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Allocation of capacitor banks in distribution systems through a modified monkey search optimization technique

Felipe G. Duque, Leonardo W. de Oliveira *, Edimar J. de Oliveira, André L.M. Marcato, Ivo C. Silva Jr.

Department of Electrical Engineering, Federal University at Juiz de Fora (UFJF), Juiz de Fora, MG, Brazil

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ABSTRACT

This work presents a methodology for the allocation of fixed capacitor banks in electrical power distribution systems by applying a bio-inspired optimization technique. The goal is to optimize the distribution network operation over a planning horizon by minimizing the system losses with minimum cost of investment in capacitors. For this aim to be achieved, this work proposes improvements to the Monkey Search optimization technique to achieve a better representation of the capacitor allocation problem and to increase the computational efficiency. Distribution systems that are widespread in the literature are used to evaluate the proposed methodology.

Introduction

The allocation and control of reactive power support in electrical distribution systems (EDS) by using capacitor banks leads to an optimization problem that has been extensively investigated in the literature. The significance of this problem is supported by the benefits that come from this optimization alternative and the requirements set out by regulatory institutions such as the National Electric Energy Agency (ANEEL) to operate these systems. Technical loss reduction [1], power factor correction and improvement of the system voltage profiles are among the capacitor allocation benefits [2–4].

The optimal capacitor allocation in EDS consists of a combinatorial mixed-integer nonlinear programming problem. Therefore, two characteristics of this problem are the following [5]: (i) the existence of several feasible solutions, which results in a non-convex solution region with several local optimal points, thereby making it difficult to obtain the global optimal solution and (ii) a combinatorial explosion of the options of allocations as the size of the electrical grid increases.

Because of the above characteristics, the proposed methods for optimal capacitor allocation in EDS should combine the ability to obtain high quality solutions with processing times that are not prohibitive for the analysis of these systems [6]. This scenario is conducive to implementation of artificial intelligence systems, heuristic and meta-heuristic optimization techniques, including bio-inspired methods.

In this context, the application of genetic algorithms has been extensively investigated in the literature [7–9]. Tabu search [10], simulated annealing [11], artificial ant colonies [12], [13], particle swarms [14], honey bee colony algorithm [15,16], gravitational search algorithm [17], artificial neural networks [18] and fuzzy logic [19,20] have also been investigated for capacitor allocation in EDS. The compromise between the quality of solutions and the processing times, and the development of simple algorithms are challenges for these types of methods.

Reference [15] proposes the artificial bee colony algorithm for optimal capacitor placement aiming to minimize power system losses and unbalances while maintaining the nodal voltages in acceptable ranges. Using the same technique, reference [16] presents an approach to allocate fixed capacitors along radial distribution networks. High potential buses for capacitor placement are initially identified by the observations of loss sensitivity factor with weak voltage buses. The gravitational search algorithm is used in [17] for optimal capacitor placement in radial distribution systems to reduce power losses subjected to the voltage limits constraints.

Hybrid approaches that combine two optimization techniques have also been proposed. In [21], a combination of fuzzy expert system for capacitor placement and genetic algorithm for capacitor

* Corresponding author. Tel.: +55 (32)8861 1754.
E-mail address: leonardo.willer@ufjf.edu.br (L.W. de Oliveira).

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sizing is proposed to enhance voltage stability by minimizing power and energy loss.

Reference [5] presents a heuristic constructive algorithm for capacitor bank allocation in EDS. In this algorithm, there is an initial selection of a set of busbars that are candidate for allocation using sensitivity indexes based on Lagrange multipliers, which are obtained from an optimal power flow tool (OPF). This selection of the candidate busbars limits efficiently the search space, thereby contributing to reduce the processing times and increasing the quality of the solution.

Reference [22] propose a heuristic algorithm that comprises a loss sensitivity technique to select the candidate locations for the capacitor placement and a loss saving equation with respect to the capacitor currents to determine the size of the capacitors at the compensated nodes. The objective is to reduce the power loss and improve the voltage profile. However, this algorithm does not consider the costs of the capacitors and losses. These same objectives are addressed in [23] by using a simulated annealing algorithm for feeder reconfiguration and capacitor settings.

