# A Comb Filter Design Using Fractional-Sample Delay

Soo-Chang Pei and Chien-Cheng Tseng

***Abstract*— In this paper, a new comb filter design method using fractional sample delay is presented. First, the specification of the comb filter design is transformed into that of fractional delay filter design. Then, conventional finite impulse response (FIR) and allpass filter design techniques are directly applied to design fractional delay filter with transformed specification. Next, we develop a constrained fractional delay filter design approach to improve the performance of the direct design method. Finally, several design examples and an experiment of the power line interference removal in electrocadiogram (ECG) signal are demonstrated to illustrate the effectiveness of this new design approach.**

***Index Terms*—Comb filter, fractional delay filter, harmonic interference removal.**

## I. Introduction

In many applications of signal processing it is desired to remove harmonic interferences while leaving the broad-band signal unchanged. Examples are in the areas of biomedical engineering, communication and control [1]–[5]. A typical one is to cancel power line interference in the recording of electrocardiogram (ECG). Usually, this task can be achieved by the comb filter whose desired frequency response is periodic with small stopband notches at 0 Hz to remove baseline wander as well as at 50 Hz and at its higher harmonics to remove power line disturbance [1]. So far, several methods have been developed to design infinite impulse response (IIR) and finite impulse response (FIR) comb filters. When the fundamental frequency of harmonic interference is known in advance [1], [5], fixed comb filter can be used. However, when fundamental frequency is unknown or time varying, adaptive comb filters are applicable [2]–[4]. In this paper, we will focus on fixed comb filter design problem.

Recently, fractional sample delay has become an important device in numerous field of signal processing, including communication, array processing, speech processing and music technology. An excellent survey of the fractional delay filter design is presented in tutorial paper [6]. Based on this useful and well-documented device, we will establish the relation between the comb filter design problem and the fractional delay filter design problem. As a result, the comprehensive design tools of the fractional delay filter in the literature can be applied to design comb filter directly.

The paper is organized as follows. In Section II, we first transform the specification of the comb filter design into that of fractional delay filter design. Thus, the comb filter design problem becomes a fractional delay filter design one. Then, conventional FIR and allpass filter design techniques for approximation of a fractional digital delay are utilized to design comb filters. Several examples are provided to illustrate the performance of the method. In Section III, we develop a constrained fractional delay filter design approach to improve the performance of the method in Section II. Finally, an experiment of the power line interference removal in ECG signal is shown.

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## II. Comb Filter Design Using Fractional Sample Delay

Generally, the input signal of comb filter has the following form:

$$\begin{aligned} x(n) &= s(n) + \sum_{k=0}^{M} A_k \sin(k\omega_0 n + \phi_k) \\ &= s(n) + I(n) \end{aligned} \tag{1}$$

where $s(n)$ is the desired signal and $I(n)$ is harmonic interference with fundamental frequency $\omega_0$. In order to extract $s(n)$ from the corrupted signal $x(n)$ undistortedly, the specification of ideal comb filter is given by

$$H_d(\omega) = \begin{cases} 0, & \omega = k\omega_0 \quad k = 0, 1, \cdots, M \\ 1, & \text{otherwise.} \end{cases} \tag{2}$$

The purpose of this paper is to design a filter such that its frequency response approximates $H_d(\omega)$ as well as possible. To achieve this purpose, we first show that the harmonic interference $I(n)$ satisfies the following property. Define the fractional sample delay $D = 2\pi/\omega_0$ which is the period of the harmonic interference $I(n)$, then we have

$$\begin{aligned} I(n-D) &= \sum_{k=0}^{M} A_k \sin[k\omega_0(n-D) + \phi_k] \\ &= \sum_{k=0}^{M} A_k \sin(k\omega_0 n + \phi_k - 2k\pi) \\ &= I(n). \end{aligned} \tag{3}$$

This expression tells us that $I(n)$ is equal to its delayed version $I(n-D)$. Thus, if the signal $x(n)$ passes through the filter $H(z) = 1 - z^{-D}$, then its output $y(n)$ is given by

$$\begin{aligned} y(n) &= x(n) - x(n-D) \\ &= [s(n) + I(n)] - [s(n-D) + I(n-D)] \\ &= s(n) - s(n-D). \end{aligned} \tag{4}$$

