$, $\mathcal{E}\{V(x(k+1|k),k)|\mathcal{F}_k\} < \gamma$. From equations (7) and (11), it follows that

$$x(k+1|k) = Ax(k) + BF_{k-d_k}x(k - \tau_{k-d_k} - d_k)$$

As a result of (10), we obtain that

$$x(k+1|k) = x(k+1|k+1)$$

Notice that $P^{k+1}(\tau_{k+1-d_{k+1}}, d_{k+1})$, W^{k+1} and S^{k+1} are the optimal solutions to the problem in (16) with the constraints (17)-(19) at time $k+1$. On the other hand, at time $k+1$, $P^k(\tau_{k+1-d_{k+1|k}|k}, d_{k+1|k})$, W^k and S^k are one set of feasible solutions. Therefore, we have

$$\begin{aligned} & \mathcal{E}\{V(x(k+1|k+1),k+1)|\mathcal{F}_k\} \\ < \; & \mathcal{E}\{V(x(k+1|k),k)|\mathcal{F}_k\} < \gamma \end{aligned}$$

Hence the optimization problem is feasible at time $k+1$. This argument can be continued for times $k+2, k+3, \cdots$, to complete the proof.

4 STABILITY ANALYSIS

Now, we are in a position to give our main result on the stability of the model predictive control problem.

Theorem 4 *Let $x(k|k)$ be the measurement state $x(k)$ at the sampling instant k, and suppose that the optimization problem in Theorem 1 is feasible. Then the closed-loop system (10) is stochastically stable by the feasible receding horizon state feedback control law in (9), which is obtained from Theorem 1.*

Proof To show the stochastic stability of (10), we shall show that $V(x(k|k),k)$ is a strictly decreasing Lyapunov function. Assume that the optimization problems established in Theorem 1 are feasible for time instant $k = 0$. Theorem 3 then ensures that these optimization problems are also feasible for all $k > 0$. Thus, we have

$$V(x(k+1|k+1),k+1) \leqslant V(x(k+1|k+1),k) \quad (23)$$

This is because $P^{k+1}(i,r) > 0$, $P_1^{k+1}(i,r)$, $P_2^{k+1}(i,r)$, $Y^{k+1}(i,r)$, $W^{k+1} > 0$, $S^{k+1} > 0$, and $Z^{k+1}(i,r)$ are optimal values at time $k+1$, while $P^k(i,r) > 0$, $P_1^k(i,r)$, $P_2^k(i,r)$, $Y^k(i,r)$, $W^k > 0$, $S^k > 0$ and $Z^k(i,r)$, are only feasible at time $k+1$. By (13), we have

$$\begin{aligned} & \mathcal{E}\{V(x(k+1|k),k)|\mathcal{F}_k\} - V(x(k|k),k) \\ \leqslant \; & -\mathcal{E}\{\beta^{\mathrm{T}}(k|k)\Xi\beta(k|k)|\mathcal{F}_k\} \end{aligned} \quad (24)$$

with $m = 0$, $\Xi = \mathrm{diag}\{Q,\ F_{k-d_k}^{\mathrm{T}}RF_{k-d_k}\}$ and $\beta(k|k) = [x^{\mathrm{T}}(k)\ x^{\mathrm{T}}(k - \tau_{k-d_k} - d_k)]^{\mathrm{T}}$. Since the measured state $x(k+1|k+1) = x(k+1)$ equals $Ax(k) + BF_{k-d_k}x(k - \tau_{k-d_k} - d_k)$, $x(k+1|k) = x(k+1|k+1)$. Combing the inequalities (23) and (24), we can see that for any $N \geqslant 1$,

$$\begin{aligned} & \mathcal{E}\{V(x(N|N),N)|\varphi,\tau_0,d_0\} - \mathcal{E}\{V(x(0|0),0)|\varphi,\tau_0,d_0\} \\ & \leqslant -\epsilon\,\mathcal{E}\{\sum_{k=0}^{N}\|x(k|k)\|^2|\varphi,\tau_0,d_0\} \end{aligned}$$

if letting $\epsilon = \inf\{\lambda_{\min}\{\Xi\}\} > 0$ (since $Q > 0$ and $R \geqslant 0$), which implies that

$$\lim_{N\to\infty} \mathcal{E}\{\sum_{k=0}^{N} \|x(k)\|^2 | \varphi, \tau_0, d_0\}$$
$$\leqslant \frac{1}{\epsilon}\mathcal{E}\{V(x(0))|\varphi, \tau_0, d_0\} = \tilde{\Xi}(\varphi, \tau_0, d_0) \quad (25)$$

From Definition 1, the stochastic stability is obtained.

