Gas-Cap Effects in Pressure-Transient Response of Naturally Fractured Reservoirs

Ali Soud Al-Bemani, Sultan Qabous U., and **Iraj Ershaghi**, SPE, U. of Southern California

Summary

This paper presents a theoretical framework of gas-cap effects in naturally fractured reservoirs. General pressure solutions are derived for both pseudosteady-state and unsteady-state matrix-fracture interporosity flow.

Deviation from the fracture or fracture-matrix response occurs as the gas-cap effect is felt. Anomalous slope changes during the transition period depend entirely on the contrast between the fracture anisotropy parameter, λ_1, and matrix-fracture interporosity parameter, λ, and between the total gas-cap storage capacitance $(1-\omega_1)$ and oil-zone matrix storage $(1-\omega)$.

A composite double-porosity response is observed for $\omega_1 \leq \omega_{1c}$ and $1.0 \leq \lambda_1/\lambda \leq 1000$. A triple-porosity response is observed for $\omega_1 \geq \omega_{1c}$ and $140 < \omega\lambda_1/\lambda < 1.0E05$.

Introduction

Estimation of reservoir and wellbore parameters in naturally fractured reservoirs using pressure-transient tests has been the subject of many publications during the last quarter of a century. A plausible model to describe naturally fractured reservoirs is a composite system consisting of tight matrix rock blocks separated by fractures. Owing to fracture high hydraulic conductivity, the fracture pressure responds rapidly to any perturbation in the reservoir. Because of matrix blocks' low permeability, they exhibit a delayed response to any reservoir perturbations.

The concept of a double porosity was first introduced by Barenblatt *et al.*[1] Warren and Root[2] used this concept to model the wellbore pressure response of a naturally fractured reservoir. They described the pressure response by two main parameters, ω and λ, which relate the two porosity systems. The semilog plot of pressure response vs. time for this model produces a characteristic "S"-shaped transition segment with a point of inflection. Kazemi[3] used a layered system composed of thin but highly conductive layers to represent the fracture network, alternating with thicker but lower-conductivity layers to represent the matrix system, and he assumed the matrix-fracture interporosity flow occurs under the unsteady-state flow regime. Observations of Serra *et al.*[6] and Streltsova[7] indicated the absence of a point of inflection and a slope ratio of 2 (late time vs. transition segment), characteristic of the transient flow conditions. Doddy and Ershaghi[8] introduced the concept of triple-porosity naturally fractured reservoir with the assumption of unsteady-state interporosity flow regime. This study pointed out an explanation for observing multiple transition periods. Jalali-Yazdi and Ershaghi[9] and Jalali-Yazdi[10] investigated the behavior of a doubleporosity and triple-porosity naturally fractured reservoir in a combined dual flow—transient and pseudosteady-state interporosity flow regimes. This study provided an explanation for the shape of the transition segment falling between the responses of pseudosteady-state and unsteady-state interporosity flow.

During the primary production life of an oil reservoir, segregation of oil and gas within the fissures before reaching the producing wells could create a secondary gas cap if no original gas cap were present, or will join the expanding original gas-cap gas.[11,12] In both cases, the upper boundary of the producing zone cannot be considered as a no-flow boundary. Crossflow from matrix blocks to the fracture network provides support similar to that provided by a layer overlain by a low-permeability bed containing a gas cap. Recognition of the gas-cap effect in modeling pressure-transient response in naturally fractured reservoirs will aid in proper delineation of the interplaying of the two systems supporting the fracture network pressure response.

The objective of this study is to develop idealized models that predict the analytical pressure response of naturally fractured reservoirs producing from an oil zone overlain by a gas cap. The study will provide additional explanations for observing multiple transition periods and anomalous slope changes during the transition period of actual well-test data.

Copyright 1997 Society of Petroleum Engineers

Original SPE manuscript received for review 23 September 1990. Revised manuscript received 16 March 1994. Paper peer approved 22 January 1997. Paper (SPE 20565) first presented at the 1990 SPE Annual Technical Conference and Exhibition held in New Orleans, 23–26 September.

Description and Derivation of the Mathematical Models

A double-porosity naturally fractured reservoir is considered consisting of continuous orthogonal anisotropic fracture network and homogeneous isotropic matrix blocks. The reservoir consists of an oil zone with a thickness of h_o and gas zone with a thickness of h_g (**Fig. 1**). The fractures are homogeneously anisotropic, with a constant vertical to horizontal permeability ratio (k_{fz}/k_{fh}). Both pseudosteady-state and unsteady-state governing mode of interporosity flow (matrix to fracture) are investigated. The reservoir is produced at a constant and restricted flow rate, q_o, (to avoid gas coning) by a fully penetrated well and completed in the oil zone. The fluid flowing through the producing zone is single phase, slightly compressible, and with constant viscosity. Darcy's law applies to flow in the matrix as well as in the fracture. The reservoir is assumed to be infinite in the radial direction and the gravitational forces are negligible.

The governing diffusivity equation that describes pressure behavior of the anisotropic fissure system in the oil zone can be written as

$$\frac{\partial^2 p_{1f}}{\partial r^2}+\frac{1}{r}\frac{\partial^2 p_{1f}}{\partial r}+\frac{k_{fz}}{k_{fh}}\frac{\partial^2 p_{1f}}{\partial z^2}=\frac{\mu}{k_{fh}}\left[(\phi C)_{of}\frac{\partial p_{1f}}{\partial t}+(\phi C)_{oma}\frac{\partial p_{1ma}}{\partial t}\right] \quad \text{.......................} \ (1)$$

where $p_1=p(r,z,t)$, and $z=$ the reservoir vertical coordinate, its origin at the lower boundary of the oil zone (Fig. 1).

The pressure of a well open to flow through the oil thickness, h_o, is considered to be the average value, p, of the pressure, $p(r,z,t)$, along the thickness, h_o; pressures in Eq. 1 are converted to average pressures, and this equation becomes

$$T\left\{\frac{\partial^2 p_f}{\partial r^2}+\frac{1}{r}\frac{\partial p_f}{\partial r}+\frac{k_{fz}}{k_{fh}h_o}\left[\frac{\partial p_f(z)}{\partial z}\right]\Big|^{z=h_o}_{z=0}\right\}$$

$$=\left(S_{fo}\frac{\partial p_f}{\partial t}+S_{oma}\frac{\partial p_{ma}}{\partial t}\right), \quad \text{.........................} \ (2)$$

where $T=\dfrac{k_{fh}h_o}{\mu}$, $S_{fo}=(\phi C)_{fo}h_o$, and $S_{oma}=(\phi C)_{oma}h_o$.