Obviously, the harmonic interference has been eliminated in the output $y(n)$. However, $y(n)$ is not equal to $s(n)$, i.e., some distortion is included in the signal $y(n)$. In order to explain this phenomenon, Fig. 1 shows the frequency response of the filter $H(z) = 1 - z^{-D}$ and desired frequency response $H_d(\omega)$ defined in (2) with $\omega_0 = 0.22\pi$ and $M = 4$. Note that we usually choose $M = \lfloor \pi/\omega_0 \rfloor$ which denotes the largest integer smaller than or equal to $\pi/\omega_0$. It is clear that both responses have the same positions of stopband notches, but they have a large difference in the passband. In order to remove this distortion, a compensation procedure is performed as follows: It is easy to show that the zeros of the filter $H(z) = 1 - z^{-D}$ are given by

$$z_k = e^{j(2\pi/D)k}, \qquad k = \text{any integer.} \tag{5}$$

For all zeros $z_k$, we introduce the poles

$$p_k = \rho e^{j(2\pi/D)k}, \qquad k = \text{any integer.} \tag{6}$$

to eliminate the distortion in the passband of the frequency response of $H(z) = 1 - z^{-D}$. The radius of the pole $\rho$ must satisfy the inequality $0 < \rho < 1$ in order to constrain the poles to be within the unit circle. After performing this compensation, the new transfer function of the comb filter is given by

$$H_c(z) = \frac{1 - z^{-D}}{1 - \rho^D z^{-D}}. \tag{7}$$

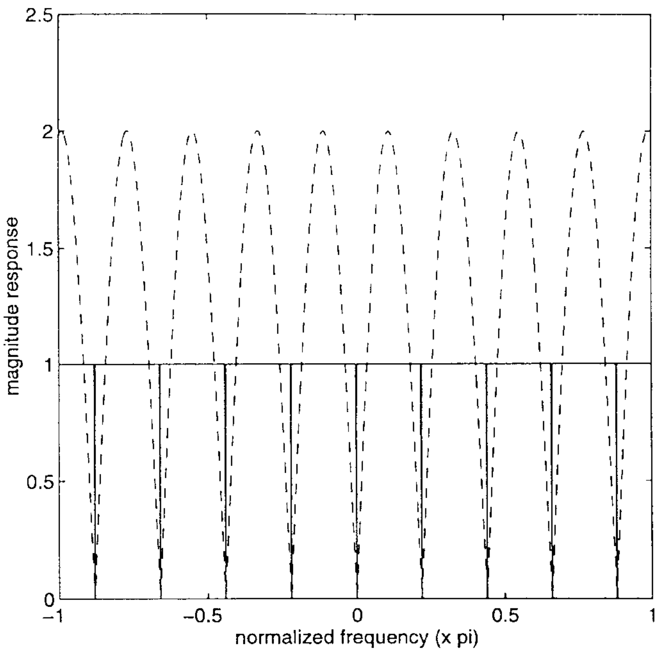



Fig. 1. The frequency response of the comb filter $H(z) = 1 - z^{-D}$ (dashed line) and the desired frequency response $H_d(\omega)$ (solid line) with $\omega_0 = 0.22\pi$ and $M = 4$.

