

# Feedback linearization control for a wind turbine driven by a variable-speed pump-controlled hydraulic servo system

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**Abstract**— The study aims to develop a feedback linearization controller applying to a novel pitch control system for a large wind turbine driven by a variable-speed pump-controlled hydraulic servo system. The variable-speed pump-controlled hydraulic servo system is composed of an AC servo motor, a constant displacement hydraulic piston pump two differential hydraulic cylinders and hydraulic circuits. Finally, the mathematic model is verified by the experimental test rig and the tracking performance in both 5<sup>th</sup> order polynomial and sinusoidal path are implemented by the proposed feedback linearization controller.

## I. INTRODUCTION

Wind energy is recognized as one of the most important renewable energies due to the increasing consumption of fossil energy. Therefore, the applications of modern wind energy have not only expanded in recent year, but also been considered as a substitution of nuclear energy in western counties such as Denmark and Germany. Besides, the development of wind farm in United Kingdom emphasizes the importance of large wind turbines (2MW).

Modern large wind turbines can be divided into three different types depending on its working design: constant speed, variable pitch control, and variable speed type. The relevant researches have been published since 1980s. A simple strategy with stall control and fixed blade was developed in 1988[1]. Freeman and Balas [2] made a system identification of the dynamic models of wind turbine with experiments. In 2000, Song et al. [3] testified the variable pitch control and variable speed wind speed wind turbine by the nonlinear and adaptive control.

For the Asian countries, which usually suffer from Typhoons and turbulence wind, pitch control type is more appropriate than others. By controlling the variable pitch angle, the rotational speed of wind turbine can be kept constant to produce rated electric power. Furthermore, the reduction of aerodynamics can prevent the rotor from shut down due to the change of blade angle. The pitch driving system of wind turbines can be classified into electrical motor driving and hydraulic driving systems. Comparing with electrical motor driving system, hydraulic driving system can not only avoid the serious erosion from gears but also enhance the robustness

due to the cylinder driven mechanism. However, a low efficiency and the high nonlinearity problems should not be neglected in hydraulic devices. Helduser[4] developed an electric-hydrostatic driven system with an AC servo motor and a constant displacement internal gear pump for saving power and motion control system. Habibi et al.[5] discussed the differences of the electro-hydraulic actuator by using gear pump and electromotor. Helbig[6] achieved a high efficiency and response characteristics in both velocity and pressure control for injection moulding machine. Therefore, we figured up a new design of hydraulic pump-controlled servo system with an AC servo motor and a constant displacement hydraulic pump, which can reach high response and energy efficiency.

In order to solve the problem that we have mentioned before, a controller should be designed as compensation. There are several control theories that have been used in hydraulic system. For instance, Kazmeier and Feldmann[7] used fuzzy control applying to the pump-controlled system with variable rotational speed for positioning control and M.H. Chiang [8] developed an adaptive fuzzy controller with self-tuning fuzzy sliding-mode compensation (AFC-STFSMC) controller to enhance the position control performance. However, a model-based controller has rarely been applied to hydraulic system especially in wind turbine system. Therefore, this study decided to develop a feedback linearization controller to solve this problem.

Linearization is one of the most fundamental and effective methods in nonlinear systems. It has played an important role in nonlinear control system theory.[9] The main concept of feedback linearization problem is to transform the nonlinear system into a linear system by designing an appropriate control signal based on system model and tracking target. Brockett [10] has verified and solved the single input case ( $m = 1$ ) while the general case ( $m \geq 1$ ) of feedback linearization problem has been solved by Jakubczyk and Respondek[11]. Subsequently, this theory was applied to a hydraulic servo system by J.H. Kwon [12] and electrohydraulic systems by Q.H. Nguyen[13]

From the above of literature survey, this study decided to build up a hydraulic servo driving system of pitch control and also compensated with a controller by using feedback linearization theory. The system model derivation would be shown in section II. Meanwhile, section III expressed the controller design. Subsequently, the system verification and simulation result could be recognized in section IV and section V.

