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On LMI conditions to design observer-based controllers for linear systems with parameter uncertainties ☆

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ABSTRACT

This paper deals with the problem of observer-based stabilization for linear systems with parameter inequality. A new design methodology is proposed thanks to a judicious use of the famous Young relation. This latter leads to a less restrictive synthesis condition, expressed in term of Linear Matrix Inequality (LMI), than those available in the literature. Numerical comparisons are provided in order to show the validity and superiority of the proposed method.

1. Introduction

Uncertainties and perturbations are frequently encountered in practical control systems because it is often very difficult to obtain exact mathematical models. This is due to environmental noises, data errors, ageing of systems, uncertain or slowly varying parameters, etc. The presence of uncertainties may cause instability and bad performances on a controlled system. Therefore, considerable efforts have been assigned to the robust stability and stabilization of linear systems with parameter uncertainties. For recent works, we refer the readers to Arcak and Kokotovic (2001), Gao, Wang, and Zhao (2003), Heemels, Daafouz, and Millerioux (2010), Ibrir and Diopt (2008) and Lien (2004a).

In some real models, state feedback control might fail to guarantee the stabilizability when some of the system states are not measurable. This is why a state observer is required and included in the feedback control (Atassi & Khalil, 2000; Hendricks & Luther, 2001; Karafyllis & Kravaris, 2005; Pagilla, King, Dreinhoefer, & Garimella, 2000; Song & Hedrick, 2004). Observer-based controllers are often used to stabilize unstable systems or to improve the system performances. The observer-based stabilization problem for both deterministic and stochastic linear systems is well characterized in Kalman et al. (1960) and Luenberger (1971). An optimal observer-based control strategy is given in both cases. Nevertheless, for uncertain systems, there is no generic algorithm. Tremendous research activities have been developed in the recent years for both linear and nonlinear systems with uncertain parameters (Gao et al., 2003; Heemels et al., 2010; Ibrir, 2008; Lin, Guan, Liu, & Shi, 2001; Lu, Tsai, Jong, & Su, 2003). However, the obtained methods remain conservative (CRUSIUS, 1999; Lien, 2004a,b).

Furthermore, there are considerable approaches in the literature dealing with the solution of the problem of output feedback controller design by directly using BMI conditions (Lens, 2009; Ostertag, 2008). However, it is well known that solving a BMI is an NP-hard problem from the complexity point of view (Toker & Osbay, 1995), which is a drawback for numerical implementations. To overcome this issue, some important and general dynamic output-feedback approaches have been presented in Scherer, Gahinet, and Chilali (1997) using relevant arguments and judicious mathematical tools. Nevertheless, in this paper we focus our study on observer-based controllers design, which presents some difficulties due to its particular structure.

Motivated by the above discussions, a new design methodology is proposed. By using the Lyapunov function approach combined with a judicious use of the Young relation, we get a new LMI synthesis methodology. This leads to a quite simple LMI condition that is numerically tractable with any LMI software. It is important to underline that the proposed LMI condition is solved without any additional restrictive conditions, namely the *a priori* choice of the Lyapunov matrix and the equality constraint (Lien, 2004a).

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Our approach is applied to the numerical example taken from Lien (2004a) aiming to provide comparisons and to show the superiority of the proposed new design methodology. A numerical evaluation of the conservatism is also provided for more than 50000 randomly generated systems.

2. Problem formulation

Consider the continuous-time uncertain linear systems described by the following equations

$$\dot{x} = \Big(A + \Delta A(t)\Big)x + Bu \tag{1a}$$

$$y = \Big(C + \Delta C(t)\Big)x \tag{1b}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the output measurement and $u \in \mathbb{R}^m$ is the control input vector. A, B and C are constant matrices of adequate dimensions. First, we consider the following assumptions:

- the pairs (A, B) and (A, C) are respectively stabilizable and detectable;
- there exist matrices $M_i, N_i, F_i(t), i = 1, 2$, of appropriate dimensions so that

$$\Delta A(t) = M_1F_1(t)N_1, \qquad \Delta C(t) = M_2F_2(t)N_2 \tag{2}$$

where the unknown matrices $F_i(t)$ satisfy the condition

$$F_i^T(t)F_i(t) \le I, \quad \text{for } i = 1, 2. \tag{3}$$

The observer-based controller we consider in this paper is under the form:

$$\dot{\hat{x}} = A\hat{x} + Bu + L\Big(y - C\hat{x}\Big) \tag{4a}$$

$$u = -K\hat{x} \tag{4b}$$

where $\hat{x} \in \mathbb{R}^n$ is the estimate of x, $K \in \mathbb{R}^{m\times n}$ is the control gain, $L \in \mathbb{R}^{n\times p}$ is the observer gain. Hence, we can write

$$\overbrace{\begin{bmatrix} x \\ e \end{bmatrix}}^{\cdot} = \begin{bmatrix} \Big(A - BK + \Delta A(t)\Big) & BK \\ \Big(\Delta A(t) - L\Delta C(t)\Big) & \Big(A - LC\Big) \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \tag{5}$$

where $e = x - \hat{x}$ represents the estimation error.

Now, consider the Lyapunov function candidate

$$V(x, e) = \begin{bmatrix} x \\ e \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} = x^TPx + e^TRe. \tag{6}$$

Notice that the Lyapunov function (6) is well known in the literature for this problem, especially in Lien (2004a) which is the main motivation of this paper. Of course, the block-diagonal structure imposed in (6) can introduce conservatism, but it is difficult to give an assessment on how much conservatism is introduced by this structure.

After calculating the derivative of V along the trajectories of (5), we have:

$$\dot{V}(x, e) = \begin{bmatrix} x \\ e \end{bmatrix}^T \underbrace{\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ (\star) & \Sigma_{22} \end{bmatrix}}_{\Sigma} \begin{bmatrix} x \\ e \end{bmatrix} \tag{7}$$

where the matrix Σ is given by (8):

$$\Sigma_{11} = (A - BK)^TP + P(A - BK) + (\Delta A)^TP + P(\Delta A) \tag{8a}$$

$$\Sigma_{12} = PBK + (\Delta A - L(\Delta C))^TR \tag{8b}$$

$$\Sigma_{22} = (A - LC)^TR + R(A - LC). \tag{8c}$$

and $(\star)$ is used for the blocks induced by symmetry.

Notice that $\dot{V}(x, e) < 0, \forall\, x \neq 0$ and $e \neq 0$ if the matrix inequality $\Sigma < 0$ holds. However, this matrix inequality is a BMI, which is hardly tractable numerically. On the other hand, linearizing the BMI $\Sigma < 0$ is a very difficult task because of the presence of the coupling term PBK; the congruence principle cannot be applied to linearize it. In this paper, we focus on the methodology of Lien (2004a) which investigated the same challenge. To clarify the different new developments of our work and the improvements with respect to existing results, we summarize, in what follows, the methodology of Lien (2004a) which concerns the same class of systems: first, in Lien (2004a, Theorem 1), the author made the particular choice of P, namely $P = I$, in order to linearize the BMI $\Sigma < 0$. In Lien (2004a, Theorem 2), to avoid the restriction in the a priori choice of $P = I$, he introduced a new matrix $\hat{P}$ satisfying the additional strong equality condition $PB = B\hat{P}$ to linearize $\Sigma < 0$. To overcome this difficulty, many research activities have been recently proposed in the literature, but deals with systems in discrete-time case (Heemels et al., 2010; Ibrir, 2008; Ibrir & Diopt, 2008). In the next section, we propose a novel manner to overcome the obstacle of the coupling PBK without equality constraint or an a priori choice of the matrix P.

3. Main theoretical results

3.1. New LMI design methodology

Theorem 1. *System (1) is asymptotically stabilizable by (4) if for a fixed scalar $\epsilon_4 > 0$, there exist two positive definite matrices $\mathcal{Z} \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times n}$, two matrices $\hat{K} \in \mathbb{R}^{n\times m}$, $\hat{L} \in \mathbb{R}^{p\times n}$ and positive scalars $\epsilon_1 > 0, \epsilon_2 > 0$ and $\epsilon_3 > 0$ so that the LMI condition (9) given in Box I is feasible. Hence, the stabilizing observer-based control gains are given by $K = \hat{K}^T\mathcal{Z}^{-1}$ and $L = R^{-1}\hat{L}^T$.*