Based on the application of bio-inspired optimization methods for the optimal allocation of fixed capacitor banks in distribution systems, this paper proposes a modified monkey search (MMS) algorithm to optimize the system operation during a planning horizon through the system losses minimization with the minimum investment cost in capacitors. Therefore, enhancements to the original Monkey Search (MS) method [24] are presented to accommodate the peculiarities of the problem and to increase the computational efficiency of the proposed modified algorithm. It is noteworthy that the application of this method for the capacitor allocation is still not extensively explored in the literature [25] and that the improvements proposed in this paper consists of contributions to this problem. These contributions made the monkey search technique easier to be applied to this practical problem. From the features of this optimization method, its application can be extended to other practical problems by making some adaptations. Case studies on systems known in the literature demonstrate the advantages of the technique proposed in this paper.

Formulation of the problem

The mathematical optimization problem associated with the optimal capacitor allocation in distribution systems to minimize the energy loss and the investment cost can be formulated as follows.

$$\text{Min OBF} = \left[\sum_{u=1}^{nt}\left(\sum_{ij=1}^{nc} ce_u \times T_u \times L_{ij,u}\right) + \sum_{i=1}^{nb} cb \times DI_{mi} \times Qb_{mi}\right] \quad (1)$$

subject to:

$$Pg_{i,u} - Pl_{i,u} + \sum_{j\in\Omega i} p_{ij,u} = 0 \quad (1.1)$$

$$Qg_{i,u} + \left[\sum_{m=1}^{Nbc} DI_{mi} \times Qb_{mi}\right] - Ql_{i,u} + \sum_{j\in\Omega i} q_{ij,u} = 0 \quad (1.2)$$

$$L_{ij,u} = g_{ij} \times \left[V_{i,u}^2 + V_{j,u}^2 - 2 \times V_{i,u}^2 \times V_{j,u}^2 \times \cos(\theta_{ij,u})\right] \quad (1.3)$$

$$V_{\min} \leqslant V \leqslant V_{máx} \quad (1.4)$$

where:

OBF	objective function
Nt	number of load levels
Nc	number of distribution branches
ce_u	energy cost (\$/kW h) for load level u
T_u	duration time (h) of load level u
$L_{ij,u}$	active power loss (kW) of branch ij at load level u
Nb	number of candidate busbars for capacitor bank allocation
Nbc	maximum number of capacitor banks per busbar
Ωi	set of the busbars connected to busbar i through distribution branches
Cb	unitary cost of the reactive power support from capacitors (\$/kVAr)
DI_{mi}	state (1, allocated or 0, not allocated) of capacitor bank m at busbar i
$Pg_{i,u}$	active power generated at busbar i at load level u
$Pl_{i,u}$	active power load at busbar i at load level u
$p_{ij,u}$	active power flow between busbars i and j at load level u
λQ_i	lagrange multiplier: reflect the sensitivity of the objective function to changes in the reactive power injection to busbar i
Qb_{mi}	power of capacitor bank m at busbar i (kVAr)
$Qg_{i,u}$	reactive power generated at busbar i at load level u
$Ql_{i,u}$	reactive power load at busbar i at load level u
$q_{ij,u}$	reactive power flow between busbars i and j at load level u
g_{ij}	conductance of branch ij
$\theta_{ij,u}$	phase angle between busbars i and j at load level u
$V_{i,u}$	voltage magnitude at busbar i at load level u
$V_{\min}$	lower voltage limit at the busbars

The objective function (*OBF*) defined in (1) involves the total cost associated with the energy loss (first term), considering different load levels u, and the investment cost in capacitor banks (second term). This function had been also used in [5]. Other terms can be added to the objective function, as the annual cost of peak power loss ($kp \times L_p$), where kp is the equivalent unitary cost of the peak power loss during one year (\$/kW-year) and L_p is the peak power loss (kW-year) [26]. In the present work, some case studies include these terms.

Eqs. (1.1) and (1.2) correspond to the balance constraints of the active and reactive power in each busbar, respectively. Eq. (1.3) is used for calculating the power loss in branch *ij*. The lower voltage limits at the busbars are considered through (1.4).