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## II. SYSTEMATIC MODEL DERIVATION

### A. Novel variable pitch control system

The novel variable pitch control system built in this study can be shown as Fig. 1. This system is composed of two differential cylinders driven by a variable-speed hydraulic pump-controlled system. More precisely, the two cylinders are supposed to be installed at the root of blade and the pump-controlled system consists of an AC servo motor and one constant displacement pump. Due to the design of closed hydraulic pump controlled circuits, the inlet and outlet volume flow of pump must remain the same. By connecting the oil hoses from the pump to the two opposite sides of two cylinders which will cause the same equivalent piston area in both inlet and outlet of pump side. Furthermore, the directions of torque generated by these two cylinders to drive the equivalent mass system are correspondent. The designed circuits of this pitch control system are illustrated in Fig. 1.

Since the overall system of a full-scale 2 MW wind turbine is difficult to achieve experimentally in laboratory, we settle a prototype experimental test rig layout and the specifications of main components are listed in Table 1.

Figure 1. Novel Variable-speed hydraulic pump-controlled system

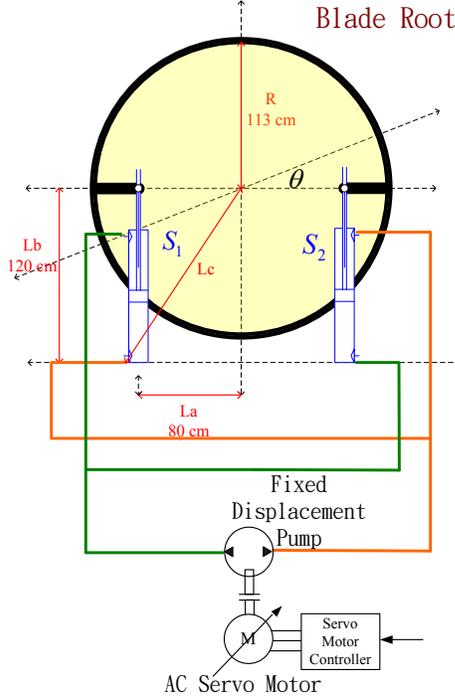


TABLE I. SPECIFICATION OF TEST RIG

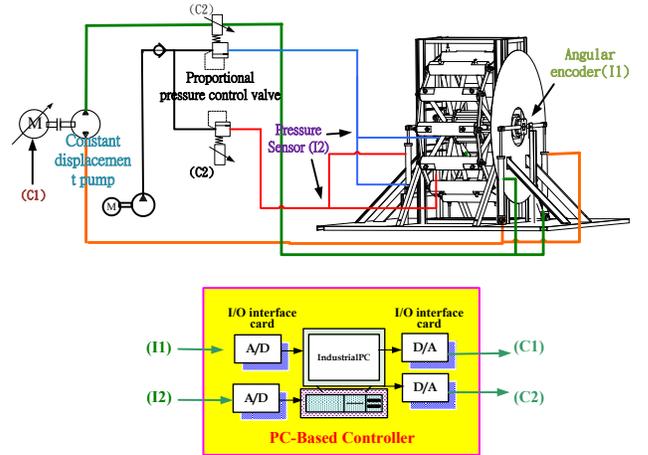
Components	Specification
AC servo motor	Input: 3w 220 V Rated rotational speed: 2000 rpm Rated output: 7.0 kW
Hydraulic pump	Swash plate axial piston pump Max. pressure: 40 MPa Fixed displacement: 12 ml/rev Max. rotational speed: 8000 rpm
Hydraulic servo cylinder	Single rod double acting cylinder Max. pressure: 21 MPa Max. stroke: 810 mm

Components	Specification
	Piston diameter: 50 mm Rod diameter: 25 mm
Optical encoder	Resolution: 20000 ppr Decoder: NI-6601
A/D D/A cards	12 bit A/D 16 CH 12 bit D/A 6 CH
Rotor mass system	Weight: 2050 kgf Moment of inertia: 1238 kg m <sup>2</sup>

### B. Mathematic Model Derivation

In correspondence with the test rig in Fig. 2, it is necessary to divide the system into different main components. The mathematic models should be derived separately to apply to the controller design

Figure 2. Design circuit of test rig



- AC servo motor

The servo motor proportionally transfers the input voltage into the rotational speed, and can be described as

$$\omega = K_a \cdot u \quad (1)$$

where  $\omega$  represents the rotational speed of AC servo motor;  $K_a$  indicates the gain of control unit and;  $u$  is the input voltage.

