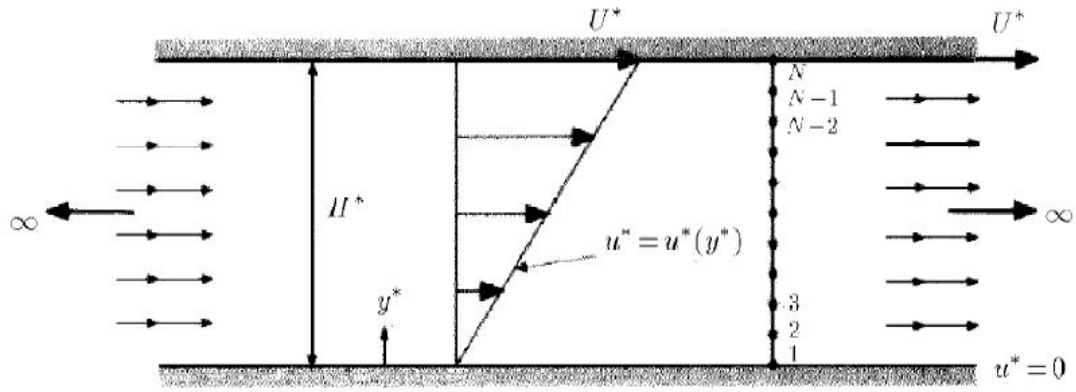
CFD1 Project#1

Report and MATLAB m files must be delivered by 30/8/96 (Deadline)

1. Consider flow between an impulsively moving and a stationary plate (unsteady Couette flow):



The governing equation is

$$\frac{\partial u^*}{\partial t^*} - \nu^* \frac{\partial^2 u^*}{\partial y^{*2}}$$

with the initial condition:

$$\text{I.C:} \qquad u^*(y^*, 0) = \begin{cases} 0, & y^* < H^* \\ U^*, & y^* = H^* \end{cases}$$

and the boundary conditions:

$$\text{B.C:} \qquad u^*(0, t^*) = 0, \qquad u^*(H^*, t^*) = U^*$$

(a) Nondimensionalize the problem using the following parameters:

$$t = \frac{t^*}{H^*/U^*}, \qquad y = \frac{y^*}{H^*}, \qquad u = \frac{u^*}{U^*}$$

and show that the governing equation leads to

$$\frac{\partial u}{\partial t} = \frac{1}{Re_H}\frac{\partial^2 u}{\partial y^2}$$

where Re_H is the Reynolds number based on the height H^* between the two plates and the velocity of the upper plate U^*

$$Re_H = \frac{U^* H^*}{\nu^*}$$

Give the appropriate nondimensional form of the initial and boundary conditions.

(b) Write a CFD code to solve the above problem using the combined method (the θ-method) for time marching and central differences for spatial discretization. Let the grid spacing be constant with spacing Δy, and constant time stepping of Δt. Thus the interior points ($j = 2, 3..., N-1$) are given by:

$$u_j^{n+1} = u_j^n + r\,(1-\theta)\left(u_{j-1}^n - 2u_j^n + u_{j+1}^n\right) + r\theta\left(u_{j-1}^{n+1} - 2u_j^{n+1} + u_{j+1}^{n+1}\right)$$

where

$$r = \frac{\Delta t}{Re_H\,(\Delta y)^2}$$

Show that converting this formula to delta operator form results in:

$$-r\theta\Delta u_{j-1}^n + (1+2r\theta)\,\Delta u_j^n - r\theta\Delta u_{j+1}^n = r\left(u_{j-1}^n - 2u_j^n + u_{j+1}^n\right)$$

in which at the boundaries:

$$\Delta u_1^n = \Delta u_N^n = 0$$

Then, one can use a tridiagonal solver to calculate Δu_j^n and update to the new solution time through:

$$u_j^{n+1} = u_j^n + \Delta u_j^n$$

Consider the three following cases (let $Re_H = 1.0$ and $N = 21$):

- $\theta = 0.0$, explicit Euler method
  Note: For this scheme, you don't need to use the tridiagonal solver!
- $\theta = 0.5$, Crank-Nicolson or Trapezoidal method
- $\theta = 1.0$, implicit Euler method

To obtain a global measure for the convergence of the solution to the steady state, calculate the L_2 norm of the change in u for each time step and plot L_2 norm vs the iteration number

$$L_2\Delta u^n = \sqrt{\frac{1}{N-2}\sum_{j=2}^{N-1}\left(\Delta u_j^n\right)^2}$$

(c) Show that with a very large time step ($\Delta t = 1 \times 10^9$), the implicit Euler scheme will converge in one time step (large L_2 first time step and then machine zero after that). For the others two methods, make a table of iterations required to reach the steady state solution as a function of r. Try to find an optimal time step to converge these methods so that the L_2 norm is reduced by a factor of 10^{-8} in the minimum number of time steps. Plot all three of the L_2 norm time histories on one log-log plot

(d) Explore the stability limits ($r \;\le\; ?$) of the explicit method and what happens when Δt is very large for the Crank-Nicolson method. To see the effect of large time step size on the time accuracy of the Crank-Nicolson method, with $r = 1, 5, 10$, and 20, make a table of velocity profiles corresponds to the same intermediate time $t = 0.3$. Compare the numerical results with the exact solution. Repeat the calculations for $r = 4000$ and plot the two intermediate transient velocity profile (one after 40 time steps and the others after 200 time steps). Do the results show the physical behavior? Any conclusions?

(e) Using the Crank-Nicolson scheme with $r = 1/2$ and 1, make a table of velocity profiles and plot the velocity distribution after 2, 12, 36, 60, 240, and 360 iterations (time steps). Repeat for the explicit Euler and implicit Euler time integration using the same time step size. Compare the numerical results with the exact solution. Give your conclusions.

Note: The exact solution of the unsteady couette flow is given by

$$\frac{u^*(y^*,t^*)}{U^*} = \frac{y^*}{H^*} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(n\pi\frac{y^*}{H^*}\right)\exp\left(-(n\pi)^2\frac{\nu^*t^*}{H^{*2}}\right)$$

(f) Now, modify the unsteady boundary condition at the top according to

$$\frac{u^*(H^*,t^*)}{U^*} = \sin\left(20\pi\frac{t^*}{H^*/U^*}\right)$$

and solve for four periods, i.e., $0 \le t \le 4$ using a reasonable time step and Crank-Nicolson time marching. Note:

$$\Delta u_N^n = u_N^{n+1} - u_N^n$$

Does the flow become periodic? Any conclusions?

(g) Prepare a report describing your program and discussing your results.