Electrical Power and Energy Systems 43 (2012) 1267–1275

Contents lists available at SciVerse ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Optimal allocation of capacitors in distribution systems using particle swarm optimization

S.P. Singh *, A.R. Rao

Department of Electrical Engineering, Institute of Technology, Banaras Hindu University, Varanasi 221 005, UP, India

ARTICLE INFO

Article history:
Received 16 September 2010
Received in revised form 11 June 2012
Accepted 13 June 2012
Available online 20 July 2012

Keywords:
Particle swarm optimization
Optimization methods
Genetic algorithm
Capacitors
Voltage profile
Power distribution

ABSTRACT

A particle swarm optimization (PSO) approach for finding the optimal size and location of capacitors is reported in this work. The proposed technique finds optimal locations for shunt capacitors from the daily load curve. In addition, it determines the suitable values of fixed and switched capacitors. A dynamic sensitivity analysis method is used to select the candidate installation locations of the capacitors to reduce the search space of this problem. In case of more than one location, the dynamic sensitivity helps in deciding other locations considering the effect of previously decided locations and values of capacitors. A simple iterative method is used to compute the power flow. The results obtained for well studied 70-bus and 135-bus systems are compared with the solutions obtained by Tabu Search (TS), Hybrid and Genetic Algorithm. It is demonstrated that the proposed PSO approach offers the global optimal solution with greater saving.

1. Introduction

Electrical energy supply from generation sites to ultimate consumers reach via the transmission, sub transmission and distribution segments of the overall power system. Such energy transfer is accompanied by network dependent power losses which have the effect of increasing the peak load on the system. It is acknowledged by all that the bulk of the power loss occurs on the distribution system which is 13% of the total power generated [1]. The reactive power accounts for a portion of these losses. Some of these losses due to reactive power can be reduced by application of shunt capacitors on primary distribution feeders to relieve capacity requirement. Hence, optimal capacitor allocation in electrical distribution networks has always been the concern of electric power utilities. Optimal capacitor allocation problem deals with determination of location, size, type and number of capacitors to be installed such that the maximum economic benefits are achieved without violating the operational constraints. Several formulations have been suggested for this problem and they have been solved by available computational techniques. A survey of these techniques by Ng et al. [1] classifies these techniques in four groups of analytical, numerical programming, heuristics, and artificial intelligence.

Analytical method in conjunction with heuristics for capacitor placement was introduced by Neagle and Samson [2] and subsequently by cook [3]. A pioneering work which determines the capacitor sizes as discrete variables using dynamic programming technique was reported by Duran [4]. More rigorous approaches were suggested in 1980's [5–8]. Grainger and Lee [5] formulated this problem as a non-linear programming problem by treating the capacitors locations and sizes as continuous variables. Fawzi et al. [6] incorporated the released substation kva and the voltage rise at light-load level into a model developed by Neagle and Samson [2]. Ponnavaikko and Prakasa Rao [7] proposed a model, which considered the load growth and the discrete nature of capacitor size, apart from those considered in [5] and used a local optimization technique. Kaplan [8] presented a formulation of feeders with multiple laterals and suggested a heuristic solution algorithm. Baran and Wu [9] presented a problem formulation similar to that of Grainger and Lee [5], a non-linear optimization problem, but incorporated the distribution power flow equation, constraints on node voltage magnitudes at different load levels and discrete nature of capacitor sizes, into the model and the resulting formulation represents a mixed-integer programming problem. Maximum saving objective of this problem and its formulation & solution as mixed integer linear problem is reported by Khodr et al. [10]. Two phase solution, formulation as conic problem and its solution by interior point method in phase-I and mixed integer linear programming formulation and solution in phase-II of this problem is suggested in [11]. A heuristic method is proposed by Segura et al. [12] in which a relaxed version of the exact mathematical model of the problem is solved using interior point method. Mixed integer non-linear programming has been

* Corresponding author. Tel.: +91 542 6702993; fax: +91 542 2368727.
E-mail addresses: spsingh.eee@itbhu.ac.in, sps5957@indiatimes.com (S.P. Singh).

0142-0615/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijepes.2012.06.059

Nomenclature

S_i^k	velocity of individual constant
rand(), *Rand*()	uniform random number between 0 and 1
x_{id}^k	current position of individual i at iteration k
pbest	best value attained by individual i
gbest	global best value i at iteration k
C_1 & C_2	acceleration of the group
w	inertia weight factor
$w_{\max}$	maximum value of inertia weight (0.9 in this study)
$w_{\min}$	minimum value of inertia weight (0.4 in this study)
$iter_{\max}$	maximum iteration number (generations)
iter	current iteration number
x	unknown power flow (depend) variables
y	known or specified (independent) variables
V_m	voltage at mth node
$V^{\min}$	minimum acceptable voltage
$V_{sys}^{\min}$	minimum system voltage in an iteration
$V^{\max}$	maximum system voltage (1.0 pu in this study)
K_{ei}	energy cost constant for load level i
K_{cj}	capacitor cost constant depending on type of capacitor placed at jth location
P_{Li}	power loss at ith load level with corresponding time duration T_i
C_j	injected kvar at jth node
T_i	duration of load level i
n	number of load levels
k	number of locations
θ_i	voltage angle at node i
θ_j	voltage angle at node j
P_j	active power at bus j
Q_j	reactive power at bus j
P_L	total power loss in the system
r_{ij}	real part of impedance between nodes i and j

suggested by Leonardo et al. [13] for capacitor placement as well as reconfiguration in order to achieve the objective of minimum energy loss operation of radial distribution network. A direct search method has been used by Ramalinga Raju et al. [14] in which a best suited node for a particular size of capacitor out of all possibilities is identified and then the capacitor is placed.