- Constant displacement piston pump

The volume flow  $Q_l$  loaded from pump to the hydraulic cylinder is the difference of outlet volume flow and pump-side leakage.

$$Q_l = D_l \omega - C_l P_l \quad (2)$$

where  $C_l$  is the leakage coefficient of the pump;  $P_l$  indicates the load pressure;  $D_l$  denotes the volumetric displacement of the axial piston pump.

- The piston cylinder system

The two piston cylinders are combined into a closed loop circuit. Both of the inlet and outlet sides of pump are connected with a larger chamber in one and a smaller chamber in the other one. Therefore, these two differential cylinders can be regarded as an equivalent symmetrical cylinder with the same chamber area therefore the continuity equation can be simplified without taking area effects into considerations for an unsymmetrical cylinder.

$$Q_l = D_l \omega - C_l P_l = A_e \dot{x} + C_l P_l + \left(\frac{V}{4\beta}\right) \dot{P}_l \quad (3)$$

$A_e$  is equivalent area of cylinder chamber;  $C_l$  is cylinder leakage coefficient;  $V$  is internal volume;  $\beta$  is considered as oil bulk modulus.

- Dynamic equation

In accordance with Newton's 2nd law, the motion equation of controlled cylinder can be derived as a general form in a rotational system

$$\sum T = A_e P \times R_l \sin \phi_2 = J \ddot{\theta} + D \dot{\theta} + C \theta \quad (4)$$

where  $J$  indicates the total Moment of inertia;  $D$  is the viscous damping coefficient;  $C$  is the spring constant and  $\sum T$  is summation of loading torque including the arbitrary torque on the rotor.

However, the Eq. (4) is a nonlinear equation which cannot be neglected. In more specific, the attacking angle of cylinder stroke and lever arm is not always perpendicular. Which depends on the rotational angle  $\theta$ . By using Trigonometric function, the equivalent equations of loading torque can be derived as follows (Eq. (5)~(8)) and the analytic figure is expressed in Fig. 3

$$L_c^2 = L_a^2 + L_b^2 \quad (5)$$

$$\phi_1 = \arctan(L_b/L_a) \quad (6)$$

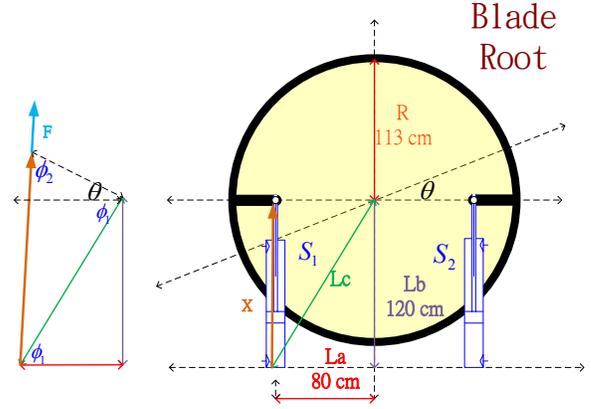
$$X^2 = L_a^2 + L_c^2 - 2L_a L_c \cos(\phi_1 + \theta) \quad (7)$$

$$\frac{X}{\sin(\phi_1 + \theta)} = \frac{L_c}{\sin(\phi_2)} \quad (8)$$

With Eq. (5)-(8), Eq.(4) can be transformed as Eq. (9)

$$\sum T = A_e P \times R_l \sin \phi_2 = J \ddot{\theta} + D \dot{\theta} + C \theta \quad (9)$$

Figure 3. Mechanical analysis of pitch control system



After combining with Eq. (1)~(9), the equivalent system equations of the variable rotational speed hydraulic pump-controlled system can be achieved as follows:

$$\omega = Gu$$

$$D_l \omega - C_l P_l = A_e \dot{X} + C_l P_l + \left(\frac{V}{4\beta}\right) \dot{P}_l \quad (10)$$

$$A_e P \times R_l \sin \phi_2 = J \ddot{\theta} + D \dot{\theta} + C \theta$$

Subsequently, we change Eq. (10) into state equations by setting state variable  $x$  to replace the system variable  $\theta$  and internal pressure  $p_l$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{D}{J} x_2 - \frac{C}{J} x_1 + \frac{A_e R}{J} \sin \phi_2 \cdot x_3 \quad (11)$$

$$\dot{x}_3 = \frac{-4\beta A_e}{V} \dot{X} - \frac{4\beta C_e}{V} x_3 + \frac{4\beta}{V} D_e u$$