The capacitor bank allocation is represented by a corresponding amount of reactive power injected into candidate busbar i according to Eq. (1.2). This amount is given by multiplying the power of each bank by the number of banks allocated at this busbar.

For each busbar i, the decision regarding the capacitor allocation is determined by the discrete variable, DI_{mi}. For illustration, it is supposed that the maximum number of banks per busbar (*Nbc*) is equal to 3 and that a single bank is allocated at busbar i. In this case, one of variables $DI_{1,i}$, $DI_{2,i}$ or $DI_{3,i}$ is equal to 1, whereas the two remaining variables are equal to zero. For treatment of the discrete variables, $DI_{m,i}$, an algorithm based on the bio-inspired optimization technique MS is proposed.

The modified monkey search algorithm

The bio-inspired optimization technique known as Monkey Search (MS) was developed in [24,27]. This technique is inspired by the behavior of a monkey searching for food in a jungle. Such a search is performed by climbing up and down trees that contain food sources. As the search continues, the monkey stores and updates in its memory the best route found. This adaptive memory is then used for achieving more promising routes among various possible alternatives.

Similar to other techniques based on heuristics and bio-inspired behaviors, the MS method seeks to determine high-quality solutions in a computationally efficient manner. The Modified Monkey Search (MMS) algorithm presented in this paper is based on the MS algorithm of [24,27]. Both algorithms associate the mechanisms of the adaptive memory and evolution of routes, as aforementioned, for applying to the search processes of combinatorial optimization problems. This association is summarized hereinafter:

(i) A tree consists of a set of nodes that are linked by paths, as pictured in Fig. 1 where the first node "A" is the root.
(ii) The root and nodes of a tree contain food sources that are related to the possible solutions for an optimization problem.
(iii) A branch of a tree is associated with a perturbation in the current solution of the search process that allows the transition to another solution, in analogy to climbing the tree from one node to another.
(iv) The adaptive memory is associated with the storage of the information acquired during scanning the solution space and is used to conduct the search process.

From the previous definitions, the paths of the tree in Fig. 1 are (i) "A–B–D", (ii) "**A–B–E**", (iii) "A–C–F" and (iv) "A–C–G". Node "A" is the initial solution and the remaining nodes correspond to the derived solutions. This tree has three levels, where the first level consists of node "A", the second level comprises nodes "B" and "C" and the third level nodes "D", "E", "F" and "G". The third level nodes form the top of this tree. The highlighted path "**A–B–E**" comprises branches "A–B" and "B–E" and nodes "A", "B" and "E".

The proposed modifications and improvements in the MS method seek to represent better the capacitor allocation problem. The search process of the proposed MMS algorithm can be divided into two steps.

(1) Step 1 – Search in the initial tree: When the proposed MMS method initiates the search process, there is no information regarding the paths to be investigated, i.e., there are no solutions stored in the memory. Thus, the search in the initial tree is full and involves all candidate paths. From the results of this full search, the adaptive memory mechanism begins to store a set of solutions that will serve as references for future trees, which are referred to as subsequent trees.

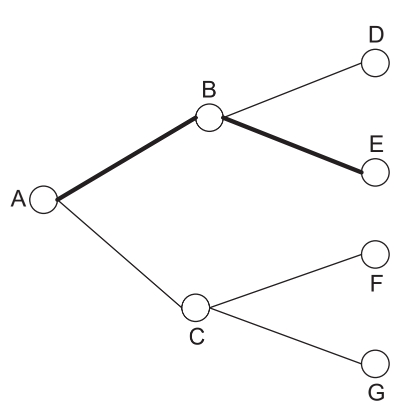

Fig. 1. The tree structure of the proposed MMS algorithm.

(2) Step 2 – Search in subsequent trees: The subsequent trees in the proposed MMS method are obtained from perturbations of the best solution found in the initial tree. From the reference memory formed in Step 1, the optimization algorithm performs a directed sweep in the subsequent trees, thereby avoiding the full search process and accelerating the investigation of these trees until a convergence criterion is met, which will be explained later.

The main aspects of the proposed MMS algorithm, including the definition of parameters, convergence criteria and perturbation mechanisms of the solutions, are addressed hereafter.