The analytical methods are very fast but they suffer from inability to escape local optima. The application of search and evolutionary techniques started in early 1990s in order to overcome this problem of analytical techniques. The evolutionary techniques, simulated annealing, Tabu Search and GA have been reported by several authors [15–29]. In this dissection, Chiang et al. [15] presented a general capacitor placement problem formulation by taking practical aspects of capacitors and the operational constraints at different load levels into consideration and solved by simulated annealing. These authors further extended this method by incorporating the cost associated with capacitor placement considering it to be a step-like function and treating the capacitor sizes and control settings as discrete variables [16]. The Tabu Search technique to find an optimal solution has been used by Huang et al. [17]. Gallego et al. [18] presented same problem using Hybrid Approach, a combination of Tabu Search and heuristics.

Genetic algorithm (GA) based method was initially introduced by Ajjarapu and Albama [19] which was further extended by Boone and Chiang [20] and later by Sundharajan and Pahwa [21] with additional features. Miu et al. [22] reported two stages GA for this problem in which the solution obtained by GA in first stage is further improved by sensitivity based heuristics at the second stage. Levitin et al. [23] included system capacity release, peak load reduction and reduction of annual energy loss in a feeder in their formulation of optimal capacitor allocation problem and solved by GA.

Further improved form of GA was applied by Kim et al. [24] for this problem that combines GA with a stochastic variant of the simplex method called elite based simplex GA (ESGA). In order to avoid local minima of GA, normally large population is desired that require high processing time. This can be overcome by use of micro genetic algorithm wherein De Souza et al. [25] applied fuzzy logic to reduce the search space and micro genetic algorithm for solution of capacitor allocation problem. Use of fuzzified multiple objective function: reducing the total cost of energy loss and capacities, increasing the margin loading of feeders and improving voltage profile and solution by GA was proposed by Hsiao et al. [26]. Ants are capable of finding the shortest path from food sources to their nests. Inspired by this behavior of real ant colonies, ant algorithm was developed. However, further improved version of this method, out detection by bird differential evolution (ADHDE) was applied by Chiou et al. [27]. This is achieved by reducing the number of mutations. Similarly, principle of plant growth process was exploited by Srinivasas Rao et al. [28] to make use of plant growth simulation algorithm for the solution of capacitor placement problem. Haghifam and Malik [29] attempted to overcome the problem of uncertainty and time variation in load by fuzzy representation of load. Final solution is obtained by GA. Their implementation of GA uses two row chromosomes to represent the capacitor values of fixed and switchable type. Szuvovivski et al. [30] suggested use of other voltage regulators along with capacitor for such applications. They solved this problem using both GA and optimal power flow.

It can be observed from above review that the initial methods of capacitor placement problem used analytical methods which are basically conventional optimization techniques. These optimization methods work on the basis of search directions generated from derivatives of the function. Therefore, it becomes imperative to express the problem in the form of continual differentiable function; otherwise, these methods loose efficiency. The later methods starting from 90s are evolutionary and AI based. Combinations of more than one method are also reported. But GA has been found to be attractive and has been widely used. However, a more recent method of particle swarm optimization (PSO) has proved to be more capable and had been applied for many optimization problems related to power system such as economic dispatch of generators [31] and reactive power and voltage control [32]. Apart from these its application for capacitor allocation also has been explored. Prakash and Sydulu [33] applied PSO technique for capacitor placement problem but in their formulation the objective function is not very clear. The PSO technique used by AlHajri et al. [34] does not reveal the constraint handling methodology. Yu et al. [35] demonstrated the application of PSO for this problem considering harmonics and demonstrated on a single and small 9 bus system. The constraints are handled using conventional penalty function method. This idea was further extended by Eajal and El-Hawary [36] to account for unbalanced operating conditions also. Demonstration of PSO is further reported by Etemadi and Fotuhi-Firuzabad [37] where in reliability cost is also included along with the cost of losses and investment in the objective function. However, unconstrained problem has been formulated and solved in this approach. Kim et al. [38] proposed non-linear interior point method to enhance the search speed of PSO which maximizes loadability.

Advantages of PSO over other existing methods for capacitor placement problem have been demonstrated in above mentioned publications [31–38]. Further exploitation of PSO for this problem with added benefits is addressed in this paper. The problem of handling inequality constraints by conventional penalty function method is resolved by using a different technique. The power flow solutions are obtained by self developed backward sweep power flow method reported in [39]. The results obtained by the proposed method for 70-bus and 135-bus test systems are compared with other methods. It is demonstrated that the proposed method out performed with less computational time. Saving in 10 years operation, losses and voltage profile are criteria for comparison. The type of capacities to be installed at various buses in also suggested in this work.

2. Problem formulation

The general capacitor placement problem is to determine the places (number and location), types and settings of capacitors to be placed on radial distribution system. The objectives are to reduce the energy loss on the system and to maintain the voltage regulation while keeping the cost of capacitors addition to a minimum.

In order to calculate the energy losses in the system, the load variations in the system for a given period of time are taken into account. It is assumed that the load variation could be approximated in discrete level and the entire load varies in a conforming manner. Vowing to these assumptions, the load duration curve is approximated by a piecewise linear function and the time period is divided into intervals during which the load level is assumed to be constant.

2.1. Mathematical representation

The objective is to minimize the sum of energy loss and the capacitor costs satisfying operational and power balance constraints. This can be mathematically expressed as:

$$\text{Minimize} \sum_{i=1}^{n} K_{ei} T_i P_{L_i} + \sum_{j=1}^{k} K_{cj} C_j \tag{1}$$

Subjected to, Power flow balance expressed as

$$F(x, y) = 0 \tag{2}$$

Limits on voltage magnitude expressed as

$$V^{\min} \leqslant V_m \leqslant V^{\max} \tag{3}$$

2.2. Sensitivity calculations

The basic objective of capacitor installation is reduction in losses. This objective can be met if capacitor is installed at a location which has maximum contribution towards loss reduction as all locations may not have identical effect. This objective can be achieved by knowing the sensitivity of active power loss to reactive power injection at a node. The buses having higher sensitivities would be candidate locations. This will in tern reduce the search space. Such a relation is described in Ref. [40] which expresses the change in active power loss of the system due to change in reactive power injection at a node as

$$\frac{\partial P_L}{\partial Q_i} = 2 \sum_{j=1}^{n} (\alpha_{ij} Q_j + \beta_{ij} P_j) \tag{4}$$

where

$$\alpha_{ij} = r_{ij} \mathrm{Cos}(\theta_i - \theta_j) / \mathrm{V}_i \mathrm{V}_j$$
$$\beta_{ij} = r_{ij} \mathrm{Sin}(\theta_i - \theta_j) / V_i V_j$$

The sensitivity of all the nodes is calculated using Eq. (4). The nodes with higher sensitivities are chosen for candidate locations.