Note that the specification of Eq. (11) is in TABLE II

TABLE II. SPECIFICATION OF EQ. (11)

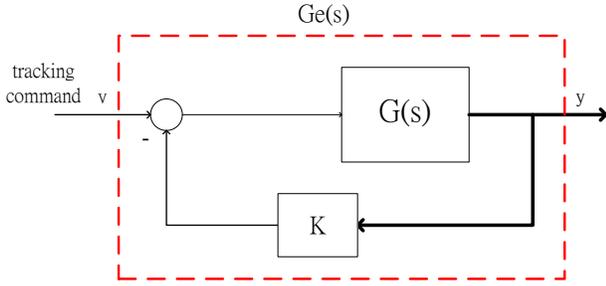
specification of Eq. (11)	
$x_1$	$\theta$
$x_2$	$\dot{\theta}$
$x_3$	$p_l$
$D_e$	$G \times D_l$
$C_e$	$C_t + C_l$
$\dot{X}$	$\frac{1}{\sqrt{R^2 + L_c^2 - 2L_a L_c \cos(\phi_1 + x_1)}} L_a L_c \sin(\phi_1 + x_1) \dot{x}_1$

### III. CONTROLLER DESIGN

To implement the novel variable pitch control system, the pitch controller design is necessary. A feedback linearization controller is developed in this study for realizing the path control of the pitch angle in the novel variable pitch control system. The basic conception of feedback linearization theory can be attributed to its literally meaning. The system can be

transformed from nonlinear to linear system due to a well-designed feedback controller. Fig. 4 shows the block diagram of feedback linearization design. The equivalent plant  $G_e(s)$  can be recognized as a linearized system plant.

Figure 4. Black diagram of feedback linearization control



To design a feedback linearization controller, the relative degree of system should be calculated. With the state equation of Eq. (11), we can know that the system external state equals to  $x_1$ . The relative degree can be calculated ( $r=0$ ) by differentiating the external state, shown in Eq. (12)

$$\begin{aligned}
 y &= x_1 \\
 \dot{y} &= \dot{x}_1 = x_2 \\
 \ddot{y} &= \dot{x}_2 = \frac{-D}{J}x_2 - \frac{C}{J}x_1 + \frac{A_e R}{J} \cos \frac{x_1}{2} \cdot x_3 \\
 \ddot{y} &= \ddot{x}_2 = \frac{-D}{J} \left( \frac{-D}{J}x_2 - \frac{C}{J}x_1 + \frac{A_e R}{J} \sin \phi_2 \cdot x_3 \right) - \frac{C}{J}x_2 \\
 &+ \frac{A_e R L_c}{J} [\cos(\phi_1 + x_1) \cdot x_2 - \sin(\phi_1 + x_1) \cdot \dot{X}] \cdot \frac{x_3}{X^2} \\
 &+ \frac{A_e R}{J} \frac{4\beta}{V} \sin \phi_2 (A \cdot \dot{X} - C_e x_3) + \frac{A_e R}{J} \sin \phi_2 \frac{4\beta}{V} D_e u
 \end{aligned} \quad (12)$$

Comparing with state Eq. (11) and Eq. (12), both of them are 3 order state equations. Which indicates the internal state is a zero order equation. Therefore we can determine the state space equations of external state in Eq. (13)

$$\begin{aligned}
 \dot{Y} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ddot{y} \\
 y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}
 \end{aligned} \quad (13)$$

Due to the third derivation form of Eq. (12) we can simplify the expression of external equation to Eq. (14)

$$\ddot{y} = \alpha(\mathbf{x})u + \beta(\mathbf{x}) \quad (14)$$

In Eq. (14) the control signal  $u$  can be easily designed. However, with the subject of this study, position control is another important target. The Eq. (14) should be changed to Eq. (15) by adding a tracking factor.

