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A Solution Method for Optimal Weight Design Problem of 10 Bar Truss Using Genetic Algorithms

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Abstract

In this paper, we formulate an optimal weight design problem of 10 bar truss for a constrained displacement and allowable stress as a statically indeterminate structure problem and solve it directly by keeping the constraint based on an improved genetic algorithm (GA). We discuss the efficiency between the proposed method and the discretized optimality criteria methods.
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Key words:**10 Bar Truss, Nonlinear Programming, Genetic Algorithm**

1 Introduction

The Finite Element Method or Analysis (FEA) can be used in all areas of engineering, but was originally used mainly for structural analysis in Aerospace and Civil Engineering. FEA's purpose is to determine static displacement and stresses and the natural frequency and mode of vibration and to analyze the stability of random time-based loading, vibration, structures and to optimize the structure for strength and weight of several different factors. When designing trusses and frames we usually use the trial-and-cut methods to determine factors such as the deflection and allowable stress of structural members. However, this method does not include the method of optimal weight design.

The mathematical model of an optimal weight design problem of the truss for minimizing objective function includes the above mentioned design factors [1],[2].

The discrepancy between analysis capability and optimization capability in structural design has prompted research into new optimality criteria methods (*i.e.*, this discrepancy has been highlighted by Berke and Khot [1]). The cause of discrepancy can be explained if we look at the quantities that determine the optimization capability of various methods used in structural design. These are summarized in the mathematical programming (MP) method, the discretized optimality criteria (DOC) method, the DOC method with fully stressed design for stress constraints (DOC-FDS) method and so on [1],[2].

Until now frequently used optimal criteria method of the Lagrange multiples based on partial differential equations. It provides only local compromised solution without a global optimal solution.

In general, it is difficult to directly solve optimality criteria methods for large discretized systems as statically indeterminated structure models. However, the application of a genetic algorithm (GA) based on the stochastic search method effectively solves the combinatorial problem making it a remarkable new tool for solving these kind of problems [3]-[7].

In this paper, we formulate an optimal weight design (OWD) problem of 10 bar truss for a constrained displacement and allowable stress of truss member as a statically indeterminate structure problem and are able to get a global solution and solve it directly by keeping the constraint by using improved GA [5]. As a result, the number of decision (design) variables do not increase and easily get the best compromised solution. We discuss the efficiency between the proposed method and a few discretized optimality criteria methods.

2 An Optimal Weight Design Problem of 10 Bar Truss

Let us consider an OWD problem of 10 bar truss with a minimum weight as shown in Fig. 1.

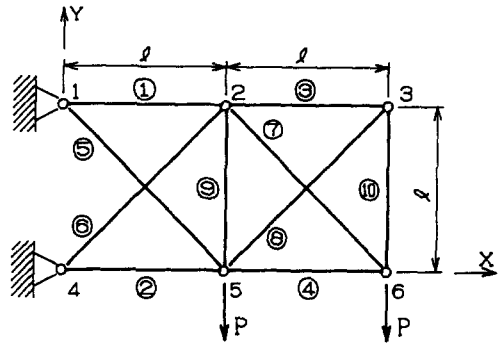

Fig. 1: An OWD problem of 10 bar truss

In Fig.1, the members' lengths are $l_{1-4,9,10} = l$ and $l_{5-8} = \sqrt{2}l$ and the members' cross sections are A_{1-10}. The node 1 and node 4 are fixed by the hinges. Therefor each node's displacements u_1, v_1 and u_4, v_4 are 0 in global coordinate. Otherwise, the truss loads f_{y5} and f_{y6} act at nodes 5 and 6 respectively.