3. Particle swarm optimization

Particle swarm optimization (PSO) is one of the evolutionary techniques which were first introduced by Kennedy and Eberhart in 1995 [41]. The method is developed from researches on swarm such as fish schooling and bird flocking. This method is capable of handling continuous state variables easily and search a solution space effectively. However, the method can be extended to treat continuous and discrete variables. The PSO algorithm uses evolutionary operators to manipulate the individuals, like in other evolutionary computational algorithms. Each individual in PSO flies in the search space with velocity which is dynamically adjusted according to flying experience of its own and its companions. Each individual is treated as a volume less particle in the *d*-dimension search space.

At each time step, the particle swarm optimization concept consists of velocity change of each particle toward its individual best (*pbest*) and global best (*gbest*) locations. Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward *pbest* and *gbest* locations.

A general engineering optimization problem can be defined as

$$\text{Minimize } f(X)$$

where *i*th particle in *d*-dimensional space is represented as

$$X_{i_i} = \{x_{i1}, x_{i2}, \ldots, x_{id}\}$$

The best previous position of the *i*th particle is recorded and represented as

$$p_{best_i} = \{p_{best_{i1}}, p_{best_{i2}}, \ldots, p_{best_{id}}\}$$

The index of the best particle among all the particles in the population is represented by *gbest*. The rate of the position change (velocity) for particle '*i*' is represented as $S_i = \{s_{i1}, s_{i2}, \ldots, s_{id}\}$

The modified velocity and position of each individual particle can be calculated using the current velocity and the distance from *pbest* to *gbest*, as expressed by the following formulae [42]

$$S_i^{k+1} = w * S_i^k + C_1 * rand() * (pbest_i^k - x_i^k) + C_2 * Rand() * (gbest^k - x_i^k) \tag{5}$$

$$\begin{aligned} &S^{\min} \leqslant S_i \leqslant S^{\max} \\ &x_i^{k+1} = x_i^k + S_i^{k+1} \end{aligned} \tag{6}$$

The parameter $S^{\max}$ in the above procedures determines the resolution, or fitness, with which regions between the present position and target position are searched. If $S^{\max}$ is too high, particles may fly past the good solutions. If $S^{\max}$ *is* too small, particles may not explore sufficiently beyond local solutions [43]. $S^{\max}$ is often set within 10–20% of the dynamic range of the variable. The constants C_1 and C_2 represent the weighting of the stochastic acceleration terms that pull each particle toward *pbest* and *gbest* positions. Low values allow particles to roam far from target regions before being tugged back. On the other hand, high values result in abrupt movement toward, or past, the target regions. Hence, the acceleration constants C_1 and C_2 are often set to be 2.0 according to past experiences.

Suitable selection of inertia weight *w* in Eq. (5) provides a balance between global and local exploration and exploitation, and on average results in less iterations required to find a sufficiently optimal solution. As originally developed, w often decreases linearly from about 0.9 to 0.4 during a run. In general, the inertia weight w is set according to the following equation

$$W = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{iter_{\max}}\right) * iter \tag{7}$$

3.1. Constraints handling

Traditionally, the objective function is regarded as the fitness function and the inequality constraints are converted to penalty functions and added to the objective function. The drawback of this method is that an excellent particle can be misjudged as inappropriate for the penalty factors. Besides, penalty parameters are usually assigned by empirical approach and are deeply affected by the problem model. For the sake of avoiding this, a binary fitness has been used: one for optimal objective and the other for the binding constraints. Optimal objective fitness is equal to the value of the expression (1) which represents the cost of energy lost and capacitors installed.

Binding constraints fitness value is adopted to scale the level of violation, calculated as follows:

$$Binding\_fitness(z) = \begin{cases} z_{\min} - z, & z < z_{\min} \\ z - z_{\max}, & z > z_{\max} \\ 0 & else \end{cases} \tag{8}$$

where z is the value of the inequality constraint, $z_{\min}$ and $z_{\max}$ are the lower and upper limits of the inequality constraints.

The fitness to binding constraints of the particles is considered first and if a particle does not satisfy the binding constraints, it is regenerated. This way feasible particles are generated that guarantee the fulfillment of binding constraints superior to infeasible particles that violate the binding constraints. Thus entering into feasible region is considered before obtaining global optimal solution. There is no need to set up the penalty parameter.

4. Implementation of PSO for optimal capacitor allocation

4.1. Coding strategy

The purpose of this PSO implementation is to determine the capacitor values (kvar) at the candidate locations during various load levels. If, there are 'n' different load levels and 'k' number of candidate locations, the PSO returns 'nk' design variables. Since practical capacitors available in market and maximum size of capacitor that can be placed as considered in this implementation are 300 and 1500 kvar respectively, the capacitor values that can be obtained from this algorithm would be any of 0, 300, 600, 900, 1200 and 1500 kvar. Since these six values of capacitors are to be represented, three binary bits for each value is suggested.

Thus the reactive power to be injected from a candidate bus is coded in binary form using three bits suitable to take care up to 1500 kvar at a bus considering each step of 300 kvar. In case, five nodes are selected as candidate locations, the string length used would be $5 * 3 = 15$ bits for single load level throughout the day. However, in present problem, the load of a day consists of three load levels. Therefore, the string's length for five locations would be three times this value resulting in a string length of $5 * 3 * 3 = 45$ bits. This way appropriate number of strings can be formed to represent a desired population size. A coded example string, 001100101011101000101110010011000010000111 00, along with decoded values of kvar is shown in Table 1. The string represents a solution/chromosome (capacitor values) for five locations at three different load levels.