During oil production through the oil zone, the expansion of the gas maintains pressure in the gas cap. For the duration of the well test, this pressure is considered as being constant (p_{fo}). The flux-boundary condition at the gas-oil interface ($z=h_o$) in fractures is described by the first-order linear relationship as follows:

$$\frac{-S_{tg}\mu}{k_{fz}}\frac{\partial p_{fo}}{\partial t}=\frac{\partial p_f(z)}{\partial z}\Big|_{z=h_o}, \quad \text{.......................} \ (3)$$

$$\frac{\partial p_f(z)}{\partial z}\Big|_{z=o}=0, \quad \text{................................} \ (4)$$

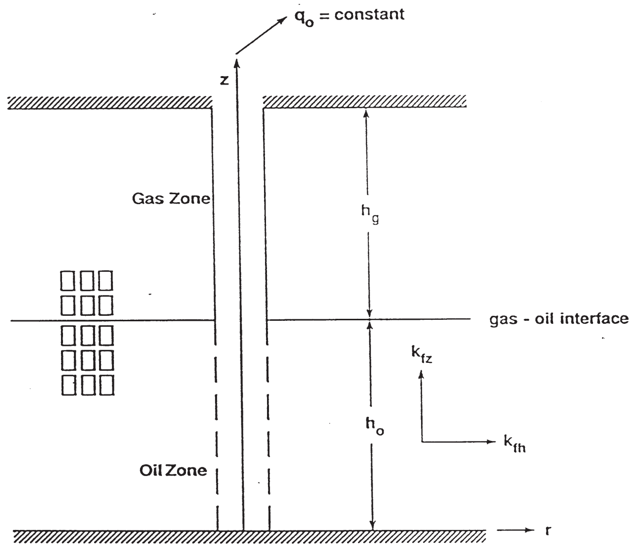

Fig. 1—Well in all oil zone overlain by, and in communication with, a gas cap of a naturally fractured reservoir (NFR).

where $S_{tg} = \left[(\phi C)_{fg} + (\phi C)_{gma}\right]h_g$.

Then Eq. 2 becomes

$$T\left(\frac{\partial^2 \overline{p}_f}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{p}_f}{\partial r}\right) = S_{fo}\frac{\partial \overline{p}_f}{\partial t} + S_{oma}\frac{\partial \overline{p}_{ma}}{\partial t} + S_{tg}\frac{\partial \overline{p}_{fo}}{\partial t}. \quad \text{(5)}$$

This is the radial diffusivity equation for the fracture with two supporting terms representing the matrix storativity, S_{mo}, and gas-cap storativity, S_{tg}.

The above equation is converted into dimensionless terms by substituting the following dimensionless definitions:

$$\Delta P_{Df} = \frac{k_{fh}h_o\Delta p}{141.2q_o\mu B_o} = \frac{2\pi\tau\Delta p}{q_o}, \quad \text{(6)}$$

$$t_D = \frac{0.000264k_{fh}t}{\left[(\phi C)_{oma} + (\phi C)_{fo}\right]\mu r_w^2} = \frac{T_t}{S_{to}r_w^2}, \quad \text{(7)}$$

$$r_D = \frac{r}{r_w}\sqrt{\frac{k_{fz}}{k_{fh}}}, \quad \text{(8)}$$

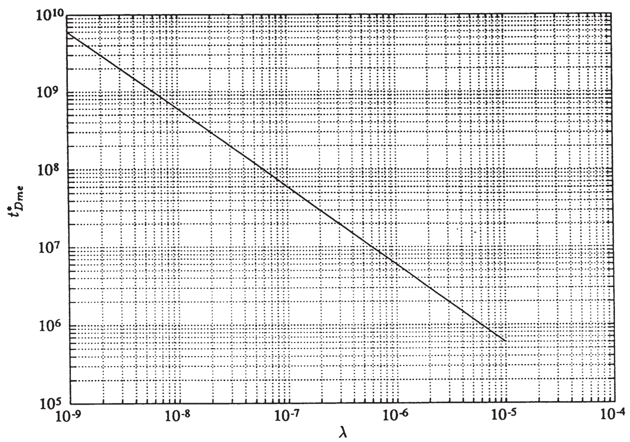

Fig. 3—Characteristic dimensionless time (t^*_{Dme}) for onset of total fracture-matrix system.

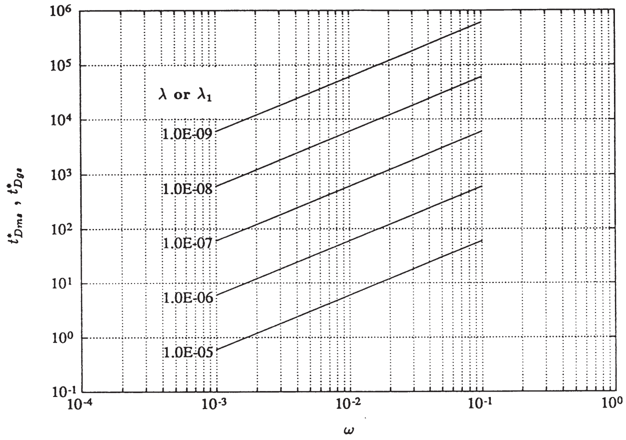

Fig. 2—Characteristic dimensionless time (t^*_{Dms} and t^*_{Dgs}) for onset of matrix and gas cap support respectively.

$$\omega = \frac{S_{fo}}{S_{to}}, \quad \text{(9)}$$

$$\omega_1 = \frac{S_{to}}{S_t}, \quad \text{(10)}$$

where $S_{to} = \left[(\phi C)_{fo} + (\phi C)_{oma}\right]h_o$ and $S_t = S_{to} + S_{tg}$.