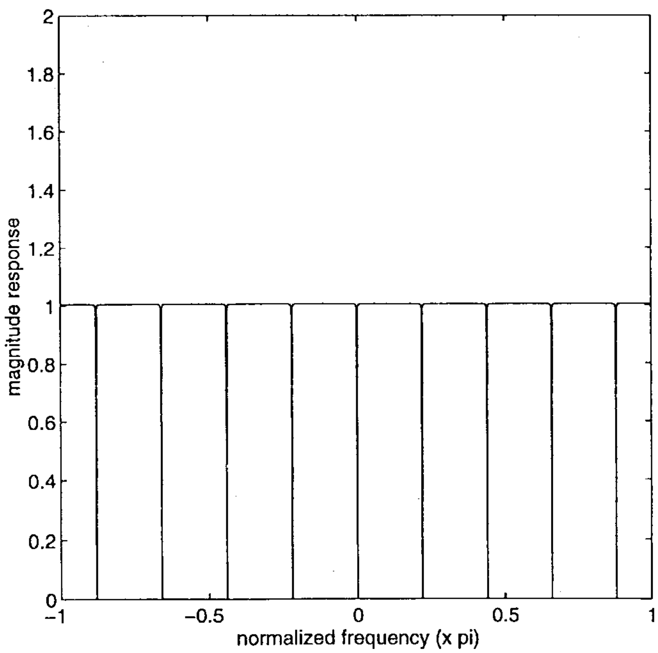



Fig. 2. The frequency response of the comb filter $H_c(z)$ with parameters $\omega_0 = 0.22\pi$ and $\rho = 0.99$.

Fig. 2 shows the frequency response of $H_c(z)$ with parameters $\omega_0 = 0.22\pi$ and $\rho = 0.99$. It is clear that the frequency response of filter $H_c(z)$ approximates $H_d(\omega)$ very well. In fact, $H_c(z)$ becomes an ideal comb filter when pole radius $\rho$ approaches unity. Moreover, Fig. 3 shows an implementation of IIR comb filter $H_c(z)$ in (7). It is clear that the entire implementation only requires a fractional sample delay $z^{-D}$. When $D$ is an integer, the delay $z^{-D}$ is implementable without requiring any design. However, when $D$ is not an integer, we need to design fractional sample delay $z^{-D}$. In [6], a comprehensive review of FIR and allpass filter design techniques for approximation

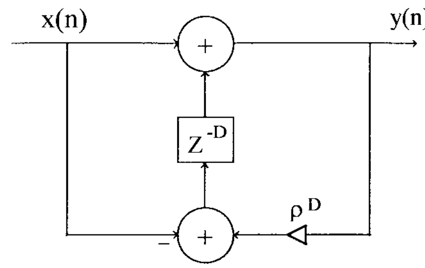



Fig. 3. The implementation of the IIR comb filter $H_c(z)$.

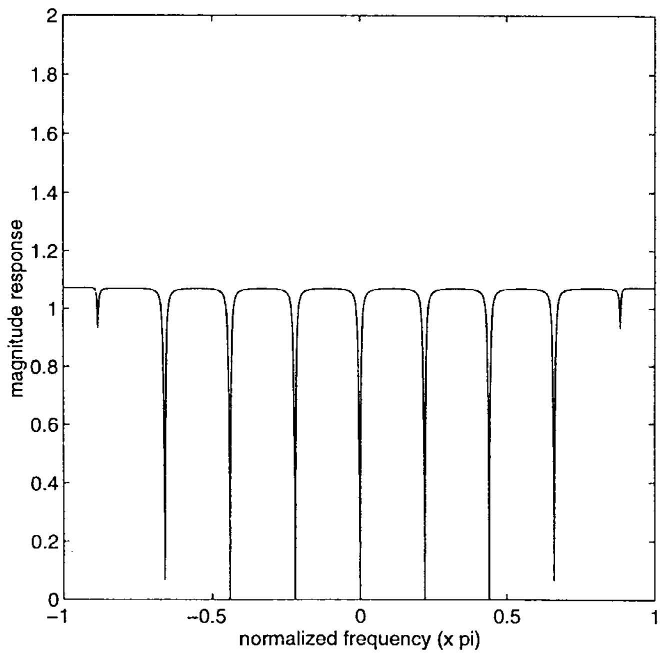



Fig. 4. The frequency response of the comb filter $H_c(z)$ designed in Example 1.

of fractional delay has been presented. Thus, we can directly use these well-documented techniques to design $z^{-D}$. Now, two examples are provided to illustrate the performance of the method. One concerns FIR design case, the other is IIR allpass filter case.