5 NUMERICAL EXAMPLE

Consider a classical angular positioning system in [5]. The control target is using the motor to rotate the antenna so that it always points to the direction of a moving object in the plane. Assume that the angular position of the antenna θ (rad), the angular position of the moving object θ_r (rad) and the angular velocity of the antenna $\dot{\theta}$ (rad $\cdot\ s^{-1}$) are measurable. The motion of the antenna can be described by the following discrete-time counterparts by discretization, using a sampling time of 0.1s and the Euler's first-order approximation for the derivative

$$x(k+1) = \begin{bmatrix} \theta(k+1) \\ \dot{\theta}(k+1) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0.1 \\ 0 & 1-0.1\alpha(k) \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.1\kappa \end{bmatrix} u(k) \quad (26)$$

where $\kappa = 0.787$ rad$^{-1}V^{-1}s^{-2}$ and $0.1s^{-1} \leqslant \alpha(k) \leqslant 10s^{-1}$. The parameter $\alpha(k)$ is proportional to the coefficient of viscous friction in the rotating parts of the antenna. Assume that $\alpha(k) = 0.1$, the initially state $x(k) = \begin{bmatrix} 0.01 & 0 \end{bmatrix}^{\mathrm{T}}$, $Q = I_{2\times 2}$ and $R = 0.001$. Then the system dynamics is shown in Fig. 2,

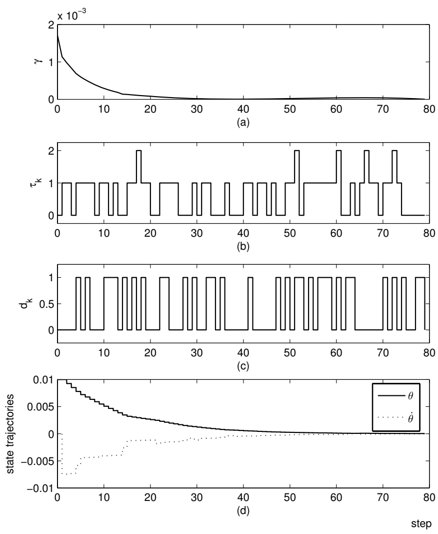

Fig. 2 The angular positioning system

where Fig. 2 (a) is the upper bound of the quadratic function in (3). Fig. 2 (b) and (c) give the changes of network-induced delays. $\tau_k = i,\ i \in \{0,\ 1,\ 2\}$ means data are delayed by iT_s on the sensor-to-controller side at time k during the transmission, where T_s is the sampling time. Similarly, $d_k = r,\ r \in \{0,\ 1\}$ means that data are delayed by rT_s on the controller-to-actuator side. By these two random serials, we assume their transition probability matrices as

$$\Lambda = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.1 \\ 0.3 & 0.2 & 0.5 \end{bmatrix},\ \Pi = \begin{bmatrix} 0.5 & 0.5 \\ 0.8 & 0.2 \end{bmatrix}$$

Fig. 2 (d) is the state trajectories, which show that the closed-loop system is stable by the controller we designed.

6 CONCLUSION

In this paper, we have studied the problem of model predictive control for networked control systems. Based on the minimization of an upper bound of the worst-case infinite horizon quadratic cost function at each sampling instant, a state feedback model predictive controller has been proposed by using the LMI approach. Only first control move is implemented to the plant. The stochastic delay-dependent stability conditions have been presented respectively for the closed-loop system resulting from the proposed controller. The numerical example shows the effectiveness of our method.

REFERENCES

[1] Chen W H, Guan Z H, Yu P. Delay-dependent stability and H_∞ control of uncertain discrete-time Markovian jump systems with mode-dependent time delays[J]. Systems & Control Letters, 2004, 52(5): 361-376.

[2] Goodwin G C, Haimovich H, Quevedo D E, Welsh J S. A moving horizon approach to networked control system design[J]. IEEE Trans. on Automatic Control, 2004, 49(9): 1427-1445.

[3] Hu S, Zhu W. Stochastic optimal control and analysis of stability of networked control systems with long delay[J]. Automatica, 2003, 39: 1877-1884.

[4] Jin Z P, Gupta V, Murray R M. State estimation over packet dropping networks using multiple description coding[J]. Automatica, 2006, 42(9): 1441-1452.

[5] Kothare M V, Balakrishnan V, Morari M. Robust constrained model predictive control using linear matrix inequalities[J]. Automatica, 1996, 32(10): 1361-1379.

[6] Krtolica R, Ozguner U, Chan H, Winkelman J, Liubakka M. Stability of linear feedback systems with random communication delays[J]. Int. J. Control, 1994, 59(4): 925-953.

[7] Lin H, Zhai G, Antsaklis P J. Robust stability and disturbance attenuation analysis of a class of networked control systems[C]. Proceedings of the 42nd IEEE Conference on Decision and Control, 2003: 1182-1187.

[8] Liu G P, Mu J X, Rees D, Chai S C. Design and stability analysis of networked control systems with random communication time delay using the modified mpc[J]. Int. J. Control, 2006, 79(4): 288-297.

[9] Nilsson J, Bernhardsson B, Wittenmark B. Stochastic analysis and control of real-time systems with random time delays[J]. Automatica, 1998, 34(1): 57-64.

[10] Rabello A, Bhaya A. Stability of asynchronous dynamical systems with rate constraints and application[J]. IEE Proc. on Control Theory and Applications, 2003, 150(5): 546-550.

[11] Richalet J. Industrial applications of model based predictive control[J]. Automatica, 1993, 29(5): 1251-1274.

[12] Tang P L, de Silva C W. Compensation for transmission delays in an ethernet-based control network using variable-horizon predictive control[J]. IEEE Trans. on Control Systems Technology, 2006, 14(4): 707-718.

[13] Zhang L, Shi Y, Chen T, Huang B. A new method for stabilization of networked control systems with random delays[J]. IEEE Trans. on Automatic Control, 2005, 50(8): 1177-1181.

[14] Zhang W, Branicky M S, Phillips S M. Stability of networked control systems[J]. IEEE Control Systems Magazine, 2001, 21(2): 84-99.