Eq. 5 becomes

$$\frac{\partial^2 \Delta p_{Df}}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial \Delta p_{Df}}{\partial r_D} = \frac{k_{fh}}{k_{fz}}$$

$$\left[\omega\frac{\partial \Delta p_{Df}}{\partial t_D} + (1-\omega)\frac{\partial \Delta p_{maD}}{\partial t_D} + \left(\frac{1-\omega_1}{\omega_1}\right)\frac{\partial \Delta p_{Dfo}}{\partial t_D}\right]. \quad \text{(11)}$$

This equation is solved for wellbore dimensionless pressure solutions in Laplace space (Appendix B):

$$\overline{\Delta P}_{wD} = \frac{K_0(x)}{sxK_1(x)}, \quad \text{(12)}$$

where $x = \sqrt{sf(s)}$. (13)

Discussion

Eq. 12 is numerically evaluated using the Stehfest algorithm,[13] and the effect of each of the two supporting systems on the fracture response is studied individually. Characteristic dimensionless time

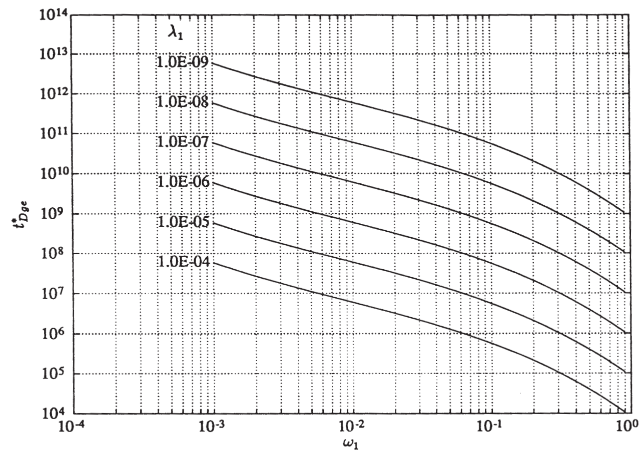

Fig. 4—Characteristic dimensionless time (t^*_{Dge}) for onset of total fracture-gas cap system.

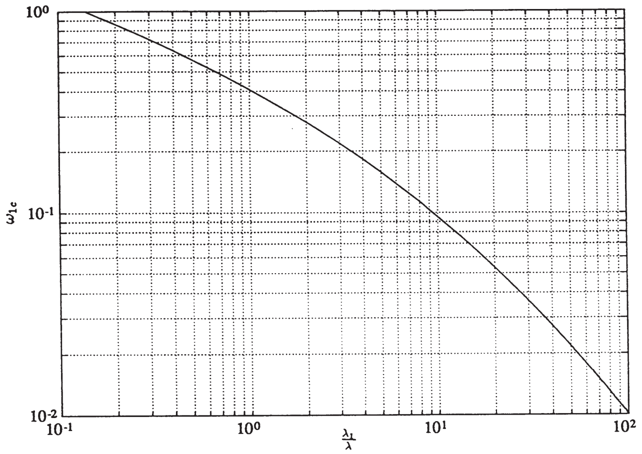

Fig. 5—Critical ω_1 for matrix-gas cap superimposition.

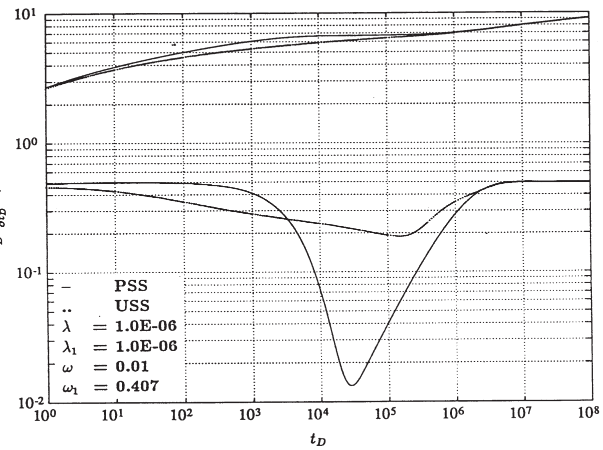

Fig. 6—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda = 1.0$ and $\omega_1 = \omega_{1c}$.

for onset of matrix and gas-cap support (**Fig. 2**), the total fracture-matrix (**Fig. 3**), and the fracture-gas-cap system (**Fig. 4**) are used in characterization of the pressure behavior.

The functionality of the wellbore dimensionless pressure presented in Eq. 12 may be shown as follows:

$$\Delta\overline{P}_{wD} = f(\omega,\omega_1,\lambda,\lambda_1,s) = \frac{K_0(x)}{sxK_1(x)}. \quad \text{(14)}$$

As fractures undergo pressure changes, matrix crossflow to the fracture network provides support similar to what is provided by the gas cap, and the combined effect forms a very complex response. The effect of the two supporting systems could be exhibited simultaneously or separately.

Superimposition of Matrix and Gas Cap. Matrix and gas-cap support will superimpose for the values of $\omega (0.001 \leq \omega < 0.1)$ and ω_1 $(0.01 \leq \omega_1 \leq 1.0)$ considered in this study when fracture anisotropy parameter to matrix-fracture interporosity parameter ratio has any value of $0.001 < \lambda_1/\lambda < 100.0$. This superimposition of the two supporting systems could either produce a double-porosity response or a resemblance to a triple-porosity system.

For the following λ ratios, the system exhibits a double-porosity response (**Fig. 5,** i.e., simultaneous onset of total matrix-fracture and gas-cap fracture systems).

1. $0.14 < \lambda_1/\lambda < 100.0$ for $\omega_1 = \omega_{1c}$.
2. $0.14 < \lambda_1/\lambda < 1.0$ (i.e., onset of matrix support appears before that of gas cap) for $\omega_1 > \omega_{1c}$ (i.e., gas-cap support terminates before that of matrix).
3. $1.0 < \lambda_1/\lambda < 100.0$ (i.e., onset of gas-cap support appears before that of matrix) for $\omega_1 < \omega_{1c}$ (i.e., matrix support terminates before that of gas cap).

For the following λ ratios, the system will produce either a triple-porosity or a double-porosity response with an anomalous slope change toward the end of the transition period.

1. $0.001 < \lambda_1/\lambda < 0.14$ (i.e., onset of matrix support appears before that of gas cap) for any value of ω_1.
2. $0.14 < \lambda_1/\lambda < 1.0$ (i.e., onset of matrix support appears before that of gas cap) for $\omega_1 < \omega_{1c}$ (i.e., matrix support terminates before that of gas cap).
3. $1.0 < \lambda_1/\lambda < 100.0$ (i.e., onset of gas-cap support appears before that of matrix) for $\omega_1 > \omega_{1c}$ (i.e., gas-cap support terminates before that of matrix).