*Example 1—FIR Fractional Delay Case:* In this example, we use Lagrange interpolation method to design an FIR filter for approximating a given fractional delay $z^{-D}$ [6]. In this method, the delay $z^{-D}$ is approximated by

$$z^{-D} \approx \sum_{n=0}^{N} h(n) z^{-n} \tag{8}$$

where filter coefficients $h(n)$ have the explicit form as

$$h(n) = \prod_{k=0,\, k\neq n}^{N} \frac{D-k}{n-k}, \qquad n = 0, 1, \cdots, N. \tag{9}$$

When the parameters are chosen as $\omega_0 = 0.22\pi$, $\rho = 0.99$, and $N = 16$, the frequency response of $H_c(z)$ is shown in Fig. 4. It is clear that the comb filter has an excellent approximation at low frequency because the Lagrange interpolation design is a maximally flat design at frequency $\omega = 0$.

*Example 2—Allpass Fractional Delay Case:* In this example, we use the maximally flat group delay allpass filter to approximate a given fractional delay $z^{-D}$ [6]. In this case, the $z^{-D}$ is approximated

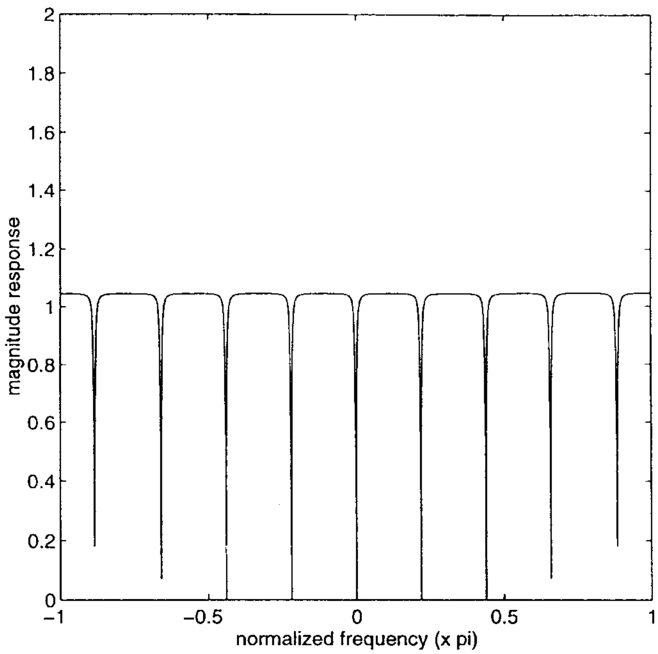



Fig. 5. The frequency response of the comb filter $H_c(z)$ designed in Example 2.

by

$$z^{-D} \approx \frac{a_N + a_{N-1}z^{-1} + \cdots + a_1 z^{-(N-1)} + z^{-N}}{1 + a_1 z^{-1} + \cdots + a_{N-1}z^{-(N-1)} + a_N z^{-N}}. \tag{10}$$

If the positive real number $D$ is split into an integer $N$ plus a fractional number $d$, i.e., $D = N + d$, the filter coefficients $a_k$ is given by

$$a_k = (-1)^k C_k^N \prod_{n=0}^{N} \frac{D - N + n}{D - N + k + n} \tag{11}$$

where $C_k^N = N!/k!(N-k)!$ is a binomial coefficient. Fig. 5 shows the frequency response of the comb filter in this design if the parameters are chosen as $\omega_0 = 0.22\pi$, $\rho = 0.99$. It is clear that the specification is well satisfied at low frequency.

## III. Comb Filter Design Based on Constrained Fractional Delay Filter Design

Although the design methods in Examples 1 and 2 provide two excellent approximations to the ideal comb filter, the frequency responses at harmonic frequencies $k\omega_0$ are not exactly zero valued. This result makes the harmonic interference $I(n)$ can not be eliminated clearly by the designed comb filter. In order to remove this drawback, some suitable constraints need to be incorporated in the design of fractional sample delay $z^{-D}$. In the following, the cases of FIR filter and allpass filter will be described in details.