Proof. We rewrite the matrix Σ as a sum of two matrices, one contains the uncertainties and an other one without the uncertainties, that is:

$$\Sigma = \begin{bmatrix} A_{11} & PBK \\ (\star) & A_{22} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ (\star) & 0 \end{bmatrix} \quad \text{where}$$

$$A_{11} = (A - BK)^TP + P(A - BK)$$

$$A_{22} = (A - LC)^TR + R(A - LC)$$

$$B_{11} = (\Delta A)^TP + P(\Delta A), \qquad B_{12} = (\Delta A - L(\Delta C))^TR.$$

We pre and post multiply Σ by the matrix $\mathrm{diag}(P^{-1}, I)$, putting $\mathcal{Z} = P^{-1}$ and by developing ΔA and ΔC we obtain:

$$\begin{aligned} \widetilde{\Sigma} = &\begin{bmatrix} \mathbb{A}_{11} & 0 \\ 0 & A_{22} \end{bmatrix} + \begin{bmatrix} BK \\ 0 \end{bmatrix} \begin{bmatrix} 0 & I \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} (BK)^T & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \mathcal{Z}N_1^T \\ 0 \end{bmatrix} F_1^T(t) \begin{bmatrix} M_1^T & 0 \end{bmatrix} + \begin{bmatrix} M_1 \\ 0 \end{bmatrix} F_1(t) \begin{bmatrix} N_1\mathcal{Z} & 0 \end{bmatrix} \\ &+ \begin{bmatrix} N_1^T \\ 0 \end{bmatrix} F_1^T(t) \begin{bmatrix} 0 & M_1^TR \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ RM_1 \end{bmatrix} F_1(t) \begin{bmatrix} N_1 & 0 \end{bmatrix} + \begin{bmatrix} -N_2^T \\ 0 \end{bmatrix} F_2^T(t) \begin{bmatrix} 0 & M_2^TL^TR \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ RLM_2 \end{bmatrix} F_2(t) \begin{bmatrix} -N_2 & 0 \end{bmatrix} \end{aligned} \tag{10}$$

where $\mathbb{A}_{11} = \mathcal{Z}\Big(A - BK\Big)^T + \Big(A - BK\Big)\mathcal{Z}$.

$$\begin{bmatrix} \overbrace{\begin{bmatrix} \mathbb{P}(\mathcal{Z},\hat{K},\epsilon_1,\epsilon_2,\epsilon_3) & 0 \\ 0 & A^TR - C^T\hat{L} + RA - \hat{L}^TC \end{bmatrix}}^{\mathcal{Q}_1} & \overbrace{\begin{bmatrix} \begin{bmatrix} B\hat{K}^T & 0 \\ 0 & I \end{bmatrix} & \begin{bmatrix} \mathcal{Z}N_1^T \\ 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ RM_1 & \hat{L}^TM_2 \end{bmatrix} \end{bmatrix}}^{\mathcal{Q}_2^T} \\ \overbrace{\begin{bmatrix} \begin{bmatrix} \hat{K}B^T & 0 \\ 0 & I \end{bmatrix} \\ \begin{bmatrix} N_1\mathcal{Z} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & M_1^TR \\ 0 & M_2^T\hat{L} \end{bmatrix} \end{bmatrix}}^{\mathcal{Q}_2} & \overbrace{\begin{bmatrix} \begin{bmatrix} -\frac{1}{\epsilon_4}\mathcal{Z} & 0 \\ 0 & -\epsilon_4\mathcal{Z} \end{bmatrix} & 0 & 0 \\ 0 & -\epsilon_1 I & 0 \\ 0 & 0 & \begin{bmatrix} -\epsilon_2 I & 0 \\ 0 & -\epsilon_3 I \end{bmatrix} \end{bmatrix}}^{\mathcal{Q}_3} \end{bmatrix} < 0 \quad (9)$$

$$\mathbb{P}(\mathcal{Z},\hat{K},\epsilon_1,\epsilon_2,\epsilon_3) = \mathcal{Z}A^T - \hat{K}B^T + A\mathcal{Z} - B\hat{K}^T + \epsilon_1 M_1M_1^T + \epsilon_2 N_1^TN_1 + \epsilon_3 N_2^TN_2$$

Box I.