For both the pseudosteady state and unsteady state, the semilog plot of dimensionless pressure vs. time exhibits two parallel straight lines. These straight lines are connected by a transition segment. The first straight line (early-time) represents the fracture-dominated response, whereas the second straight line (late-time) represents the behavior of the total reservoir. The existence, duration, and the feature of the transition curve, which connects the early to the late time, is influenced by the magnitude of ω, ω_1, λ, and λ_1. The transition period signifies the combined effect of the interporosity flow from the matrix and the gas-cap support to fracture network.

A double-porosity response can be observed when both gas cap and matrix simultaneously and equally respond to fracture pressure change, or the preceding system terminates after or together with the second system (**Figs. 6 through 10**).

A resemblance to a triple or a double porosity with an anomalous slope change toward the end of the transition period could either be an indication of simultaneous response both gas cap and matrix to

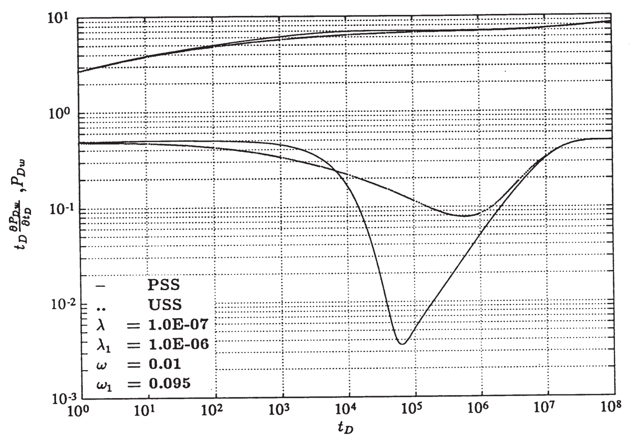

Fig. 7—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda > 1.0$ and $\omega_1 = \omega_{1c}$.

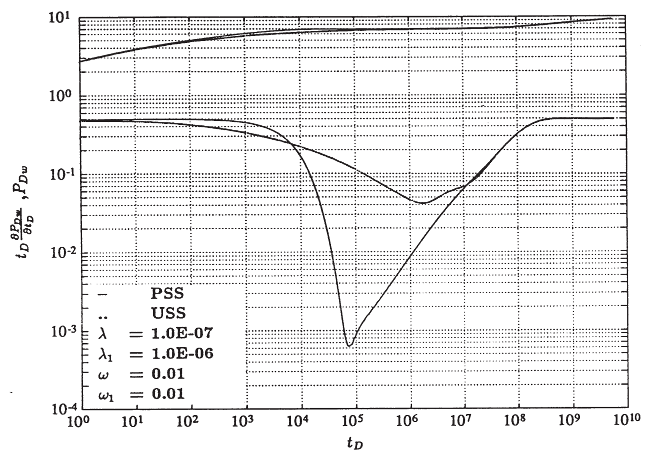

Fig. 8—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda > 1.0$ and $\omega_1 < \omega_{1c}$.

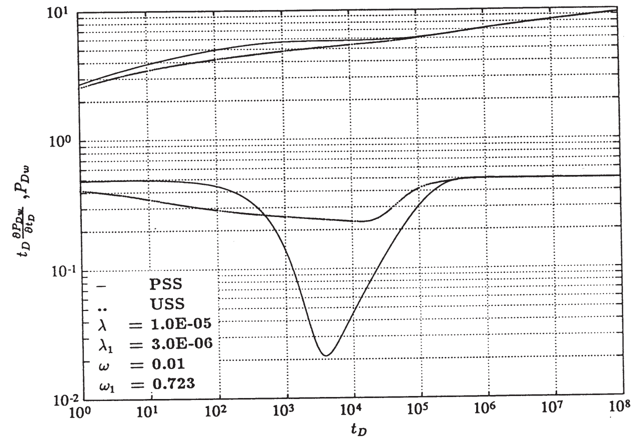

Fig. 9—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda < 1.0$ and $\omega_1 = \omega_{1c}$.

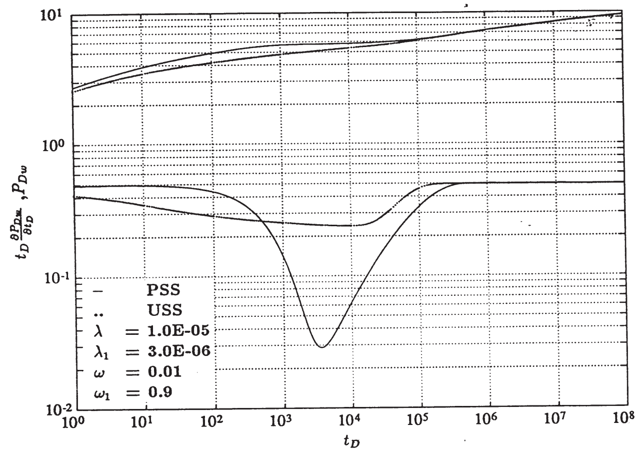

Fig. 10—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda < 1.0$ and $\omega_1 > \omega_{1c}$.

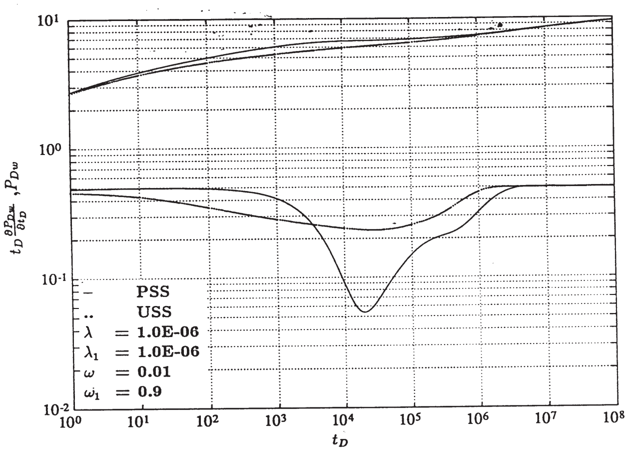

Fig. 11—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda = 1.0$ and $\omega_1 > \omega_{1c}$.

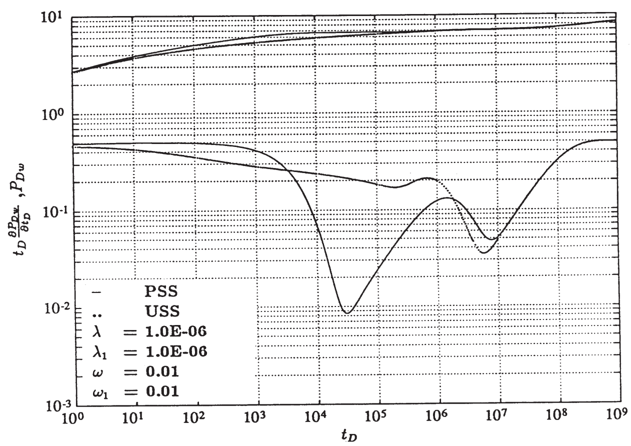

Fig. 12—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda = 1.0$ and $\omega_1 < \omega_{1c}$.