### *A. FIR Fractional Delay Filter Design*

In this subsection, we will design an FIR filter to approximate the fractional sample delay $z^{-D}$ implemented in Fig. 3. The transfer function of a causal $N$th-order FIR filter can be represented as

$$H(z) = \sum_{n=0}^{N} h(n)z^{-n}.$$

The frequency response of the FIR filter is given by

$$H(\omega) = \mathbf{h}^t \mathbf{e}(\omega) = \mathbf{e}^t(\omega)\mathbf{h} \tag{12}$$

where vectors $\mathbf{h}$ and $\mathbf{e}(\omega)$ are

$$\begin{aligned} \mathbf{h} &= [h(0) \quad h(1) \quad \cdots \quad h(N)]^t \\ \mathbf{e}(\omega) &= [1 \quad e^{-j\omega} \quad \cdots \quad e^{-jN\omega}]^t. \end{aligned} \tag{13}$$

Since $h(n)$ is real valued, the frequency response $H(\omega)$ is conjugate symmetric, i.e.,

$$H(-\omega) = H(\omega)^*. \tag{14}$$

For fractional delay filter design, the desired frequency response $F_d(\omega)$ is chosen as $e^{-jD\omega}$. In this paper, the filter coefficients $\mathbf{h}$ are obtained by minimizing the following least squares error:

$$J(\mathbf{h}) = \int_{\omega \in (R^+ \cup R^-)} |H(\omega) - F_d(\omega)|^2 \, d\omega \tag{15}$$

where frequency bands $R^+ = [0, \alpha\pi]$ and $R^- = [-\alpha\pi, 0]$. Using the conjugate symmetric property of $H(\omega)$ and $F_d(\omega)$, the error $J(\mathbf{h})$ can be rewritten as the quadratic form:

$$J(\mathbf{h}) = \mathbf{h}^t\mathbf{Q}\mathbf{h} - 2\mathbf{h}^t\mathbf{p} + c \tag{16}$$

where matrix $\mathbf{Q}$, vector $\mathbf{p}$, and scalar $c$ are real and given by

$$\begin{aligned} \mathbf{Q} &= 2\int_{\omega \in R^+} \mathrm{Re}[\mathbf{e}(\omega)\mathbf{e}^H(\omega)] \, d\omega \\ \mathbf{p} &= 2\int_{\omega \in R^+} \mathrm{Re}[F_d(\omega)\mathbf{e}^*(\omega)] \, d\omega \\ c &= 2\int_{\omega \in R^+} |F_d(\omega)|^2 \, d\omega = 2\alpha\pi. \end{aligned} \tag{17}$$

The $H$ denotes the Hermitian conjugate transpose operator, and $\mathrm{Re}(\cdot)$ stands for the real part of a complex number. In order to make comb filter be exactly zero valued at the harmonic frequencies $k\omega_0$, the following constraints are considered in the design procedure:

$$H(k\omega_0) = e^{-jDk\omega_0} \qquad k = 0, 1, \cdots, M \tag{18}$$

where $M = \lfloor \pi/\omega_0 \rfloor$. After some maniputation, these constraints can be written in vector matrix form $\mathbf{C}\mathbf{h} = \mathbf{f}$, where real valued matrix $\mathbf{C}$ and vector $\mathbf{f}$ are given by

$$\begin{aligned} \mathbf{C} = \{&\mathrm{Re}[\mathbf{e}(0)], \mathrm{Re}[\mathbf{e}(\omega_0)], \mathrm{Im}[\mathbf{e}(\omega_0)], \cdots, \mathrm{Re}[\mathbf{e}(M\omega_0)], \\ &\mathrm{Im}[\mathbf{e}(M\omega_0)]\}^t \\ \mathbf{f} = [&1, \cos(D\omega_0), -\sin(D\omega_0), \cdots, \cos(DM\omega_0), \\ &-\sin(DM\omega_0)]^t \end{aligned}$$

where $\mathrm{Im}(\cdot)$ stands for the imaginary part of a complex number. Based on the above description, the design problem becomes

$$\begin{aligned} &\text{Minimize} \quad \mathbf{h}^t\mathbf{Q}\mathbf{h} - 2\mathbf{h}^t\mathbf{p} + c \\ &\text{Subject to} \quad \mathbf{C}\mathbf{h} = \mathbf{f}. \end{aligned}$$

Using the Lagrange multiplier method, the optimal solution of this constrained problem is given by

$$\mathbf{h} = \mathbf{Q}^{-1}\mathbf{p} - \mathbf{Q}^{-1}\mathbf{C}^t(\mathbf{C}\mathbf{Q}^{-1}\mathbf{C}^t)^{-1}[\mathbf{C}\mathbf{Q}^{-1}\mathbf{p} - \mathbf{f}]. \tag{19}$$

Now, we use an example to examine the performance of this design method.