Now, by applying the Young relation (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994) in a judicious manner to retrieve the variable $\mathcal{Z}K^T$, we get the following inequality:

$$\begin{aligned} \widetilde{\Sigma} \le & \begin{bmatrix} \mathbb{A}_{11} & 0 \\ 0 & A_{22} \end{bmatrix} + \epsilon_4 \begin{bmatrix} BK \\ 0 \end{bmatrix} \mathcal{Z} \begin{bmatrix} BK \\ 0 \end{bmatrix}^T + \frac{1}{\epsilon_4} \begin{bmatrix} 0 \\ I \end{bmatrix} \mathcal{Z}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}^T \\ & + \frac{1}{\epsilon_1} \begin{bmatrix} \mathcal{Z}N_1^T \\ 0 \end{bmatrix} \begin{bmatrix} \mathcal{Z}N_1^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} \epsilon_1 M_1M_1^T & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \epsilon_2 N_1^TN_1 & 0 \\ 0 & 0 \end{bmatrix} \\ & + \frac{1}{\epsilon_2} \begin{bmatrix} 0 \\ RM_1 \end{bmatrix} \begin{bmatrix} 0 \\ RM_1 \end{bmatrix}^T + \begin{bmatrix} \epsilon_3 N_2^TN_2 & 0 \\ 0 & 0 \end{bmatrix} \\ & + \frac{1}{\epsilon_3} \begin{bmatrix} 0 \\ \hat{L}^TM_2 \end{bmatrix} \begin{bmatrix} 0 \\ \hat{L}^TM_2 \end{bmatrix}^T \end{aligned} \quad (11)$$

where $\epsilon_1, \epsilon_2, \epsilon_3$ and ϵ_4 are positive constants. Using the change of variables $\hat{K} = \mathcal{Z}K^T$ and $\hat{L} = L^TR$, the right hand side of inequality (11) takes the following form:

$$\mathcal{Q}_1 - \mathcal{Q}_2\mathcal{Q}_3^{-1}\mathcal{Q}_2^T \quad (12)$$

where $\mathcal{Q}_1$, $\mathcal{Q}_2$ and $\mathcal{Q}_3$ are given in (9). Finally, from Schur's Lemma (see, e.g. Boyd et al., 1994), we deduce that the inequality $\widetilde{\Sigma} < 0$ is satisfied if the LMI (9) is feasible.

Remark 2. Notice that the matrix inequality (9) is an LMI if the scalar variable $\epsilon_4 > 0$ is fixed *a priori*. In order to overcome the difficulty of the choice of ϵ_4, we proceed as in Li and Fu (1997, Remark 5) by using the gridding method. This latter consists to scale ϵ_4 by defining $\kappa = \frac{\epsilon_4}{1+\epsilon_4}$ (and then $\epsilon_4 = \frac{\kappa}{1-\kappa}$). We know that $\epsilon_4 > 0$ if and only if $\kappa \in]0, 1[$. Then, we assign a uniform subdivision of the interval $]0, 1[$ and we solve the LMI (9) for each value of this subdivision. On the other hand, using some additional inequality constraints, we can obtain an LMI on ϵ_4. For instance, we can use the following constraint and approximation

$$\mathcal{Z} > \alpha I \quad \text{and} \quad -\frac{1}{\epsilon_4} I \le -(2-\epsilon_4)I \quad (13)$$

leading to an LMI on α and $\beta = \alpha\epsilon_4$. Indeed, thanks to (13), we have

$$-\frac{1}{\epsilon_4}\mathcal{Z} \le -(2-\epsilon_4)\mathcal{Z} \le -(2-\epsilon_4)\alpha I = -(2\alpha-\beta)I$$

and

$$-\epsilon_4\mathcal{Z} \le -\epsilon_4\alpha I = -\beta I.$$

Hence, to have the resulting LMI, it suffices to replace in (9) the blocks $\frac{1}{\epsilon_4}\mathcal{Z}$ and $\epsilon_4\mathcal{Z}$ by $(2\alpha-\beta)I$ and βI, respectively.

3.2. On the necessary conditions of the proposed design methodology

This part is devoted to some remarks about the feasibility of the proposed LMI condition. A discussion on the necessary conditions for the feasibility of (9) is provided. It should be noticed that the necessary condition for the feasibility of the LMI (9) is $\mathcal{Q}_1 < 0$, which is equivalent to the stabilizability and detectability of the system (1) without uncertainties. However, in Lien (2004a, Theorems 1 and 2), the necessary condition for the feasibility of (8) and (10) is analytically more conservative than the stabilizability of (A, B) for some systems.