4.2. Implementation of algorithm

The PSO for above discussed problem of capacitor placement can be implemented using following steps.

1. Determination of Candidate Locations
   a. Input the distribution system branch impedance values and bus real and reactive power data.
   b. Find Sensitivity Factors.
   c. Select the buses with higher sensitivity factors as candidate locations.
2. Optimization by particle swarm algorithm
   a. Input PSO control data
   b. Initialize population with random strings.
   c. Outer Loop: while Gen ⩽ Max. Gen.
   d. Enter the inner loop.
   i For each string decode into a test configuration.
   ii For each load demand: low, medium and high
   - Apply load demand
   - Call distribution load flow solver.
   - Check voltage constraints.
   - Determine real power loss and energy loss.

   iii Compute capacitor cost.
   iv Compute total cost function. (fitness function)
   v Calculate the previous best performance of each particle and save it as pbest.
   vi Calculate the best performance of al the particles and save it as gbest.
   vii Calculate the velocity of each particle using *pbest* and *gbest.*
   viii Update each particle.
   ix If so obtained particle satisfies all constraints and is better than previous values then change *pbest* to new value.
   x Inner Loop: while (pop number ⩽ pop size).
   xi Calculate the best value of all *pbest* and save as *gbest.*
   xii Undo generation.

   e. The obtained value is *gbest*, the global solution obtained by algorithm, decode it and find the capacitor values at different load conditions.
   f. Calculate the savings obtained with resultant solution.

5. Simulation results

The proposed particle swarm optimization technique based solution methodology for capacitor placement has been implemented in C++ using Pentium III 450 MHz, 256 MB computer.

Table 1
Example of coding/decoding a string/chromosome.

Load level	L1					L2					L3				
Location	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Chromosome	001	100	101	011	101	000	101	011	010	001	100	001	000	011	100
Value (kvar)	300	1200	1500	900	1500	0	1500	900	600	300	1200	300	0	900	1200

Table 2
Duration and energy cost per year for 70 bus system.

Load level	L_1	L_2	L_3
Load level value	0.625	1.0	1.25
Duration time (h)	1000	6760	1000
Energy loss cost $K_e^{(\mathrm{NT\$/KW\ h})}$	0.7	1.78	2.95

The results for the 70-bus and 135-bus test systems have been obtained and reported in this section. The population size of 100 and $iter_{\max}$ as 100 have been chosen for PSO for both the test cases.

The annual demand curve is approximated by 360 identical daily demand variation curves. This demand variation curve is used to calculate energy loss. The entire load of a day has been divided into three load levels L_1, L_2, and L_3 with time periods T_1, T_2, and T_3.

The maximum range of capacitor to be installed at a candidate location is taken as 1500 kvar. The capacitors are regarded as discrete variables and as multiples of standard bank (300 kvar). The investment cost of fixed type capacitor is NT$ 56,300 per bank and switched type capacitor is NT$ 74,900 per bank. The cost is taken in NT$ for fare comparison with Tabu Search and Hybrid methods reported in [17,18] respectively. The minimum acceptable voltage taken for this study is 0.9 pu.

5.1. Choice of capacitor type

If the reactive power requirement at a bus remains same for all load levels, a fixed valued capacitor equal to the reactive power requirement of the bus is installed at that bus. On the other hand if the reactive power requirement at a bus varies with load levels, a switchable capacitor equal to reactive power requirement at highest load level is installed at that bus.

5.2. 70-Bus system

The data for this system is taken from [16]. The load levels L_1, L_2 and L_3, time periods T_1, T_2 and T_3 and energy cost data are given in Table 2. Ten year planning horizon has been considered with yearly load growth rate of 9.55% for the first three years as considered in Ref. [17]. Since the peak load has reached the maximum capacity of 5000 kW of feeders, after that the load is assumed constant till the end of planning horizon.

A sensitivity analysis as described in Section 2.2 is incorporated into algorithm to determine the candidate locations for placing the capacitor in the distribution system. A priori estimation of these candidate locations helps to reduce the search space of the optimization problem. According to sensitivity calculations, the five buses, in descending order of their sensitivities are 50, 53, 17, 10, and 43.

5.2.1. Choice of number of locations

In order to ascertain the optimal number of locations, the number of locations of capacitors is varied according to descending order of their sensitivities. The system losses after capacitor allocation are recorded as tabulated in Table 3. These losses are shown for a single load level of L_2 (normal load) in this table. The system loss without capacitors at this load level is 225 kW. It is observed from this table that the losses in the system decreases as the number of locations of capacitors installation increase. However, the decrease in losses slowly saturates as the number of locations increase. But the installation cost of these capacitors increases with increase in number of locations causing reduction in saving. Therefore, the number of locations should be chosen corresponding to maximum saving. In order to ascertain the long term impact, the savings for ten years operation with respect to number of locations were obtained as shown by bar chart in Fig. 1. It is obvious from this figure that the maximum saving is achieved corresponding to four locations. Therefore, capacitors should be installed at four locations of 50, 53, 17 and 10. This procedure is called optimal capacitor allocation using static sensitivity since the effect of capacitors installed at previously decided locations have not been incorporated while deciding the next location.

5.2.2. Effect of dynamic sensitivity

A priori selection of locations based on sensitivity values called static sensitivity have been used in TS [17] and Hybrid [18]

Table 3
Effect of variation in number of locations for 70-bus at normal load.