Parameters of the initial tree

The initial tree involves a set of solutions obtained from a search process that starts from a single solution, node "A" in Fig. 1. For the capacitor bank allocation problem, this initial solution, the root of the initial tree, is not random and corresponds to the base case, i.e., to the condition without any capacitor bank allocation.

The search procedure in the initial tree is exhaustive due to the lack of prior information regarding the solution space. However, in the proposed MMS algorithm, the convergence of the initial tree is obtained when all paths are covered.

Each tree has a binary-structured system, where at each node one of two perturbation options can be selected and each choice leads to a new node. From node "A" of Fig. 1, node "B" can be selected through the upper branch or node "C" via the lower branch. The codification for the decisions taken in this structure can be "0" for the lower branch and "1" for the upper branch. Thus, the covered path "A–B–E" is represented by the binary code as 1–0, which means that this path consists of the upper branch from "A" ("A–B") followed by the lower branch from "B" ("B–E").

The depth (h) of a tree is defined by the number of levels minus 1. So the depth of the tree in Fig. 1 is given by $h = 3 - 1 = 2$. The number of paths (c) of a tree is limited by parameter h according to Eq. (2).

$$c = 2^h \tag{2}$$

where:

c	number of possible paths in the tree
h	height of the tree

Thus, the number of paths in the tree of Fig. 1 is equal to $c = 2^h = 2^2 = 4$.

The routes taken in the MMS algorithm include processes of climbing up and down a tree. Each climbing down process occurs in the opposite direction of the previous stage of climbing up, i.e., covering the same nodes, however, in the opposite direction.

The parameter "depth" of the tree (h) has great importance because it determines the number of paths to be investigated and consequently, the number of solutions to be evaluated. The choice of this parameter should consider the following aspects:

(i) Elevated h value – THE greater the depth of the tree, the greater is the number of paths and candidate solutions. As the parameter h increases, the chance of obtaining a high quality solution increases, but the processing time also increases.

(ii) Low h value – a low value of h implies shorter computational times for convergence of the MMS; however, it limits the search space and thus affects the quality of the solution.

Based on these aspects, for choosing a value for h that is adequate for each specific problem, there is a compromise between the quality of the solution and the computational requirements.

One advantage of the proposed MMS is the reduced number of parameters to be adjusted. For example, once the parameter h is choosen, the parameter c is a function of h as shown in Eq. (2).

Adaptive memory

The adaptive memory of the proposed MMS algorithm consists of a list of the ten best solutions found. This memory is formed during the search in the initial tree and it is updated during the searches in the subsequent trees.

To explain the formation and updating of the adaptive memory, consider that after the search in the initial tree, the memory is given by memo_{m1} of Eq. (3), which is formed by solutions $m_{1,m1}$ to $m_{10,m1}$ arranged in descending order of quality.

$$\text{memo}_{m1} = [m_{1,m1}, m_{2,m1}, m_{3,m1}, m_{4,m1}, m_{5,m1}, m_{6,m1}, m_{7,m1}, m_{8,m1}, m_{9,m1}, m_{10,m1}] \tag{3}$$

where:

memo_{m1} initial adaptive memory
$m_{n,mi}$ solution stored in position n during exploration of tree mi; $mi = m1$ for the initial tree

The vector of Eq. (3) is obtained by an exhaustive search in the initial tree ($m1$). The best solution of this vector is $m_{1,m1}$ and it is defined as the root of the first subsequent tree ($m2$). This solution is perturbed until the convergence criterion of the subsequent tree is reached, which will be explained later. The process of updating the memory is performed every time a solution better than that belonging to the set [$m_{1,m1}$: $m_{10,m1}$] is found. Hence, the new solution is inserted in this set at a position defined according to its quality and the subsequent values are shifted to the right. The value in the last position, $m_{10,m1}$, is discarded and replaced by the value stored in $m_{9,m1}$, $m_{9,m1}$ receives the value of $m_{8,m1}$, and so on, until the position after the new solution. In this update mechanism, the size of the vector memo remains the same, with ten positions.