To illustrate this statement, we consider the particular class of systems with a matrix $B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$, where B_1 is of appropriate dimension. Now, we write

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \qquad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}, \qquad K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}.$$

The equality constraint $PB = B\hat{P}$, required in Lien (2004a, Theorem 2, Eq. (11)), leads to $P_{12} = B^{\perp}PB = 0$ with $B^{\perp} = \begin{bmatrix} 0 & I \end{bmatrix}$, which means that the Lyapunov matrix P is diagonal. On the other hand, the LMI (10) in Lien (2004a) means that

$$\left(A - BK\right)^T P + P\left(A - BK\right) < 0,$$

which can be rewritten under the detailed form

$$\begin{bmatrix} \mathcal{T} + \mathcal{T}^T & A_{21}^TP_{22} + P_{11}\left(A_{12} - B_1K_2\right) \\ (\star) & A_{22}^TP_{22} + P_{22}A_{22} \end{bmatrix} < 0,$$

where

$$\mathcal{T} = P_{11}\left(A_{11} - B_1K_1\right).$$

Hence, A_{22} must be Schur stable to guarantee

$$A_{22}^TP_{22} + P_{22}A_{22} < 0. \quad (14)$$

Notice also that the feasibility of the LMI (10) in Lien (2004a) requires

$$A_{22}^T + A_{22} < 0 \quad (15)$$

since, in particular, we have $P_{22} = I$. To sum up, for this type of systems, in addition to the stabilizability and the detectability, the necessary condition for the feasibility of (8) and (10)–(11) in Lien (2004a) is the Schur stability of A_{22} in the sense of (14) and (15), respectively. This shows analytically the superiority of the proposed design methodology at least for this particular class of systems.

4. Numerical comparisons and conservatism evaluation

4.1. Numerical examples

In the sequel, we give two simple numerical examples in order to show the superiority of our design method.

4.1.1. Example 1: on the necessary conditions

Consider the following example without uncertainties:

$$A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

As shown in the previous section, the LMI design methodology presented in Lien (2004a) does not work on this simple example. Indeed, for $P = I$ or to satisfy the equality constraint $PB = B\hat{P}$ required in Lien (2004a), we should have necessary $A_{22}^T P_{22} + P_{22}A_{22} = 6P_{22} < 0$ which contradicts the definition of $P_{22} > 0$. Otherwise, by using Matlab LMI toolbox, our LMI (9) provides the following observer and controller gains:

$$K = \begin{bmatrix} 7.2159 & 19.1095 \end{bmatrix}, \qquad L = \begin{bmatrix} 4.2678 \\ 19.7414 \end{bmatrix}.$$

4.1.2. Example 2: about the example of Lien (2004a)

We take the same example than that of Lien (2004a). The system has the following parameters:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 1 & -2 & -5 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$\Delta A(t) = \begin{bmatrix} 0 & 0 & a(t) \\ 0 & b(t) & 0 \\ c(t) & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \qquad \Delta C(t) = \begin{bmatrix} 0 & d(t) & 0 \end{bmatrix}$$

where $a(t) \leq \alpha$, $b(t) \leq \beta$, $c(t) \leq \gamma$ and $d(t) \leq \delta$. The uncertainties can be rewritten under the form (2) with

$$F_1(t) = \begin{bmatrix} \frac{a(t)}{\alpha} & 0 & 0 \\ 0 & \frac{b(t)}{\beta} & 0 \\ 0 & 0 & \frac{c(t)}{\gamma} \end{bmatrix}, \qquad N_1 = \begin{bmatrix} 0 & 0 & \alpha \\ 0 & \beta & 0 \\ \gamma & 0 & 0 \end{bmatrix}$$

$$M_1 = M_2 = I, \; F_2(t) = \frac{d(t)}{\delta}, \; N_2 = \begin{bmatrix} 0 & \delta & 0 \end{bmatrix}.$$

In Table 1, we test the feasibility of LMIs (8), (10)–(11) in Lien (2004a) and our proposed LMI (9). The superiority of our approach to that in Lien (2004a) is quite clear from this table. Assuming that $\alpha = \beta = \gamma = \delta$, our design methodology provides solutions for all $\alpha \leq 5.54$ with $\epsilon_4 = 0.01$, while the design methods given in Lien (2004a) fail for $\alpha > 1.36$. In fact, notice that for this example we have used the gridding technique to tackle the constant ϵ_4 of LMI (9), and not the LMIs of Remark 2.