*Example 3—Constrained FIR Filter Case:* In this example, the design parameters are chosen as $\alpha = 0.9$, $\omega_0 = 0.22\pi$, $\rho = 0.999$, and $N = 16$. The frequency response of the designed comb filter $H_c(z)$ is shown in Fig. 6. It is clear that the frequency response of the comb filter is exactly zero valued at harmonic frequencies $k\omega_0$ and almost has unity gain at the remaining frequencies.

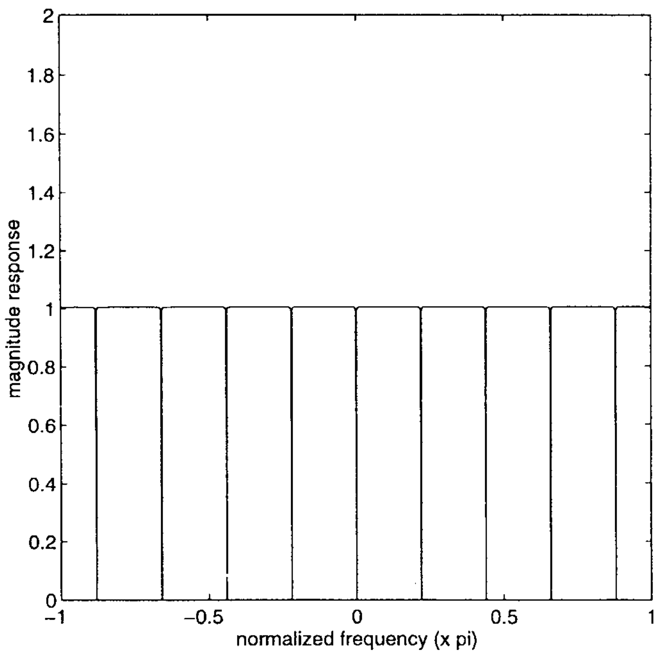



Fig. 6. The frequency response of the comb filter $H_c(z)$ designed in Example 3.

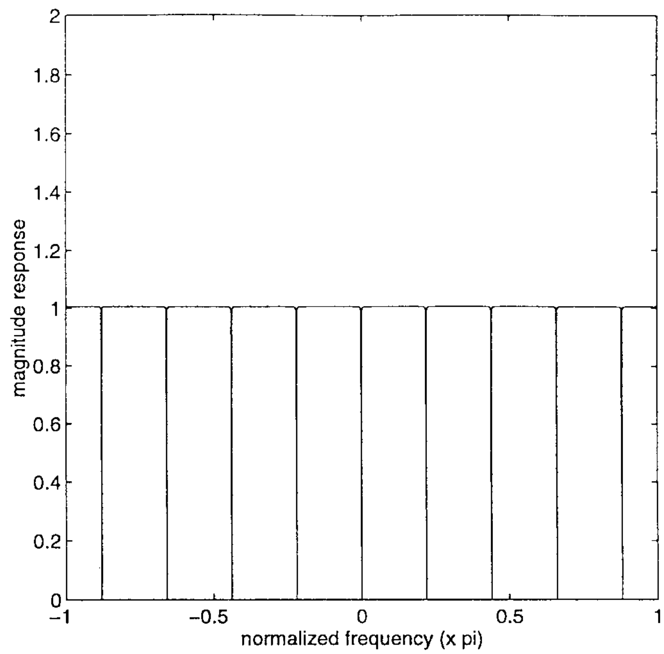



Fig. 7. The frequency response of the comb filter $H_c(z)$ designed in Example 4.