No. of Locations		1	2	3	4	5
Selected buses	Static sensitivity	50	50, 53	50, 53,17	50, 53, 17, 10	50, 53,17, 10, 43
	Dynamic sensitivity	50	50, 16	50, 16,10	50, 16, 10, 39	50, 16, 10, 39, 4
Losses (kw)	Static sensitivity	161.488	153.690	147.402	143.9768	143.976
	Dynamic sensitivity	161.488	147.67	146.05	143.415	143.415

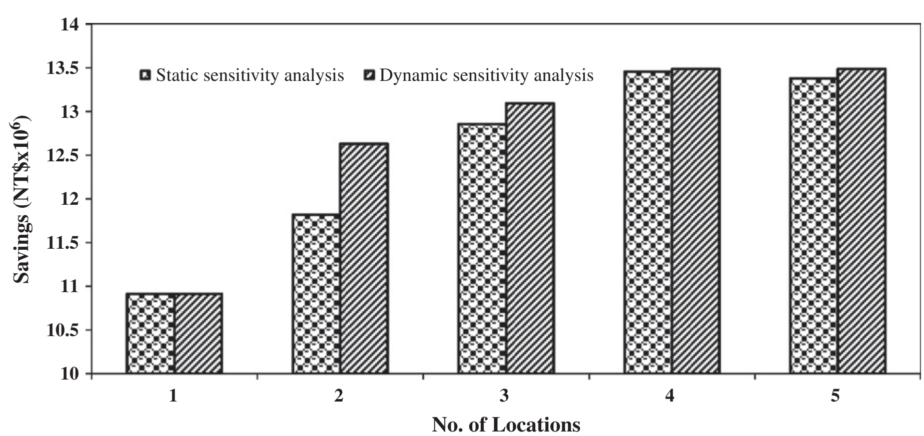

Fig. 1. Savings vs no. of locations (70-Bus).

Table 4
Comparative results for 70-bus system.

Method	Optimal locations	L_1	L_2	L_3	Optimal size (kvar) and type	
					Fixed	Switched
Tabu search [17]	11	300	600	600	300	300
	50	300	900	900	300	600
	53	300	300	300	300	–
	V^{min} (p.u)	0.9564	0.9271	0.9075	**Total = 1800**	
	Loss (kw)	56.073	152.248	245.629		
Hybrid method [18]	10	0	300	300	–	300
	16	300	300	300	300	–
	38	300	300	300	300	–
	50	900	900	1200	900	300
	53	000	300	300	–	300
	V^{min} (p.u)	0.9590	0.9315	0.9122	**Total = 2400**	
	Loss (kw)	55.0	144.00	234.00		
GA	10	0	300	600	–	600
	17	300	300	300	300	–
	50	600	900	1200	–	1200
	53	300	300	300	300	–
	V^{min} (p.u)	0.9603	0.9313	0.9142	**Total = 2400**	
	Loss (kw)	54.79	143.97	233.64		
PSO static sensitivity	50	600	900	900	600	300
	53	0	300	600	–	600
	17	300	300	300	300	–
	10	300	300	300	300	–
	V^{min} (p.u)	0.9612	0.9323	0.9148	**Total = 2100**	
	Loss (kw)	54.79	143.97	233.64		
PSO dynamic sensitivity	50	900	1200	1200	900	300
	16	300	300	300	300	–
	10	0	300	300	–	300
	39	0	600	600	–	600
	V^{min} (p.u)	0.9603	0.9313	0.9142	**Total = 2400**	
	Loss (kw)	54.79	143.41	231.55		

methods. But whenever a capacitor is placed at a location, the system conditions change, as a result, the very next location to be selected may not be same as the one obtained in static sensitivity analysis. In order to incorporate this effect, dynamic sensitivity has been calculated and used. Accordingly, the initial installation is done at node 50 having highest sensitivity in this case. The values of capacitors are determined by PSO. The sensitivities at all nodes are calculated again following this installation. The node having highest sensitivity, according to fresh sensitivity calculations following earlier allocation, is selected for second location. This process is repeated till optimal number of locations is determined. The optimal number of locations is that number at which maximum saving is achieved. The optimal locations so determined in this case are 50, 16, 10, and 39 as against 50, 53, 17, and 10 in case of static sensitivity. Only two nodes 50 and 10 are common in both the procedures. The losses corresponding to various locations for placement using dynamic sensitivity are also shown in Table 3 and savings are plotted in Fig. 1 along with static sensitivity. The losses are less and savings are more in case of dynamic sensitivity as shown Table 3 and Fig. 1 respectively.

5.2.3. Comparison with Tabu Search, Hybrid and GA methods

The performance of the proposed PSO method has been compared with TS, Hybrid and GA. The bases of comparison are number of locations, total capacitance required (fixed and switchable), system losses and saving achieved. These values for all the methods are tabulated in Table 4 except saving which is shown in Fig. 3 for ten years. This table divulge that the TS method suggests only three locations and total capacitance of 1800 kvar where as all other methods propose four locations and higher capacitor values

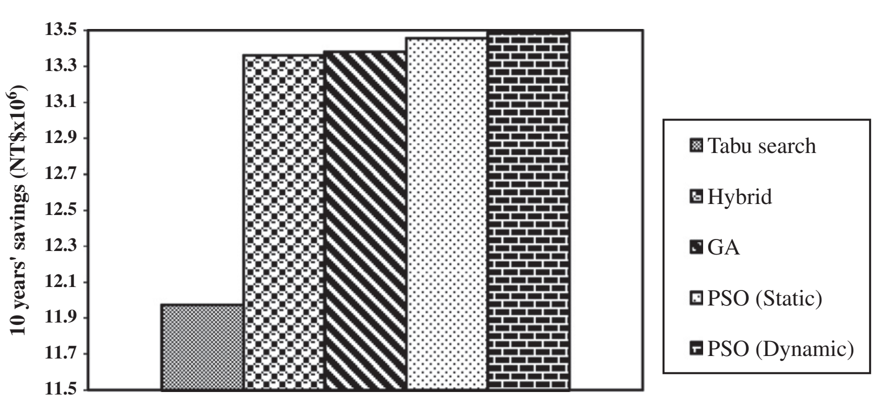

Fig. 2. Savings obtained by different methods (70-Bus).