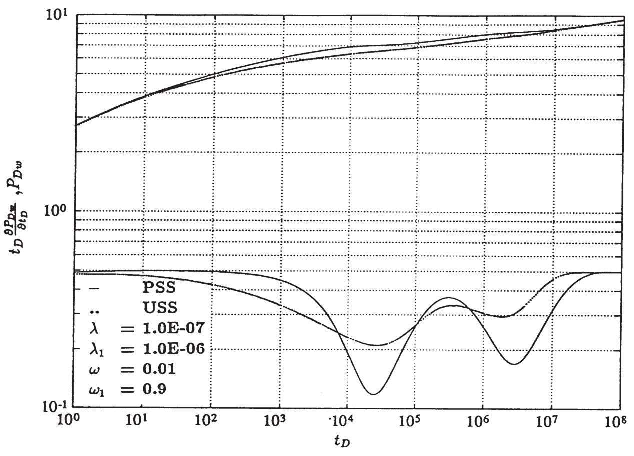

Fig. 13—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda > 1.0$ and $\omega_1 > \omega_{1c}$.

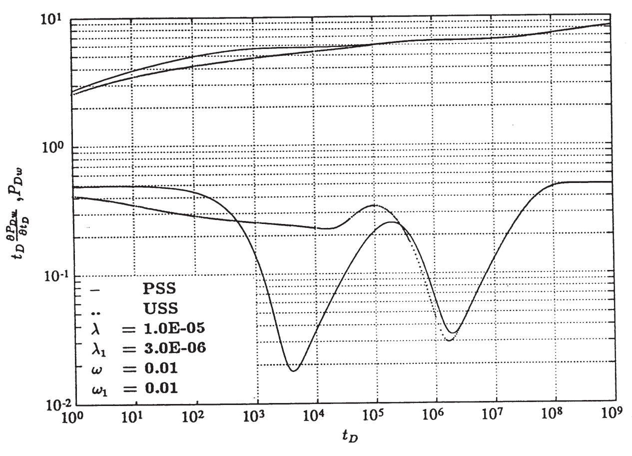

Fig. 14—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda < 1.0$ and $\omega_1 < \omega_{1c}$.

fracture pressure change and one of the support systems terminating before the other, or the preceding support system terminates before the other (**Figs. 11 through 15**).

Matrix and Gas Cap Act Separately. Matrix and gas-cap support will act separately for the values of ω and ω_1 considered in this study when fracture anisotropy parameter to matrix-fracture interporosity parameter ratio is in the range of $\lambda_1/\lambda \leq 0.001$ or $\omega\lambda_1/\lambda \leq 140$ and $\omega_1 > \omega_{1c}$ (**Fig. 16**). The pressure response represents the support of the active system to the fracture-dominated flow period, followed by the support of the latent system to the total active-fracture infinite-acting flow period.

1. Matrix could act as an active supporting system compared to gas cap if $\lambda_1/\lambda \leq 0.001$, whereas gas-cap support appears after an infinite acting response of the total matrix-fracture system (**Fig. 17**).

2. Gas cap could act as an active supporting system compared to matrix if $\omega\lambda_1/\lambda \leq 140$ and $\omega_1 > \omega_{1c}$ (Fig. 6), whereas matrix support

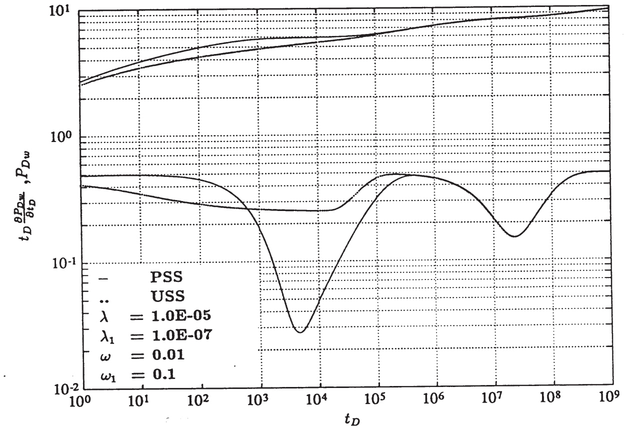

Fig. 15—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda < 0.14$.

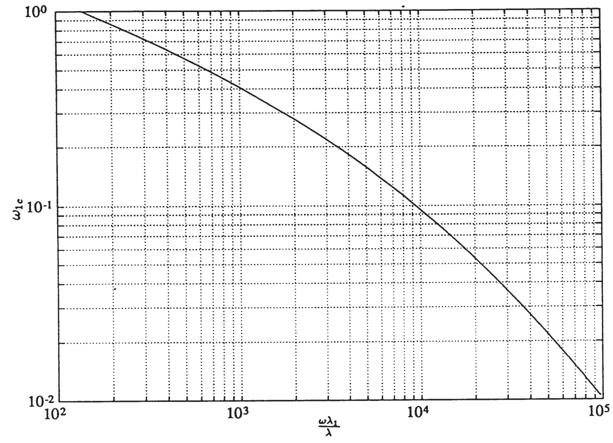

Fig. 16—Critical ω_1 above which gas cap is an active supporting system.

appears after an infinite acting response of the total gas-cap fracture system (**Fig. 18**).

Dimensionless pressure solutions exhibit three parallel straight lines on a semilog plot of dimensionless pressure vs. time. Therefore, the pressure response will display five segments compared to three segments in a double-porosity system. Similar to the double-porosity response, the early-time and the late-time straight lines represent the pressure response of the fracture-dominated flow period and the total reservoir, respectively. The intermediate straight line depicts the composite response of the active-fracture system. The early transition period represents the response of the active supporting system to fracture-dominated flow period, whereas the late transition period represents the behavior of the latent supporting system to the total active-fracture flow period.

The possibility of observing $\lambda_1/\lambda < 1.0$ is very small. As a result, a resemblance to triple-porosity behavior in a dual porosity naturally fractured reservoir producing from an oil zone overlain by a gas cap (i.e., $1.0 < \lambda_1/\lambda < 100.0$ and $\omega_1 > \omega_{1c}$) is mainly caused by gas-cap support developing before the matrix support.