### *B. Allpass Fractional Delay Filter Design*

It is easy to show that the phase response $\theta_A(\omega)$ of the allpass filter in (10) can be written as

$$\theta_A(\omega) = -N\omega + 2\ \arctan\left[\frac{\sum_{k=1}^{N} a_k\ \sin(k\omega)}{1+\sum_{k=1}^{N} a_k\ \cos(k\omega)}\right]. \qquad (20)$$

The purpose of this subsection is to design an allpass filter such that the $\theta_A(\omega)$ approximates the prescribed phase response $-D\omega$, that is, we want to achieve the following specification:

$$\theta_A(\omega) = -D\omega, \qquad \omega \in [0,\ \alpha\pi]. \qquad (21)$$

Substitute (20) into (21), we obtain the expression [7]

$$\sum_{k=1}^{N} a_k\ \sin[\beta(\omega) + k\omega] = -\sin[\beta(\omega)] \qquad (22)$$

where $\beta(\omega) = -1/2(-D\omega + N\omega)$. Define two vectors

$$\begin{aligned}\mathbf{a} &= [a_1 \quad a_2 \quad \cdots \quad a_N]^t\\ \mathbf{b}(\omega) &= \{\sin[\beta(\omega)+\omega] \quad \sin[\beta(\omega)+2\omega] \quad \cdots \quad \sin[\beta(\omega)+N\omega]\}^t,\end{aligned} \qquad (23)$$

then (22) can be rewritten as

$$\mathbf{a}^t\mathbf{b}(\omega) = -\sin[\beta(w)]. \qquad (24)$$

In this paper, we will minimize the following least squares error to obtain optimal filter coefficients $\mathbf{a}$

$$\begin{aligned}J(\mathbf{a}) &= \int_0^{\alpha\pi} |\mathbf{a}^t\mathbf{b}(\omega) + \sin[\beta(\omega)]|^2\, d\omega\\ &= \mathbf{a}^t\mathbf{Q}\mathbf{a} - 2\mathbf{p}^t\mathbf{a} + c\end{aligned} \qquad (25)$$

where matrix $\mathbf{Q}$, vector $\mathbf{p}$ and scalar $c$ are given by

$$\begin{aligned}\mathbf{Q} &= \int_0^{\alpha\pi} \mathbf{b}(\omega)\mathbf{b}(\omega)^t\, d\omega\\ \mathbf{p} &= -\int_0^{\alpha\pi} \mathbf{b}(\omega)\ \sin[\beta(\omega)]\, d\omega\\ c &= \int_0^{\alpha\pi} \sin[\beta(\omega)]^2\, d\omega.\end{aligned} \qquad (26)$$

In order to make comb filter be exactly zero valued at harmonic frequencies $k\omega_0$, the following constraints are incorporated in the design:

$$\mathbf{a}^t\mathbf{b}(k\omega_0) = -\sin[\beta(kw_0)], \qquad k = 1, 2, \cdots, M \qquad (27)$$

where $M = \lfloor \pi/\omega_0 \rfloor$. After some maniputation, these constraints can be written in vector matrix form $\mathbf{C}\mathbf{a} = \mathbf{f}$, where real valued matrix $\mathbf{C}$ and vector $\mathbf{f}$ are given by

$$\begin{aligned}\mathbf{C} &= [\mathbf{b}(\omega_0),\ \mathbf{b}(2\omega_0),\ \cdots,\ \mathbf{b}(N\omega_0)]^t\\ \mathbf{f} &= \{-\sin[\beta(\omega_0)],\ -\sin[\beta(2\omega_0)],\ \cdots,\ -\sin[\beta(N\omega_0)]\}^t.\end{aligned}$$

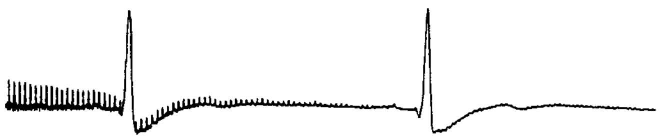
Using the Lagrange multiplier method, the optimal solution of this constrained problem is also given by

$$\mathbf{a} = \mathbf{Q}^{-1}\mathbf{p} - \mathbf{Q}^{-1}\mathbf{C}^t(\mathbf{C}\mathbf{Q}^{-1}\mathbf{C}^t)^{-1}(\mathbf{C}\mathbf{Q}^{-1}\mathbf{p} - \mathbf{f}). \qquad (28)$$

Finally, we use an example to investigate the performance of this design method.