As an example, during the search process in tree $m2$, solutions $m_{1,m2}$ and $m_{2,m2}$ found in $m2$ are better than solutions $m_{3,m1}$ and $m_{6,m1}$ found in tree $m1$ and included in the initial memory of Eq. (3), respectively. In this case, the new configuration of the adaptive memory, updated after searching tree $m2$, is shown in Eq. (4).

$$\text{memo}_{m2} = [m_{1,m1}, m_{2,m1}, \boldsymbol{m_{1,m2}}, m_{3,m1}, m_{4,m1}, m_{5,m1}, \boldsymbol{m_{2,m2}}, m_{6,m1}, m_{7,m1}, m_{8,m1}] \tag{4}$$

where memo_{mi} is the adaptive memory obtained after convergence of tree mi and the solutions highlighted with gray shades updated memo_{m2}.

It is observed that the updating memory procedure is not restricted only to solutions better than $m_{1,m1}$. In the example above, $m_{1,m2}$ and $m_{2,m2}$ are not better than $m_{1,m1}$ but are better than $m_{3,m1}$ and $m_{6,m1}$, respectively. This updating strategy allows faster convergence of the algorithm and consequently increases its computational efficiency in relation to the Monkey Search in [24,27], which limit the updating procedure to solutions better than $m_{1,m1}$. Thus, the proposed updating strategy is an improvement of the MMS algorithm.

Perturbation mechanism of the current solution

The procedure performed to modify the optimal solution contained in the root of the tree is defined as a perturbation. In the problem of capacitor bank allocation, this perturbation comprises some unit increases or decreases in the number of capacitor banks in randomly selected candidate busbars. The numbers of unit increments and decrements are also random and can vary from one to three. To exemplify this perturbation mechanism, a system with 10 busbars is considered, and after searching tree $m1$, the best solution found, $m_{1,m1}$, is given by Table 1.

According to Table 1, the solution given by $m_{1,m1}$ after searching tree $m1$ establishes 1 capacitor bank at busbars 1, 2 and 10, two banks at busbar 6 and three banks at busbar 8. Supposing that the random values for the numbers of unit increments and decrements are equal to 1 and 2, respectively, and that the randomly selected busbars are busbar 2 for the unit increment and busbars 6 and 8 for the unit decrements, solution $m_{1,m1}$ after this perturbation is given by Table 2, where the highlighted positions refer to the values updated after the perturbation mechanism.

The perturbation mechanism must meet the constraint of the maximum number of banks per busbar (*Nbc*). Therefore, if one busbar that already contains *Nbc* capacitor banks allocated is selected for increment, another busbar should be selected until this constraint is met. Similar reasoning holds for the case in which a busbar without capacitor allocation is selected for decrement of its number of banks. Also in this case, another busbar should be selected until the constraint of the minimum number of banks (zero) is satisfied.

Convergence

The convergence criterion of a tree in the proposed MMS algorithm differs from the respective criterion in the MS [24,27]. Convergence of a tree in the MS algorithm is achieved when all the branches of this tree are covered (exhaustive search for branches). This criterion applies to both the initial tree and subsequent trees. In the proposed MMS, the convergence criterion for the initial tree differs from the criteria for the subsequent trees, as described here.

Criterion for the initial tree: The convergence of the initial tree is obtained when all paths of this tree are covered (exhaustive search for paths). The concept of path was previously described.

Criterion for subsequent trees: The convergence of a subsequent tree is obtained when at least one of the following conditions is met:

(i) The solution obtained by a perturbation is better than the root solution of the tree, i.e., the quality of a perturbed solution ($m_{n,mi}$) is greater than the quality of the best solution ($m_{1,\text{mi}}$). In the problem of optimal capacitor allocation, this condition is met when $OBF(m_{n,mi}) < OBF(m_{1,\text{mi}})$.
(ii) All paths of the subsequent tree are covered.

In addition to the convergence criterion for a tree that allows the transition to another tree, there is a global convergence criterion for the proposed MMS algorithm, which also differs from the respective criterion of the MS. In the MS algorithm, the global convergence is achieved when all the memory values are sufficiently close. In the proposed MMS algorithm, the global convergence is achieved when at least one of the following conditions is met:

Table 1
Capacitor allocation defined by $m_{1,m1}$ after searching tree $m1$.