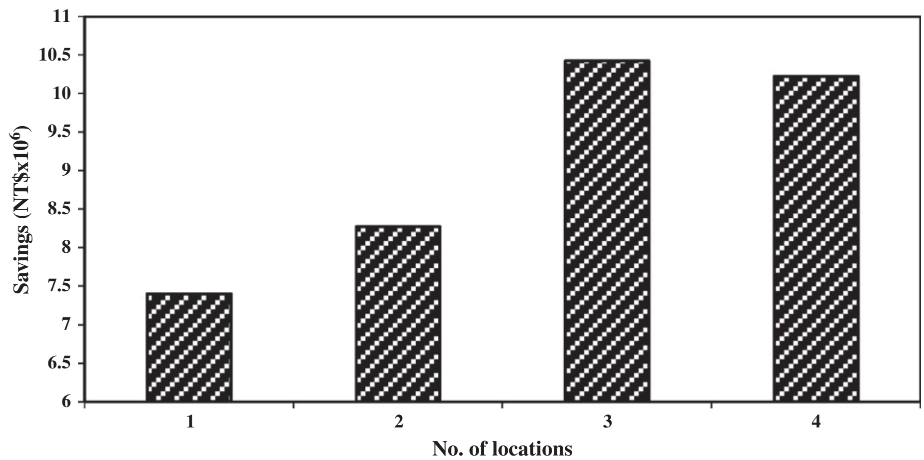

Fig. 3. Savings vs no. of locations (135-Bus).

Table 5
Effect of variation in number of locations for 135-bus at normal load.

No. of Locations	1	2	3	4
Chosen buses	155	155, 43	155, 43, 20	155, 43, 20, 130
Losses (kw)	323.923	317.531	302.263	301.862

than that of TS. However, the voltage profiles in other methods are better and losses are lower as evident in Table 4. The saving is least by TS as compared to other methods. All other methods Hybrid, GA and PSO dynamic suggest capacitor allocation of 2400 kvar in total at four different locations in exception to 2100 kvar by PSO static. The voltage profiles and losses obtained using GA, PSO static and PSO dynamic are comparable with superior voltage profile and lower losses compared to hybrid method. However, losses in case of PSO dynamic are marginally lower resulting in greater saving compared to other methods as seen in Fig. 2. Thus PSO dynamic out performed other methods.

5.3. 135-Bus system

This network is a part of the distribution system of Tres Lagoas, Brazil and the data is obtained from the authors of Ref. [18]. The test conditions of the system are same as documented by the study conducted by Gallego et al. [18] using hybrid method. The load levels L_1, L_2 and L_3 are 0.5, 1.0, and 1.8 respectively and duration of these load levels and energy costs are same as shown in Table 2.

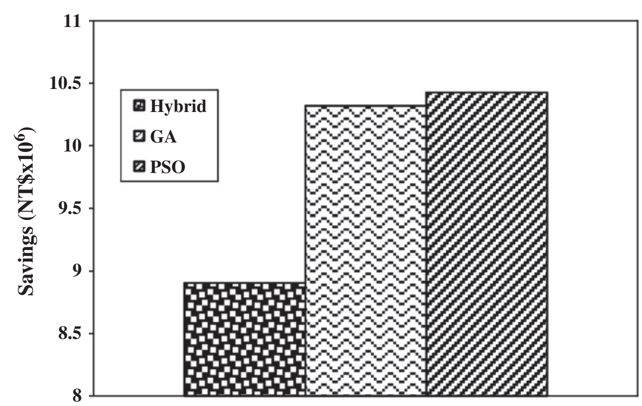

Fig. 4. Savings obtained by different methods (135-Bus).

The sensitivity calculation is performed in the beginning to determine the location of capacitors. The sensitivities of the first four nodes in descending order of their sensitivities are 155, 43, 20, and 130. Finally the dynamic sensitivity has been used to determine the suitable locations due to its advantages demonstrated earlier for 70-bus system.

Following the selection of locations for installation of capacitors, particle swarm optimization is used for determination of values of capacitors at each location. The losses corresponding to various locations are tabulated in Table 5 for normal loading, L_2,

Table 6
Comparative results for 135-bus system.

Method optimal locations		L_1	L_2	L_3	Optimal size (kvar) and type	
					Fixed	Switched
Hybrid method [18]	20	300	600	600	300	300
	43	600	600	600	600	–
	155	600	1200	1200	600	600
	Loss (kw)	118.78	313.53	501.53	**Total = 2400**	
GA method	20	600	600	600	600	–
	43	600	900	900	600	300
	155	600	900	1200	600	600
	Loss (kw)	116.78	302.67	495.86	**Total = 2700**	
PSO	20	600	900	900	600	300
	43	900	900	900	900	–
	155	900	900	900	900	–
	Loss (kw)	115.16	302.26	494.25	**Total = 2700**	

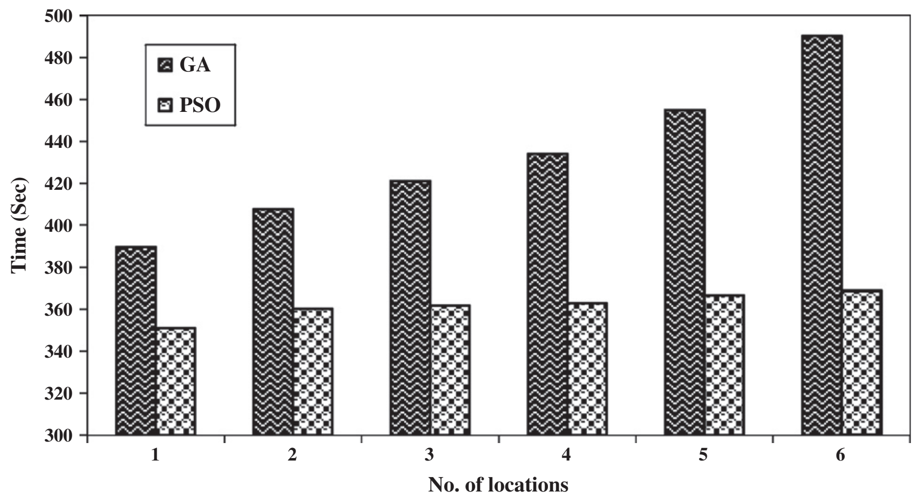

Fig. 5. Comparison of computation time of GA and PSO (135-Bus).