*Example 4—Constrained Allpass Filter Case:* In this example, the design parameters are chosen as $\alpha = 0.9$, $\omega_0 = 0.22\pi$, $\rho = 0.999$, and $N = \lfloor 2\pi/\omega_0 \rfloor = 9$, The frequency response of $H_c(z)$ is shown in Fig. 7. It is clear that the frequency response of the designed comb filter is exactly zero valued at harmonic frequencies $k\omega_0$ and almost has unity gain at the remaining frequencies.

(a)



(b)

Fig. 8. Power line interference removal in ECG signal. (a) Input waveform of the comb filter. (b) Output waveform of the comb filter.

## IV. Application Example

A major problem in the recording of ECG is that the measurement signals degraded by the power line interference. One source of interference is electrical field characterized by noise concentrated at the fundamental frequency 60 Hz. The other source is magnetic field which is characterized by high harmonic content. The harmonics are due to the nonlinear characteristics of transformer cores in the power supply [8]. Thus, to use comb filter to reduce interference becomes an important subject in ECG measurement.

In this example, we utilize the comb filter designed by the method in Example 4 to remove power line interference. The samples used here have 8 bits and the sampling rate is 600 Hz. Fig. 8(a) shows the input waveform that is ECG signal corrupted by harmonic interference with fundamental frequency 60 Hz. The specification of comb filter is chosen as

$$H_d(\omega) = \begin{cases} 0, & \omega = 0.2k\pi \quad k = 0, 1, \cdots, 5 \\ 1, & \text{otherwise.} \end{cases} \tag{29}$$

Fig. 8(b) shows the waveform of comb filter output with zero initial. From this result, it is obvious the interference has been removed by our comb filter except some transient states appear at the beginning.

## V. Conclusion

In this paper, a new comb filter design method using fractional sample delay has been presented. First, the specification of the comb filter design is transformed into that of fractional delay filter design. Then, the FIR and allpass filter design techniques are directly used to design fractional delay filter with transformed specification. Next, we develop a constrained fractional delay filter design approach to improve the performance of the direct design method. Finally, several design examples and an experiment of the power line interference removal in ECG signal are demonstrated to illustrate the effectiveness of this new design approach.

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# A Complete Pipelined Parallel CORDIC Architecture for Motion Estimation

Jie Chen and K. J. Ray Liu

***Abstract*—In this paper, a novel fully pipelined parallel CORDIC architecture is proposed for motion estimation. Unlike other block matching structures, it estimates motion in the discrete cosine transform (DCT) transform domain instead of the spatial domain. As a result, it achieves high system throughput and low hardware complexity as compared to the conventional motion estimation design in MPEG standards. That makes the proposed architecture very attractive in real-time high-speed video communication. Importantly, the DCT-based nature enables us not only to efficiently combine DCT and motion estimation units into a single component but also to replace all multiply-and-add operations in plane rotation by CORDICs to gain further savings in hardware complexity. Furthermore this multiplier-free architecture is regular, modular, and has solely local connection suitable for VLSI implementation. The goal of the paper is to provide a solution for MPEG compatible video codec design on a dedicated single chip.**

## I. Introduction

Because of the simplicity of the block matching motion estimation (BKM-ME), it has been adopted in MPEG and H.263 standards. However, the computational complexity of BKM-ME is very high, i.e. $O(N^4)$ for a $N \times N$ block, hence high hardware complexity. To reduce the computational complexity, some simplified block search methods (such as logarithmic search, three-step search, etc.) and the corresponding structures have been proposed. Those methods pick several displacement candidates out of all possible displacement values in terms of minimum mean absolute difference values of the reduced number of pixels and still require two or more sequential steps to find suboptimal estimates. A good review paper about VLSI architectures for video compression can be found in [1]. Besides

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