$$u = \frac{1}{\alpha(x)} (-\beta(x) + v + \ddot{y}) \quad (15)$$

Where  $v$  is indicated as a tracking command as Fig. 5 and which has to be properly specified. Therefore, we design it with a tracking error factor  $E$  and a state feedback gain  $K_r$  calculated from the eigenvalues of Eq. (13) to match the desired poles.

$$u = \frac{1}{\alpha} (-\beta + \mathbf{K}_r \mathbf{E} + y^{(r)}_m) \quad (16)$$

#### IV. SIMULATION RESULT

This section will contain two kinds of simulation result. One is model verification in time domain, another is control simulation result. The experimental data will be compared with simulation result in open loop response and the validity of tracking performance can be verified in control simulation result

##### A. Model Verification

This study contains both experiment and simulation. The validity of simulation result needs to be verified. However, the design of hydraulic pump-controlled system cannot be identified in frequency domain due to the constraint of its characteristics. Therefore, this section compare with internal pressure and rotational angle with two different step input.

Fig. 5 shows the comparison result with input signal  $u=1$ , while  $u=2.5$  is illustrated in Fig. 6.

Figure 5. Model verification with step input 1 V: (a) Comparison of pressure (b) comparison of angle

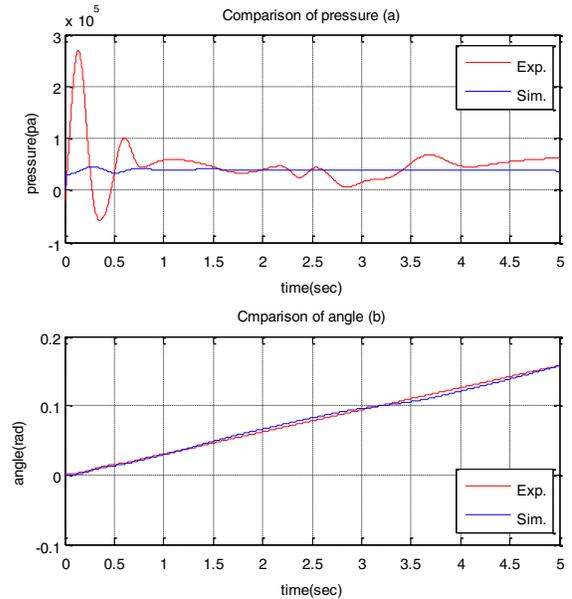
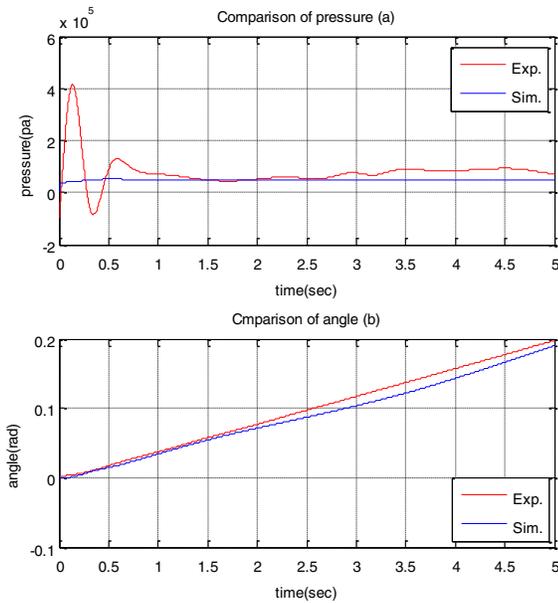


Figure 6. Model verification with step input 2.5 V: (a) Comparison of pressure (b) comparison of angle



Both Fig. 5a and Fig.6a show the correspondence with experiment and simulation result in internal pressure. Meanwhile, Fig. 5b and Fig.6b show the consistence with experiment and simulation result in dynamic response due to the movement of rotational angle. The differences in internal pressure can be explained by the following reasons