and savings for 10 years are plotted in Fig. 3. As the number of locations increases, investment cost increases according to increase in capacitor values but the losses in the system decrease and savings increase up to certain value beyond which reverse trend is observed. Thus, as the locations increase, the savings obtained increase up to certain locations after which the savings decrease. It is seen from Fig. 3 that maximum saving is achieved for three locations. Further increase in number of locations causes decrease in savings. It is to be noted that these results have been achieved using PSO with dynamic sensitivities.

5.3.1. Comparison with Hybrid Method and Genetic Algorithm

The results of present study are compared with hybrid [18] and GA methods. Three locations are reported for hybrid method also and these locations are same (20, 43 and 155) as obtained for PSO dynamic. The losses and total capacitor values (fixed and switchable) are tabulated in Table 6. This table reveals that the capacitor values of 2400 kvar are required in hybrid method as against 2700 kvar in GA and PSO. However, losses are less in case of GA and PSO compared to hybrid method and least in case of PSO. This reduction in losses reminds greater saving. In view of this, the savings were obtained for these three methods for 10 years and are displayed in Fig. 4. It is obvious from this figure that the saving is least in case of hybrid and maximum in case of PSO. Thus the PSO dynamic has outperformed once again for this system also.

5.3.2. Computational time

It can be observed from the foregoing discussions that the performance of GA and PSO were found to be superior as compared to other methods. Evolutionary nature of both the methods is responsible for this which helps them in avoiding local minima. However, GA involves complex operators like reproduction, crossover and mutation which require time consuming process of swapping of strings. On the contrary, PSO is a simple technique in which a string is modified using dynamically changing velocity. Thus requiring less computational burden compared to GA. The comparison of execution time of PSO and GA with respect to number of locations is shown in Fig. 5. As the number of locations increases, the length of the string of each chromosome increases resulting in more computational burden in GA. Since PSO does not involve any complex operations mentioned above, its computational burden increases marginally with number of locations as demonstrated in this figure. Thus the computational time is very less in case of PSO compared to GA.

6. Conclusions

The out come of the work carried out in this paper can be summarized as under:

(i) Formulation of Optimal Capacitor placement problem and its solution using particle swarm optimization method is reported.
(ii) Static and Dynamic sensitivity analysis are carried out for selection of the locations of capacitors. Dynamic sensitivity yields improved results.
(iii) Performance of PSO has been studied on two widely referred test systems namely 70-Bus and 135-Bus.
(iv) Comparison of PSO is carried out with other methods like Tabu Search, Hybrid Method and Genetic Algorithm
(v) Results demonstrate that PSO yields greater savings, lesser losses and better voltage profile compared to other methods.
(vi) Computation time of PSO is very less when compared with Genetic Algorithm.
(vii) Type of capacitors (fixed or switchable) to be installed is also suggested.
(viii) Further possible applications of the PSO for combinatorial optimization problem in power systems are thus encouraged.

References

[1] Ng HN, Salama MMA, Chikhani AY. Classification of capacitor allocation techniques. IEEE Trans Power Delivery 2000;15(2):387–92.
[2] Neagle NM, Samson DR. Loss reduction from capacitors installed on primary feeders. AIEE Trans 1956;75:950–9.
[3] Cook RF. Optimizing the application of shunt capacitors for volt-ampere control and loss reduction. AIEE Trans 1961;80:430–44.
[4] Duran H. Optimal number, location, and size of shunt capacitors in radial distribution feeders: a dynamic programming approach. IEEE Trans Power Apparat Syst 1968;PAS-87(9):1769–74.
[5] Grainger JJ, Lee SH. Optimal size and location of shunt capacitors for reduction of shunt capacitors on radial distribution feeders. IEEE Trans Power Apparat Syst 1981;PAS-100:1105–18.
[6] Fawzi TH, EI-Sobki SM, Abdel-Halium MA. A new approach for the application of shunt capacitors to the primary distribution feeders. IEEE Trans Power Apparat Syst 1983;PAS-102 (1):10–3.
[7] Ponnavaikko M, Prakasa Rao KS. Optimal choice of fixed and switched shunt capacitors on radial distribution feeders by the method of local variations. IEEE Trans Power Apparat Syst 1983;PAS-102(6):1607–14.
[8] Kaplan M. Optimization of number, location, size, control type and control setting of shunt capacitors on radial distribution feeders. IEEE Trans Power Apparat Syst 1984;PAS-103(9):2659–63.
[9] Baran ME, Wu FF. Optimal sizing of capacitors placed on radial distribution system. IEEE Trans Power Delivery 1989;4(1):735–43.