Number of busbars	1	2	6	8	10
Number of banks	1	1	2	3	1

Table 2
Optimal capacitor banks allocation after the perturbation.

Number of busbars	1	2	6	8	10
Number of banks	1	**2**	**1**	**2**	1

(i) The difference between the objective functions of the solutions of the last and first positions of the adaptive memory is less than or equal to a tolerance ε. Thus, this condition is achieved when $OBF(m_{10,mi}) - OBF(m_{1,mi}) \leqslant \varepsilon$ for a given tree *mi*.
(ii) A maximum number of trees (nt_{max}) are covered.

Intensification process

The proposed MMS algorithm stores the paths that lead to the best solutions of the adaptive memory shown in Eq. (4) during the optimization procedure. Then, the intensification process intensifies the search process in these paths after a predefined number of perturbations that lead to them (*nperi*). In this sense, the MMS can evaluate solutions with the allocation of any total number of capacitor banks in the beginning of the search process. During this process, if a candidate solution is included in the adaptive memory *nperi* times, the total number of capacitor banks will be fixed at the value that has been obtained in this solution. For example, if the solution of Table 1 gets into the adaptive memory *nperi* times during the search process, the total number of capacitor banks of the subsequent candidate solutions will be fixed at 8.

From the aforementioned aspects of the proposed MMS algorithm, Table 3 summarizes the differences between this approach and the MS technique of [24,27]. These differences arose from the modifications and improvements to represent better the capacitor allocation problem.

Flowchart of the proposed MMS algorithm

Fig. 2 presents the flowchart of the proposed MMS algorithm. The steps of this algorithm are described as follows.

Step 1: Input Data. In this step, the distribution system data are obtained and the MMS parameters are defined.
Step 2: Climbing up the initial tree. This step consists of exploring the initial tree from its root, which corresponds to the base case, without any capacitor bank allocation. The root is successively perturbed until the top is reached. The root and each node achieved in the tree are solutions for the optimal capacitor allocation evaluated each one through the optimization problem formulated in (1). The quality or fitness of each solution is inversely proportional to its objective function (OBF) associated with the total cost of loss and investment.
Step 3: Initialization of the adaptive memory (memo). The *n* better solutions found in the initial tree are stored in memo in descending order of fitness. The first element of memo is named *ibest*. In the proposed algorithm, *n* is set to 10.
Step 4: Beginning of search in the subsequent tree *mi*. Perturbation of *ibest* to generate two new nodes or solutions through the perturbation mechanism described before.
Step 5: Choosing the new current solution. The node generated at Step 4 that presents the better fitness is chosen.
Step 6: Evaluation of the new current solution chosen at Step 5. Two situations can occur: (i) if the chosen solution presents *fitness* better than *ibest*, this solution replaces *ibest* and the convergence of tree *mi* is achieved, in this case counter *i* is incremented and a new tree *mi* begins to be explored from Step 4; (ii) otherwise, the adaptive memory is updated if the new current solution is better than at least one solution of memo, in this case the algorithm goes to Step 7.
Step 7: Global convergence criteria evaluation. In this step, the global convergence criteria previously described are assessed. If at least one of the presented conditions is achieved, the algorithm is ended. Otherwise, it goes to Step 8.
Step 8: The algorithm verifies if the top of tree *mi* was achieved. If the answer is 'yes', it means that no solution better than *ibest* has been found from the root to the top of *mi*. In this case, the algorithm returns to Step 4 to perform a new perturbation