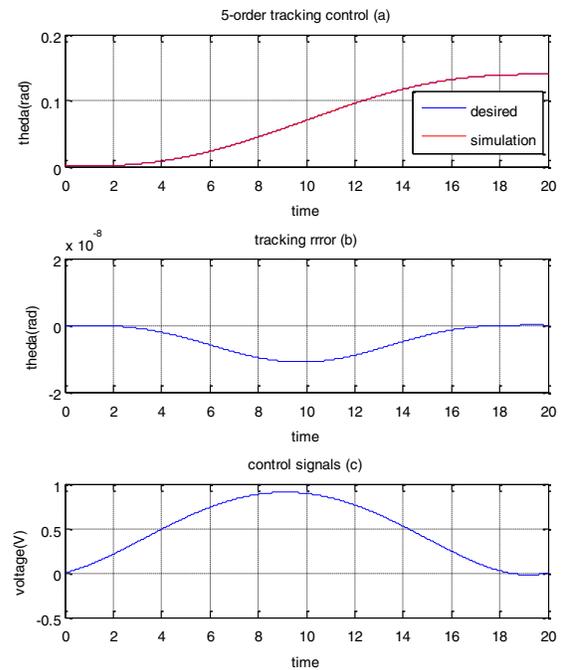
- The system leakage is unmeasurable. It was supposed to be a constant and time invariant value in simulation. However, the pressures converge to each other after few seconds.
- The pressure sensor that we used in an analog input pressure sensor. The high frequency noise is serious when the signal sent back to computer. Therefore, a filter is used to reduce the noise which may affect the accuracy slightly.

### B. Tracking performance

The simulations of tracking control of the variable pitch control system were firstly implemented for confirming the performance of the novel variable pitch control system. To achieve a smooth motion to the designed pitch angle, a path-positioning control was developed, including path control during the motion and positioning control at the designed pitch angle.

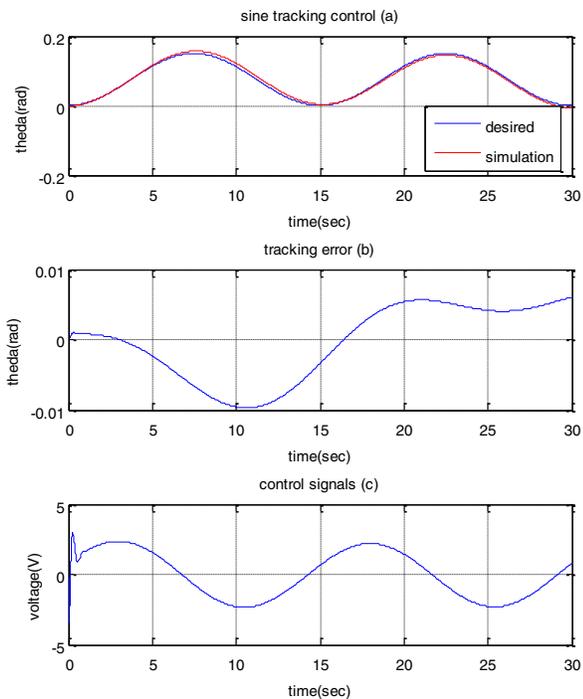
To implement the path tracking control and positioning control simultaneously, a jerk-free path profile of the 5th order polynomial is given. Fig. 7 shows the simulation results of the path-positioning control with a pitch angle stroke of 8 degree in 20 seconds, including the pitch angle response in Fig. 7a, the control pitch angle error in Fig. 8b expressed the tracking error is under 1% in simulation, and control signals are in Fig. 7c.

Figure 7. 5<sup>th</sup> order polys tracking: (a) tracking performance (b) tracking error (c) input signals



Besides, the path control of a sinusoidal path with an amplitude of 8 degree and frequency with 0.66 Hz in 30 seconds is implemented for verifying the bi-directional motion control performance of the novel pitch control system, as illustrated in Fig. 8. The simulation results shows the path tracking control of the pitch angle can be achieved and the control error can be kept under 3%.

Figure 8. Sinusoidal tracking: (a) tracking performance (b) tracking error (c) input signals



## V. CONCLUSION

This study proposed a novel pitch control system of wind turbines driven by variable speed pump-controlled hydraulic system. To achieve the pitch control in this system, a model based controller with feedback linearization was developed and which showed the excellent accuracy and robustness in path-positioning control, including a 5th order polynomial and a sinusoidal profile. Furthermore, the validity of simulation model was substantiated by comparing with experimental test rig out. Besides, to further improve the study, the Coulomb bearing friction force and the dynamics of the AC servo motor should be considered in the future work. Moreover, it is necessary to achieve the experiment with model-based controller built in this study.

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