[10] Khodr HM, Vale Zita A, Ramos Carlos. Optimal cost-benefit for the location of capacitors in radial distribution systems. IEEE Trans Power Delivery 2009;24(2):787–96.
[11] Jabr RA. Optimal placement of capacitors in a radial network using conic and mixed integer linear programming. Electric Power Syst Res 2008;78:94–948.
[12] Silvio Segura, Romero Ruben, Marcos Rider J. Efficient heuristic algorithm used for optimal capacitor placement in distribution systems. Int J Electr Power Energy Syst 2010;32:71–8.
[13] de Oliveira Leonardo W, Carneiro Sandoval Jr, de Oliveira Edimar J, Pereira JLR, Silva Jr Ivo C, Costa Jeferson S. Optimal reconfiguration and capacitor allocation in radial distribution systems for energy losses minimization. Int J Electric Power Energy Syst 2010;32(8):840–8.
[14] Raju Ramalinga M, Murthy Ramachandra KVS, Ravindra K. Direct search algorithm for capacitive compensation in radial distribution systems. Int J Electric Power Energy Syst 2012;42(1):24–30.
[15] Chiang HD, Wang JC, Cockings O, Shin HD. Optimal capacitor placement in distribution systems part I: a new formulation and the overall problem. IEEE Trans Power Delivery 1990;5(2):634–42.
[16] Chiang HD, Wang JC, Cockings O, Shin HD. Optimal capacitor placement in distribution systems part II: solution algorithms and numerical results. IEEE Trans Power Delivery 1990;5(2):643–9.
[17] Huang YC, Yang HT, Huang CL. Solving the capacitor placement problem in a radial distribution system using Tabu search approach. IEEE Trans Power Syst 1996;11(4):1868–73.
[18] Gallego RA, Monticelli AJ, Romero R. Optimal capacitor placement in radial distribution networks. IEEE Trans Power Syst 2001;16(4):630–7.
[19] Ajjarapu V, Albanna Z. Application of genetic based algorithms to optimal capacitor placement. In: Proceedings of First International Forum on Neural Networks to Power Systems; 1991. p. 251–5.
[20] Boone G, Chiang HD. Optimal capacitor placement in distribution systems by genetic algorithm. Int J Electrical Power Energy Syst 1993;15(3):155–62.
[21] Sundhararajan S, Pahwa A. Optimal selection of capacitor for radial distribution systems using a genetic algorithm. IEEE Trans Power Syst 1994;9(3):1499–507.
[22] Miu KN, Chiang HD, Darling G. Capacitor placement, replacement and control in large-scale distribution systems by a GA-based two-stage algorithm. IEEE Trans Power Syst 1997;12(3):1160–5.
[23] Levitin G, Kalyuzhny A, Shenkman A, Chertov M. Optimal capacitor allocation in distribution systems using a genetic algorithm and a fast energy loss computation technique. IEEE Trans Power Syst 2000;15(2):623–8.
[24] Kim KH, Rhee SB, Kim SN, You SK. Application of ESGA hybrid approach for voltage profile improvement by capacitor placement. IEEE Trans Power Syst 2003;15(4):1516–22.
[25] De Souza BA, Alves HN, Ferreira HA. Micro genetic algorithms and Fuzzy logic applied to the optimal placement of capacitor banks in distribution networks. IEEE Trans Power Syst 2004;19(2):942–7.
[26] Hsiao YT, Chen CH, Chien CC. Optimal capacitor placement in distribution systems using a combination Fuzzy-GA method. Electric Power Syst Res 2004;26:501–8.
[27] Chiou JP, Chang CF, Su CT. Ant direction hybrid differential evolution solving large capacitor placement problems. IEEE Trans Power Syst 2004;19(4):1794–800.
[28] Rao Srinivasas R, Narasimham SVL, Ramalingaraju M. Optimal capacitor placement in a radial distribution system using plant growth simulation algorithm. Int J Electric Power Energy Syst 2011;33(5):1133–9.
[29] Haghifam MR, Malik OP. Genetic algorithm based approach for fixed and switch able capacitors placement in distribution systems with uncertainty and time varying loads. Proc IET Gener Transm Distrib 2007;1(2):244–52.
[30] Szuvovivski I, Fernandes TSP, Aoki AR. Simultaneous allocation of capacitors and voltage regulators at distribution networks using genetic algorithms and optimal power flow. Int J Electric Power Energy Syst 2012;40(1):62–9.
[31] Gaing ZL. Particle swarm optimization to solving the economic dispatch considering the generator constraints. IEEE Trans Power Syst 2003;18(3):1187–95.
[32] Yoshida H, Kawata K, Fukuyama Y. A particle swarm optimization for reactive power and voltage control considering voltage security assessment. IEEE Trans Power Syst 2000;15(4):1232–9.
[33] Prakash K, Sydulu M. Particle swarm optimization based capacitor placement on radial distribution systems. In: IEEE power engineering society general meeting 2007; C1–C5. p. 24–8.
[34] Alhajri MF, AlRashidi MR, El-Hawary ME. A novel discrete particle swarm optimization algorithm for optimal capacitor placement and sizing. In: Canadian conference on electrical, and computer engineering, 22–24 April 2007. p. C-1286–C-1289.
[35] Yu X-m, Xiong XY, Wu YW. A PSO based approach to optimal capacitor placement with harmonic distortion consideration.. Electric Power Syst Res 2004;71(1):27–33.
[36] Eajal AA, El-Hawary ME. Optimal capacitor placement and sizing in unbalanced distribution systems with harmonics consideration using particle swarm optimization. IEEE Trans Power Delivery 2010;25(3):1734–41.
[37] Etemadi AH, Fotuhi-Firuzabad M. Distribution system reliability enhancement using optimal capacitor placement. Proc IET Gener Transm Distrib 2008;2(5):621–31.
[38] Kim T, Lee Y, Lee B, Song H, Kim T. Optimal capacitor placement considering voltage stability margin based on improved PSO algorithm. In: 5th International conference on intelligent system applications to power system, ISAP; 2009. 09 art no. 5352890.
[39] Afsari M, Singh SP, Raju GS, Rao GK. A fast power flow solution of radial distribution networks. Electric Power Compon Syst 2002;30(10):1065–74.
[40] Elgerd OI. Electric energy systems theory: McGraw-Hill; 1970.
[41] Kennedy J, Eberhart R. Particle swarm optimization. In: Proceedings of IEEE international conference on neural networks, vol. IV. Perth, Australia, November 29–December 1 1995. p. 1942–8.
[42] Kennedy J, Eberhart R. A discrete binary version of the particle swarm optimization. In: Proceedings of IEEE international conference on neural networks, vol. IX, Perth, Australia. p. 4104–8.
[43] Yshi, Eberhart R. Empirical study of particle swarm optimization. In: Proceedings of the 1999 congress on evolutionary computation. Piscataway; 1999. p. 1945–50.