We formulate the optimal design problem with a satisfied displacement and allowable stress with a minimized 10 bar truss weight $W(A)$. It is the following linear programming (LP) problem:

$$\min \quad W(A) = \rho \sum_{i=1}^{10} l_i A_i \tag{1}$$

$$\text{s. t.} \quad G_i = \sigma_i \leq b_i, (i = 1, 2, .., 10) \tag{2}$$

$$G_k = v_k \leq b_k, (k = 2, 3, 5, 6) \tag{3}$$

$$A_i^L \leq A_i \leq A_i^U, \quad (i = 1, 2, .., 10), \tag{4}$$

$$\sigma_i^L \leq \sigma_i \leq \sigma_i^U, \quad (i = 1, 2, .., 10), \tag{5}$$

$$v_k^L \leq v_k \leq v_k^U, \quad (k = 2, 3, 5, 6), \tag{6}$$

where

$$\sigma_i = \epsilon_i E, (i = 1, 2, .., 10) \tag{7}$$

$$\begin{bmatrix} u_k \\ v_k \end{bmatrix} = \{F\} [K]^{-1}, (k = 2, 3, 5, 6) \tag{8}$$

Where $W(A)$ is the weight function, ρ is density of truss' member, $\sigma_i (i = 1, \ldots 10)$ is stress of truss' members, b_i is allowable stress of truss' members, $v_k (k = 2, 3, 5, 6)$ is displacement of y component for each node, b_k is allowable displacement of y component for each node, ϵ_i is strains of each member, E is the Young modulus, A_i^L and A_i^U are lower/upper limit values of the design variable, and $(v_k^L, \sigma_i^L)/(v_k^U, \sigma_i^U)$ are lower/upper limits of allowable stress and deflection of each member.

3 Representation and Evaluation Function

3.1 Representation and Initialization

In this paper, each design variable of 10 bar truss is represented by the gene of chromosome V_k which is the integer-valued decision variable $V_k = [A_1, A_2, \cdots, A_{10}]$. *pop_size* denotes the size of the population. The initial population is randomly generated by satisfying the constraints (2) to (3).

3.2 Evaluation Function

Generally, in GA the fitness value of a chromosome is obtained by the evaluation function. In this paper, the evaluation function eval(V_k) is defined using the objective function (1).

When the chromosome V_k does not satisfy eqs. (1) to (3), the evaluation function is given the following equation:

$$eval(V_k) = \begin{cases} W(A_1, A_2, \cdots, A_{10}); \\ \quad G_i \leq b_i, \ i = 1, 2, \cdots, 10 \\ \quad G_k \leq b_k, \ k = 2, 3, 5, 6 \\ M; \quad \text{otherwise} \end{cases} \tag{9}$$

where, M is a large positive number.

In this definition, the search for a solution occurs only inside the feasible region. This fact may not constitute a serious obstacle for small-scale problems where the feasible region is relatively large.

When considering large-scale problems on the other hand, many times the feasible region is extremely small. This is accentuated by the condition that only integer-valued solutions are acceptable, causing most of the generated chromosomes to be unfeasible solutions. Consequently, it is not possible to produce high-quality chromosomes, and the search efficiency deteriorates.

Yokota *et al.* [5] reported an improved GA to cope with such a problem. That is, for the constraint condition, the following measure is introduced:

$$d_i = \begin{cases} 0; \quad G_{i+k} \leq b_{i+k}, \ i = 1, 2, \cdots, 10, \\ \qquad\qquad\qquad k = 2, 3, 5, 6 \\ (G_{i+k} - b_{i+k})/b_{i+k}; \quad \text{otherwise} \end{cases} \tag{10}$$

The measure d_i adopted here indicates how much the left-hand side of the expression corresponding to the constraint condition exceeds the right-hand side, and the evaluation function is defined as follows:

$$eval(V_k) = W(A_1, A_2, \cdots, A_{10}) \left(1 - \frac{1}{10} \sum_{i=1}^{10} d_i \right) \tag{11}$$

This improved GA includes information about unfeasible chromosomes as near as possible to the feasible region in the evaluation function.

We can select the best chromosome with the following equation:

$$V^* = \underset{V_k}{\operatorname{argmin}}\{eval(V_k)\} \tag{12}$$

$$mineval = \min\{eval(V_k)\} \tag{13}$$

Where the "arg" of argmin is an abbreviation for argument and argmin means that we adopt the chromosome whose evaluation returns the lowest value.

4 Algorithm

The procedure for solving the OWD problem of 10 bar truss by genetic algorithms is proposed in this section.