Conclusions

Pressure response of an oil zone overlain by a gas cap is characterized by a specific response that could behave as a double-porosity system, a triple-porosity system, and a system between a double- and triple-porosity system.

Based on the definition of λ_1, a triple-porosity response in a double-porosity naturally fractured reservoir producing from an oil zone overlain by a gas cap is mainly caused by a gas-cap support influencing the system earlier than the matrix. Failure to run the well test long enough for the matrix-fracture interporosity to appear will result in inaccurate estimation of the reservoir parameters.

In a double-porosity behavior, ignoring the effect of the gas cap will cause an overestimation of the matrix storativity. The abnormal behavior toward the end of the transition period should not be ignored, as shown in this study; this could be caused by the superimposition of matrix response and the gas-cap response owing to pressure changes in the fracture network.

Nomenclature

c_{fg} = gas zone fracture compressibility, psi^{-1}
c_{fo} = oil zone fracture compressibility, psi^{-1}
c_{gma} = gas zone matrix compressibility, psi^{-1}
c_{oma} = oil zone matrix compressibility, psi^{-1}
$f(s)$ = defined by Eq. 14 and 15
$\bar{l}_{ma}$ = matrix block characteristic length, ft
h_g = gas zone thickness, ft
h_o = oil zone thickness, ft
$I_0(x)$ = modified Bessel function, first kind, zero order
k_{fh} = fracture horizontal permeability
k_{fz} = fracture vertical permeability, md
k_{ma} = matrix permeability, md
$K_0(x)$ = modified Bessel function, 2nd kind, zero order
$K_1(x)$ = modified Bessel function, 2nd kind, first order
ΔP_{Df} = pressure drop in fracture, dimensionless
ΔP_{maD} = pressure drop in matrix, dimensionless
ΔP_{Dw} = wellbore pressure drop, dimensionless
$\bar{p}_f$ = average value of the fracture pressure, $p_f(z)$, psi
$\bar{p}_{ma}$ = average value of the matrix pressure, $p_{ma}(z)$, psi
$\bar{p}_w$ = average value of the wellbore pressure, $p_w(z)$, psi
Δp = pressure change, psi
q_o = volumetric oil flow rate, stb/day
r = radial distance, ft

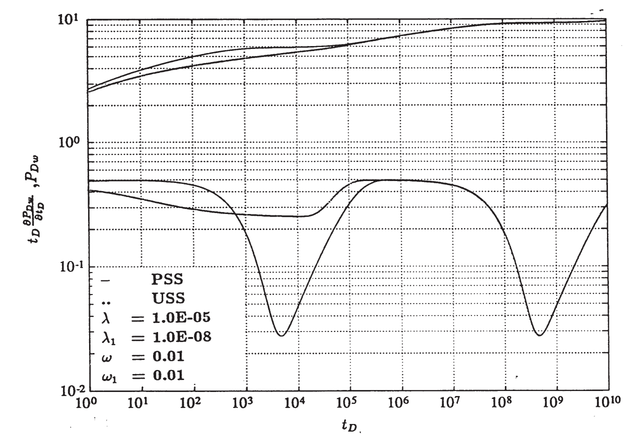

Fig. 17—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda = 0.001$.

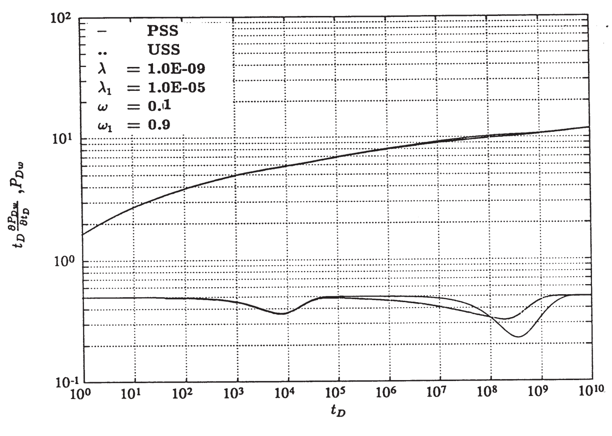

Fig. 18—Pressure response of a NFR producing an oil zone overlain by a gas cap for $\lambda_1/\lambda > 140$ and $\omega_1 > \omega_{1c}$.

r_D = radial distance, dimensionless
r_w = wellbore radius, ft
s = Laplace parameter
S_{fo} = oil zone fracture storativity
S_{oma} = oil zone matrix storativity
S_t = total reservoir storativity
S_{tg} = total gas zone storativity
S_{to} = total oil zone storativity
t = time, hours
t_D = time, dimensionless
x = defined by Eq. 13
y = defined by Eq. B-2
z = reservoir vertical coordinate, ft
z_D = reservoir vertical coordinate, dimensionless
z_{ma} = matrix vertical coordinate, ft
z_{maD} = matrix vertical coordinate, dimensionless
α = shape factor, ft^{-2}
B_o = formation volume factor, rb/stb
λ = matrix-fracture interporosity parameter, dimensionless
λ_1 = fracture anisotropy parameter, dimensionless
μ = fluid viscosity, cp
T = transmissibility of the horizontal fractures in the oil zone, md·ft/cp
ϕ_f = bulk fracture porosity, fraction
ϕ_m = bulk matrix porosity, fraction
ω = oil zone fracture storativity with respect to total oil zone storativity, dimensionless
ω_1 = oil zone storativity with respect to total reservoir storativity, dimensionless

Acknowledgments

The first author acknowledges Sultan Qaboos U. for supporting his graduate study at the U. of Southern California. The authors appreciate the support provided by Center for Study of Fractured Reservoirs supported by Texaco, Phillips, Unocal, Oryx, Elf Aquitaine, and Shell Western Operations.