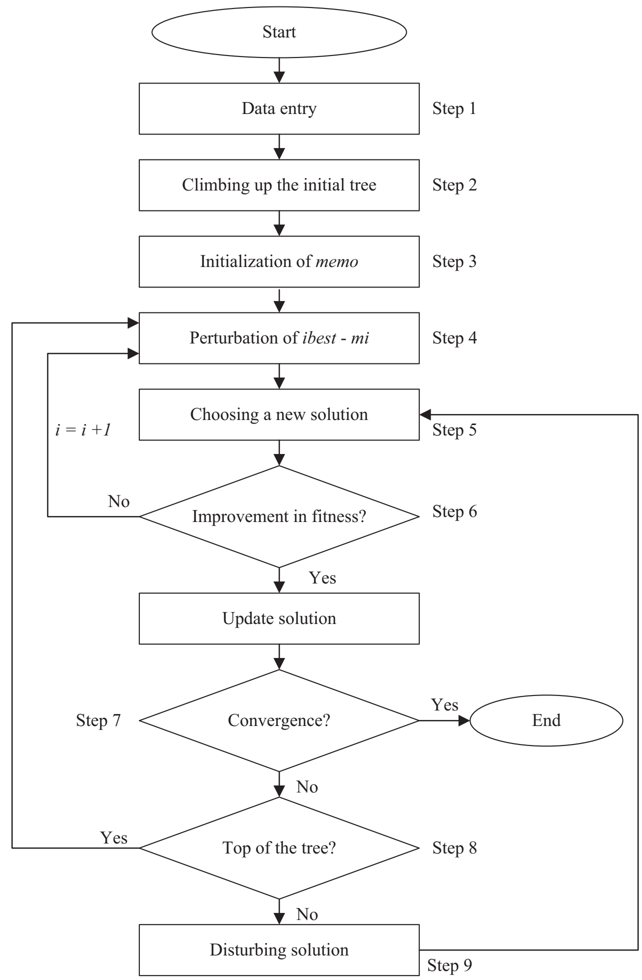

Fig. 2. Flowchart of the proposed MMS algorithm.

Table 3
Comparison between the proposed MMS and the MS algorithms.

Algorithm/ criterion	Proposed MMS	MS
Trees	Initial ≠ subsequent	Initial = subsequent
Search in subsequent trees	Depends on the convergence of the tree	Full
Convergence of the tree	All paths are covered	All branches are covered
Path	Root to top	Root to top or Intermediate node to top
Root solution	The best solution of the adaptive memory	Any solution stored in the adaptive memory
Intensification process	There is an intensification process to improve the solution	There is no intensification process

process in *ibest*. This procedure is the climbing down process. Otherwise, i.e., if the top has not been achieved, the algorithm remains in the climbing up process at Step 9.
Step 9: Perturbation of the current solution. As previously described, the perturbation mechanism generates two new solutions candidate to the new current solution. From this generation, the algorithm goes to Step 5.

Case studies

To evaluate the proposed MMS algorithm, case studies were performed using the following systems: (i) Tutorial Case, a system of 15 busbars [28]; (ii) Case 1, a system of 33 busbars [29]; (iii) Case 2, a system of 34 busbars [30]; (iv) Case 3, a system of 85 busbars [28]; (v) Case 4, a system of 69 busbars [31]; and Case 5, a system of 476 busbars [32]. The parameters of the proposed MMS algorithm for these studies are:

- The parameter "depth" is given by $h = 8$, which results in a total number of paths in each tree $c = 2^h = 2^8 = 256$ according to Eq. (2).
- The initial total number of capacitor banks is equal to 15.
- Tolerance ε for the global convergence is equal to zero.
- The maximum number of trees for the global convergence ($nt_{\max}$) is equal to 20.
- The number of times that a solution must get into the adaptive memory (*nperi*) for fixing the total number of capacitor banks of the candidate solutions is equal to 100 (intensification process).

Other meta-heuristic optimization techniques were developed to be compared with the proposed MMS approach. These techniques are the Monkey Search (MS) [24,27], Genetic Algorithm (GA) and the Simulated Annealing (SA). The GA parameters were obtained from references [5]: (i) crossover probability 95%, (ii) mutation probability 2%, (iii) population size 300, (iv) number of generations 100, (v) convergence criterion based on number of generations, (vi) elitism, (vii) decimal coding, (viii) roulette selection, and (ix) two point crossover. The SA parameters for all the case studies were obtained from reference [23]: (i) Boltzmann constant 1, (ii) initial temperature 30, (iii) maximum number of iterations 300, (iv) cooling rate 0.95.

All simulations using the proposed MMS and the developed MS, GA and SA algorithms were performed using a 3.40-GHz Intel Corei7–2600 processor with 4 GHz RAM. The constraint related to minimum voltage of 0.9 pu in all busbars is applied to all cases.