[Step 1] Parameter setting

The size of population: *pop_size*, probability of crossover:P_c, probability of mutation: P_m, and maximum number of generation: *maxgen* are set.

Additionally, the generation counter *gen* and the initial evaluation function value *mineval* are both zero.

[Step 2] Generation of the initial chromosome population

The initial chromosome V_k consists of elements for variables $A_1, A_2, \cdots, A_{10}$ as follows:

$$V_k = [A_1, A_2, \cdots, A_{10}], k = 1, ..., pop_size \tag{14}$$

These variables generate V_k randomly satisfying the constraint eqs. (2) to (6).

If chromosomes satisfying the constraint eqs. (2) to (6) can not be found, then it is assumed that there are no feasible solutions for the problem, and the algorithm terminates.

Table 1: The range of variable and coefficient values

$11.5 \leq A_1 \leq 12.5$	$8.0 \leq A_2 \leq 9.0$
$0.1 \leq A_3 \leq 1.0$	$5.5 \leq A_4 \leq 6.5$
$5.5 \leq A_5 \leq 6.0$	$8.0 \leq A_6 \leq 9.0$
$8.0 \leq A_7 \leq 9.0$	$0.1 \leq A_8 \leq 1.0$
$0.1 \leq A_9 \leq 1.0$	$0.1 \leq A_{10} \leq 1.0$
$E = 10^7$	$\rho = 0.1$
$\lvert\sigma\rvert \leq 25000$	$\lvert v_6\rvert \leq 5.0$
$l_{1-4,9,10} = 360$	$P = 10^5$
$l_{5-8} = 360\sqrt{2}$	

[Step 3] Calculation of fitness value

The generation counter is updated ($gen = gen + 1$), and the fitness chromosome is calculated from the evaluation function (12).

[Step 4] Genetic operators

(1) Crossover

Let the number of chromosomes generated be $ccnt = 0$. Create the random number r_k ($k = 1, 2, ..., pop_size$) from the range [0,1]. Select V_k that satisfies $r_k < P_c$. Make a pair of V_k and set $ccnt = ccnt + 2$. Choose the position for crossover at random and exchange each other's elements. Let offspring be V'_{ccnt-1} and V'_{ccnt} respectively.

(2) Mutation

Let the number of chromosomes generated by mutation be $mcnt = 0$. Generate the random number r_k ($k = 1, 2, ..., pop_size + n * ccnt$) from the range [0,1]. Select a gene that satisfies $r_k < P_m$ and set $mcnt = mcnt + 1$. There is no increase in the value of $mcnt$ in the case of a mutation in the same chromosome. Mutate the element that is selected in the region of eqs. (4) to (6).

Let the chromosome that is newly generated be $V_{ccnt+mcnt'}$.

(3) Selection

Calculate the evaluation functions $eval(V'_l)$, ($l = 1, 2, ..., ccnt + mcnt$) that are newly generated. Select the chromosomes among the parents V_k and V'_l starting with the highest values and choosing the subsequently highest variables. The number to be selected is pop_size. Let these chromosomes be the V_k for the generation where the chromosome, that has the same structure, is not selected.

[Step 5] Calculate the best chromosome

Update the best chromosome V^* and the corresponding evaluation function from eqs. (2) to (6).

[Step 6] Next generation

If $gen \leq maxgen$, then go to Step 3. Otherwise output V^* and stop.

5 Numerical Example

Usually, many designers decided each dimension for A_i, v_k and σ_i by experimenting until they obtained the truss load at node 5,6, deflection of each member and allowable stress in constraint.

In this paper, when we are given the design parameters, we discuss the optimal 10 bar truss design problem for minimizing the weight as shown in Fig.1 and Table 1.