References

1. Barenblatt, G.I., Zheltov, Yu. P., and Kochina, I.N.: "Basic Concepts in the Theory of Seepage of Homogeneous Liquids in Fissured Rocks [strata]," *J. Appl. Math. Mech.* (USSR) (1960) **24,** no. 5, 1286.
2. Warren, J.E. and Root, P.J.: "The Behavior of Naturally Fractured Reservoirs," *SPEJ* (September 1963) 245; *Trans.,* AIME, **228**.
3. Kazemi, H.: "Pressure Transient Analysis of Naturally Fractured Reservoirs with Uniform Fracture Distribution," *SPEJ* (December 1969) 451.
4. de Swaan, A.O.: "Analytical Solution for Determining Naturally Fractured Reservoirs Properties by Well Testing," *SPEJ* (June 1976) 117.
5. Najurrieta, H.L.: "A Theory for Pressure Transient Analysis in Naturally Fractured Reservoirs," *JPT* (July 1980) 1241.
6. Serra, K.V., Reynolds, A.C., and Raghavan, R.: "New Pressure Transient Analysis Methods for Naturally Fractured Reservoirs," paper SPE 10780 presented at the 1982 SPE California Regional Meeting, San Francisco, 24–26 March.
7. Streltsova T.D. : "Well Pressure Behavior of a Naturally Fractured Reservoir," *SPEJ* (October 1983) 769.
8. Abdassah, D. and Ershaghi, I.: "Triple-Porosity Systems for Representing Naturally Fractured Reservoirs," *SPEFE* (April 1986) 113.
9. Jalali-Yazdi, Y. and Ershaghi, I. : "A Unified Type Curve Approach for Pressure Transient Analysis of Naturally Fractured Reservoirs," paper SPE 16778 presented at the 1987 SPE Annual Technical Conference and Exhibition, Dallas, 27–30 September.
10. Jalali-Yazdi, Y.: "Pressure Transient Behavior of Heterogeneous Naturally Fractured Reservoirs," PhD Thesis, U. of Southern California (September 1987).
11. Peaceman D.W.: "Convection in Fractured Reservoir—The Effect of Matrix-Fissure Transfer on the Instability of a Density Inversion in a Vertical Fissure," SPEJ (October 1976) 269; *Trans.,* AIME, **261**.
12. Streltsova, T.D.: "Pressure Transient Analysis for Afterflow-Dominated Producing From a Reservoir With a Gas Cap," *JPT* (April 1981) 743.
13. Stehfest H.: "Algorithm 368: Numerical Inversion of Laplace Transform," D-5 Communication of the ACM, 13 (1), (January 1970) 47.

Appendix A—Derivation of the Diffusivity Equation

Transform Eq. 11 into Laplace space with the use of the following initial condition:

$$\Delta P_{Df} = \Delta P_{maD} = \Delta P_{Dfo} \text{ for all } r_D . \quad \text{(A-1)}$$

Eq. 11 becomes

$$\frac{\partial^2 \Delta\overline{P}_{Df}}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial \Delta\overline{P}_{Df}}{\partial r_D} = \frac{k_{fh}}{k_{fz}},$$

$$\left[\omega s\Delta\overline{P}_{Df} + (1-\omega)s\Delta\overline{P}_{maD} + \frac{(1-\omega_1)}{\omega_1}s\Delta\overline{P}_{Dfo}\right]. \quad \text{(A-2)}$$

To solve this equation for $\Delta\overline{P}_{Df}$, the relationship between $\Delta\overline{P}_{maD}$ and $\Delta\overline{P}_{Dfo}$ to $\Delta\overline{P}_{Df}$ is obtained as follows.

A. Relationship between $\Delta\overline{P}_{maD}$ and $\Delta\overline{P}_{Df}$. For the case of pseudosteady-state matrix-fracture interporosity flow condition, it is assumed that[2]

$$-(\phi C)_{oma}\frac{\partial p_{ma}}{\partial t} = \alpha\frac{k_{ma}}{\mu}(p_{ma} - p_f);$$

as a result,

$$-(\phi C)_{oma}\frac{r_w^2\mu}{k_{fh}}\frac{\partial p_{ma}}{\partial t} = \alpha\frac{r_w^2 k_{ma}}{k_{fh}}(p_{ma} - p_f). \quad \text{(A-3)}$$

As defined by Warren and Root,[2]

$$\lambda = \frac{\alpha r_w^2 k_{ma}}{k_{fh}}, \quad \text{(A-4)}$$

then $(\phi C)_{oma} h_o \dfrac{r_w^2}{\tau}\dfrac{\partial p_{ma}}{\partial t} = \lambda(p_f - p_{ma})$. (A-5)

Converting the above equation into dimensionless terms, it becomes

$$(1-\omega)\frac{\partial \Delta P_{maD}}{\partial t_D} = \lambda(\Delta P_{Df} - \Delta P_{maD}). \quad \text{(A-6)}$$

Taking Laplace transform of the above equation, the relationship between $\Delta\overline{P}_{maD}$ and $\Delta\overline{P}_{Df}$ is obtained

$$\Delta\overline{P}_{maD} = \frac{\lambda}{\lambda + (1-\omega)s}\Delta\overline{P}_{Df}. \quad \text{(A-7)}$$

For the case of unsteady-state matrix-fracture interporosity flow condition, the matrix pressure distribution, p_m, is assumed to be represented by the following diffusivity equation:[3-7]

$$\frac{\partial^2 p_{ma}}{\partial z_{ma}^2} = \frac{(\phi C)_{oma}\mu}{k_{ma}}\frac{\partial p_{ma}}{\partial t}. \quad \text{(A-8)}$$

Define $z_{maD} = \dfrac{z_{ma}}{l_{ma}}$, (A-9)

where l_{ma} = thickness of matrix block, which contributes flow to horizontal fracture and z_m = vertical coordinate of the matrix block.

Eq. A-8 is converted into dimensionless terms:

$$\lambda\frac{\partial^2 p_{maD}}{\partial z_{maD}^2} = (1-\omega)\frac{\partial p_{maD}}{\partial t_D}. \quad \text{(A-10)}$$

Transforming the above equation into Laplace space with the following conditions:

1. initial condition:

$\Delta P_{maD} = 0$, for all r_D.