4.2. Numerical study and comparisons

This subsection is dedicated to show the superiority of our proposed new design methodology. We will evaluate numerically the conservatism of the LMI method considered in this paper and their relation to the detectability and stabilizability conditions. For this, we consider linear systems without uncertainties in order to demonstrate that our design method is not restrictive compared to the necessary conditions. The influence of the number of inputs is also addressed.

Table 1
Superiority of the proposed LMI methodology.

Method	LMI (8) in Lien (2004a)	LMIs (10)–(11) in Lien (2004a)	LMI (9) $\epsilon_4 = 0.01$
α_{max}	1.32	1.36	5.54

Table 2
Percentage of systems for $n = 3$, $m = 2$, $p = 1$.

LMI (8) in Lien (2004a)	LMI (10)–(11) in Lien (2004a)	LMI (9)
40.4%	70.4%	100%

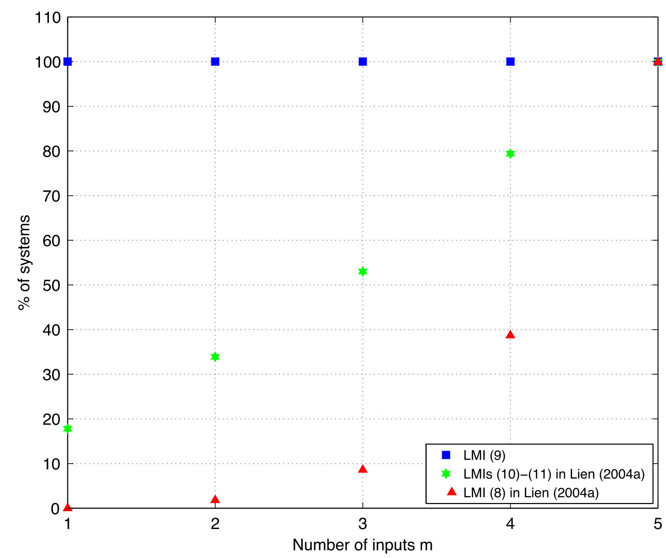

Fig. 1. Percentage of systems for different values of m with $n = 5$, $p = 1$.

4.2.1. Evaluation of the conservatism

We have randomly generated 1000 stabilizable and detectable systems of dimension $n = 3$, $m = 2$ and $p = 1$. The results are summarized in Table 2. The condition (9) of Theorem 1 provides an observer-based controller for 100% of these systems. Nevertheless, the LMIs (8) and (10)–(11) in Lien (2004a) succeeded for 40.4% and 70.4%, respectively.

4.2.2. Relationship with the number of inputs

The approach of Lien (2004a) depends on the input matrix B. Indeed, the *a priori* choice of the Lyapunov matrix $P = I$ and the introduction of the strong equality constraint $PB = B\hat{P}$ are due to the presence of the bilinear term PBK. This latter depends on the matrix B, and then on the number of inputs m. For this reason, this part is devoted to the influence of the number of inputs on the feasibility of the LMIs (8), (10)–(11) in Lien (2004a). We have randomly generated 1000 detectable and stabilizable systems of dimension $n = 5$, $p = 1$ and m ranging from 1 to n. The results are presented in Fig. 1, which gives the percentage of systems for which the different methods addressed in this note succeeded for each value of m. It is well clear, from Fig. 1, that our proposed design methodology succeeded for 100% of the systems and for each $m \leq n$. However, the results obtained by LMIs (8), (10)–(11) in Lien (2004a) depend on the value of m.

Remark 3. It should be noticed that more than 50000 randomly generated detectable and stabilizable systems are tested. Indeed, we have used 1000 randomly generated systems for each $m = 1, \ldots, 5$ and for 10 different values of ϵ_4. Our proposed design method is found feasible for 100% of them. This shows, numerically, the superiority of our LMI approach.

5. Conclusion

In this paper, a linear matrix inequality approach to design observer-based controllers for uncertain linear systems is addressed. We have shown that a judicious use of the Young relation led to a less restrictive LMI condition. A comparison study of the results established in this note with respect to those given in Lien (2004a) and Lien (2004b) shows the superiority of the proposed design methodology.

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