We solve the OWD problem of the 10 bar truss by using our proposed algorithm. We set up the following initial parameters for the numerical example:

$$pop_size = 20,\ P_c = 0.4,\ P_m = 0.1,\ maxgen = 1000$$

Table 2: The result of calculation and former methods

	improved GA	DCOC	Dual	DOC-FSD
A_1	12.131896	12.161173957	12.161173956	12.126576172
A_2	8.794619	8.707029023	8.707029026	8.827450732
A_3	0.100000	0.100000000	0.100000000	0.100000000
A_4	6.065801	6.040579884	6.040579884	6.046585281
A_5	5.100000	5.560164853	5.560164853	5.564322434
A_6	8.539911	8.573640198	8.573640196	8.497882192
A_7	8.575261	8.542669996	8.542669996	8.551162911
A_8	0.100000	0.100000000	0.100000000	0.100000000
A_9	0.100000	0.100000000	0.100000000	0.100000000
A_{10}	0.100000	0.100000000	0.100000000	0.100000000
W(lb)	2118.626	2139.105	2139.105	2139.198

Table 3: The result of stress σ_i, and axial force f_i

	σ_i	f_i
l_1	166.2779	20215.11096
l_2	-2249.6584	-19784.88904
l_3	475.6522	47.56522
l_4	-1640.7454	-9952.43478
l_5	2713.3182	13837.92279
l_6	-1691.6275	-1446.34846
l_7	1641.3341	14074.86824
l_8	-672.6738	-67.26738
l_9	2626.7618	262.67618
l_{10}	475.6522	47.56522

We apply the proposed method to this problem and repeat the process for 1,000 generations. At the $648th$ generation of the chromosome and constraint design variables, the following results are obtained:

$$V^* = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \end{bmatrix} = \begin{bmatrix} 12.093310 \\ 8.799735 \\ 0.100000 \\ 6.074495 \\ 5.100000 \\ 8.538694 \\ 8.594261 \\ 0.100000 \\ 0.100000 \\ 0.100000 \end{bmatrix}$$

The above values obtained are the best compromised solutions. The corresponding evaluation function value, that is, the minimized weight of 10 bar truss, was $W(A_1, A_2, \cdots, A_{10}) = 2118.64$.

6 Evaluation

By applying the method proposed in this paper to the numerical example described in the previous section, with population size equal to 20, crossover and mutation probabilities set to 0.4 and 0.1, respectively, and maximum number of generations set at 1,000, it was possible to obtain results shown in Table 2. And we also include the specification for the former method values in Table 2.

After comparing the 10 bar truss figures and our calculation results, the weight of 10 bar truss (W) improved by 20.45 lb (1%). We infer this is due to the fact that the improved GA is a global solution while three discretized optimality criteria methods by DCOC, Dual and DOC-FSD method (include Lagrange multiples) sometimes falls on local solutions.

Therefor we obtained stress σ_i, axial force f_i and deflection u_k, v_k as shown in Table 3 and Table 4.

The results shown in Table 3 confirm that the values of σ_i for $l_{3,8,10}$ are negligible and therefor can be removed without modifying the system.

Table 4: The result of deflection u_k and v_k of each node

node	u_k	v_k
1	0	0
2	0.606673	-1.817000
3	0.768973	-4.83595
4	0	0
5	-0.827898	-2.78003
6	-1.422710	-4.99826

In order to keep the deflection v_k of node 6 below -5.0 in (Table 4), the improved GA has enlarged the area of A_1 (Table 2) and thus the small value of σ_i of l_1 (Table 3).

In this improved GA method, when a chromosome is not contained in the feasible region, it will include information about the unfeasible region's chromosome in the evaluation function in order to improve its search efficiency. We obtained a calculation time of 11 sec.

7 Conclusion

In this paper, we formulate an OWD problem of 10 bar truss for a constrained allowable stress and deflection of the dimension of each member as a statically indeterminate structure problem and solve it directly by keeping the constraint by using improved GA. We discuss the efficiency of the proposed method. As a result, the number of decision (design) variables did not increase, and easily got the best compromised solution.

When a chromosome is not contained in the feasible region, it will include information about the infeasible region's chromosome in the evaluation function in order to improve its search efficiency.

As a result, we obtained a minimum weight of $W(A_1, A_2, \cdots, A_{10}) = 2118.64$, then the weight of 10 bar truss (W) improved by 20.45lb (1%). and confirmed the efficiency of the proposed method.

Here we also confirm that $l_{3,8,10}$ are unnecessary to the system and therefor do not need to be included them in future calculations.

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