2. boundary conditions:

$\Delta P_{maD} = \Delta P_{aD}$ at z_{maD}–0 for all t_D,

$\dfrac{\partial \Delta P_{Dm}}{\partial z_{mD}} = 0$ at $z_{maD} = \dfrac{l_{ma}}{2}$ for all t_D, the relationship between $\Delta\bar{P}_{Dm}$ and $\Delta\bar{P}_{Df}$ is obtained as

$$\Delta\bar{P}_{maD} = \sqrt{\frac{\lambda}{(1-\omega)s}}\tanh\left[\sqrt{\frac{(1-\omega)s}{\lambda}}\right]\Delta\bar{P}_{Df}. \quad \text{(A-11)}$$

B. Relationship between $\Delta\bar{P}_{Dfo}$ and $\Delta\bar{P}_{Df}$. It is assumed that the crossflow at the gas-oil interface in fractures is proportional to the difference between the average drawdown over the thickness of the oil zone and the drawdown at the interface[12]:

$$S_{tg}\frac{\partial p_{fo}}{\partial t} = \frac{-k_{fz}}{\mu h_o}\left[p_f(z)\right]_{z=0}^{z=h_o} = \frac{-k_{fz}}{\mu h_o}\left(p_{fo} - p_f\right). \quad \text{(A-12)}$$

Converting the above equation into dimensionless terms results in

$$\frac{S_{tg}}{S_{to}}\frac{\partial \Delta P_{Df}}{\partial t_D} = \frac{r_w^2 k_{fz}}{h_o^2 k_{fh}}\left(\Delta P_{Df} - \Delta P_{Dfo}\right). \quad \text{(A-13)}$$

Define $\lambda_1 = \dfrac{r_w^2 k_{fz}}{h_o^2 k_{fh}}$, (A-14)

then $$\frac{(1-\omega_1)}{\omega_1}\frac{\partial \Delta P_{Dfo}}{\partial t_D} = \lambda_1\left(\Delta P_{Df} - \Delta P_{Dfo}\right). \quad \text{(A-15)}$$

Transforming the above equation into Laplace space, the relationship between $\Delta\bar{P}_{Dfo}$ and $\Delta\bar{P}_{Df}$ is obtained:

$$\Delta\bar{P}_{Dfo} = \frac{\lambda_1\omega_1}{\lambda_1\omega_1 + (1-\omega_1)s}\Delta\bar{P}_{Df}. \quad \text{(A-16)}$$

Substituting $\Delta\bar{P}_{Dm}$ and $\Delta\bar{P}_{Dfo}$ in Eq. A-2, the following dimensionless diffusivity equation in Laplace space is obtained:

$$\frac{\partial^2 \Delta\bar{P}_{Df}}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial \Delta\bar{P}_{Df}}{\partial r_D} = \frac{k_{fh}}{k_{fz}} s f(s)\Delta\bar{P}_{Df}. \quad \text{(A-17)}$$

For pseudosteady-state matrix fracture interporosity flow mode, $f(s)$ is defined as

$$f(s) = \omega + \frac{(1-\omega)\lambda}{\lambda + (1-\omega_1)s} + \frac{(1-\omega_1)\lambda_1}{\lambda_1\omega_1 + (1-\omega_1)s}, \quad \text{(A-18)}$$

and for unsteady-state matrix fracture interporosity flow mode, $f(s)$ is defined as

$$f(s) = \omega + \sqrt{\frac{(1-\omega)\lambda}{s}}\tanh\left[\sqrt{\frac{(1-\omega)s}{\lambda}}\right] + \frac{(1-\omega_1)\lambda_1}{\lambda_1\omega_1 + (1-\omega_1)s}. \quad \text{(A-19)}$$

Appendix B—Derivation of the Solutions in Laplace Space

Eq. A-17 is a zero-order modified bessel equation that has the following general solution:

$$\Delta\bar{P}_{Df}(r_D, s) = AI_0(r_D y) + BK_0(r_D y), \quad \text{(B-1)}$$

where $$y = \sqrt{\frac{k_{fh}}{k_{fz}} s f(s)}. \quad \text{(B-2)}$$

A and B are coefficients determined by applying the inner and outer boundary conditions of the oil zone. I_0 and K_0 are the zero-order modified Bessel function of the first and second kind, respectively.

Solving for A and B.

1. Inner boundary condition $(r = r_w)$.

The well is assumed to have a vanishingly small radius and produces at a constant rate q_o under no effect of skin damage or wellbore storage $(p_w = p_f)$. The constant flow rate solution is expressed as

$$\left(r\frac{\partial p_f}{\partial r}\right)_{r=r_w} = \frac{-q_o}{2\pi T}. \quad \text{(B-3)}$$

In dimensionless terms,

$$\frac{\partial \Delta\bar{P}_{Df}}{\partial r_D} = -\sqrt{\frac{k_{fh}}{k_{fz}}}. \quad \text{(B-4)}$$

Applying the Laplace transformation of the above equation, it becomes

$$\frac{\partial \Delta\bar{P}_{Df}}{\partial r_D} = -\frac{1}{s}\sqrt{\frac{k_{fh}}{k_{fz}}}. \quad \text{(B-5)}$$

2. Outer boundary condition.

During the limited time of well test, reservoir is assumed infinitely large. As a result,

$$\lim_{r\to\infty}\left(\frac{\partial p_f}{\partial r}\right)_r = 0 \quad \text{(B-6)}$$

or $$\frac{\partial \Delta\bar{P}_{Df}}{\partial r_D} = 0. \quad \text{(B-7)}$$

Then, from inner and outer boundary conditions, the coefficients are

$$A = 0 \quad \text{(B-8)}$$

and $$B = \frac{1}{s\sqrt{\frac{k_{fz}}{k_{fh}}}\sqrt{sf(s)}\,K_1\left(r_D\sqrt{sf(s)}\right)}. \quad \text{(B-9)}$$

Substituting the values of A and B in Eq. B-1, a dimensionless wellbore pressure drop, in Laplace domain, of a naturally fractured reservoir producing from an oil zone overlain by a gas cap (at

$r = r_w$ or $r_D = \sqrt{\dfrac{k_{fz}}{k_{fh}}}$) is obtained as follows:

$$\Delta\bar{P}_{Dw} = \frac{K_0(x)}{sxK_1(x)}, \quad \text{(B-10)}$$

where $$x = \sqrt{sf(s)}. \quad \text{(B-11)}$$

SPEFE

Ali Soud Al-Bemani is a lecturer in petroleum and mining engineering at Sultan Qabous U. in Oman. Previously, he worked as a production engineer in Oman. His area of research interest is the modeling of pressure-transient tests. He holds a PhD degree in petroleum engineering from the U. of Southern California. **Iraj Ershaghi** is a professor and director of the Petroleum Engineering Program at the U. of Southern California. His areas of research include pressure-transient testing, geothermal systems, and reservoir characterization. He holds a BS degree from U. of Tehran and MS and PhD degrees from the U. of Southern California, all in petroleum engineering. Ershaghi received the 1996 SPE Distinguished Member Award, the 1983 SPE Distinguished Award for Petroleum Engineering Faculty, and was a member of the Editorial Review Committee during 1981–83, 1987–89, and 1995–97.

Al-Bemani

Ershaghi