Tutorial case: 15 busbar system

The system of 15 busbars of [28] was used as a tutorial system to clarify the steps of the proposed MMS algorithm. This system has a substation (SS), 14 load busbars and nominal voltage of 11 kV. Fig. 3 illustrates the diagram of this system.

In this case, a unique load level (1.0 pu) is considered. The energy cost for this level is 0.06 \$/kW h, the investment cost in capacitors is 4.0 \$/kVAr, the duration time is 1 year and all load busbars are candidate for capacitor allocation. For this study, a maximum of three capacitor banks of 200 kVAr per busbar was considered. The active losses for the base case, without capacitor allocation, is 61.79 kW or 541,311.00 kW h. The cost formulated in (1) for the objective function is \$ 32,478.66 for the base case.

From the base case, the search in the initial tree starts. This search continues until all paths of the initial tree, involving 256 solutions, are covered, in accordance with the convergence criteria for this tree. After this search, the ten best solutions found are selected to initialize the adaptive memory (memo). Table 4 presents the active losses for the ten best solutions found in the initial tree.

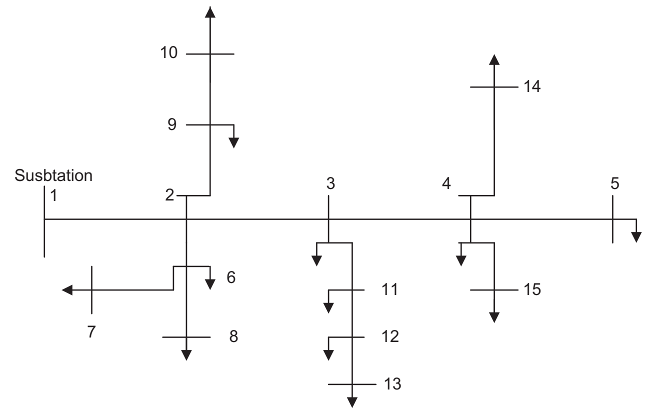

Fig. 3. 15 Busbar system.

The next step consists of searching the subsequent trees looking for improving the adaptive memory. This search is not full, i.e., it does not cover all paths in the subsequent trees. Table 5 presents the number of the evaluated solutions and the cost of the best solution (*ibest*) of each tree.

From Table 5, it can be observed that the total cost modeled in the *OBF* (1) decreased from \$ 32,478.66 (base case) to \$ 21,175.90 after the search in the initial tree ($m1$). The root of tree $m2$ is the best solution found in $m1$ and so on. Starting in the root of $m2$, ten perturbation processes are performed in this tree until a solution better than its root is achieved, which is the convergence criterion for $m2$. Then, ten solutions are evaluated in $m2$ before its convergence criterion is met. The same reasoning applies for the other trees. The final solution given by the proposed algorithm is that found after searching the last tree ($m9$) and presents a total cost of \$20,220.96. Fig. 4 presents the convergence of the proposed algorithm for this tutorial case in which the x-axis gives the

Table 4
Tutorial case: solutions of the initial adaptive memory.

Position	Costs (\$)
$m_{1,m1}$	21,175.90
$m_{2,m1}$	21,229.42
$m_{3,m1}$	21,240.06
$m_{4,m1}$	21,322.87
$m_{5,m1}$	21,367.37
$m_{6,m1}$	21,497.35
$m_{7,m1}$	21,661.31
$m_{8,m1}$	21,676.33
$m_{9,m1}$	21,780.89
$m_{10,m1}$	21,902.45

Table 5
Tutorial case: number of evaluated solutions and the best solution of each tree.

Tree (m_i)	Evaluated solutions	Cost of the *ibest* (\$)
$m1$	256	32,478.66 → 21,175.90
$m2$	10	21,175.90 → 21,110.34
$m3$	14	21,110.34 → 21,097.01
$m4$	16	21,097.01 → 21,068.62
$m5$	20	21,068.62 → 21,015.76
$m6$	35	21,015.76 → 20,945.40
$m7$	96	20,945.40 → 20,682.30
$m8$	158	20,682.30 → 20,471.15
$m9$	254	20,471.15 → 20,220.96

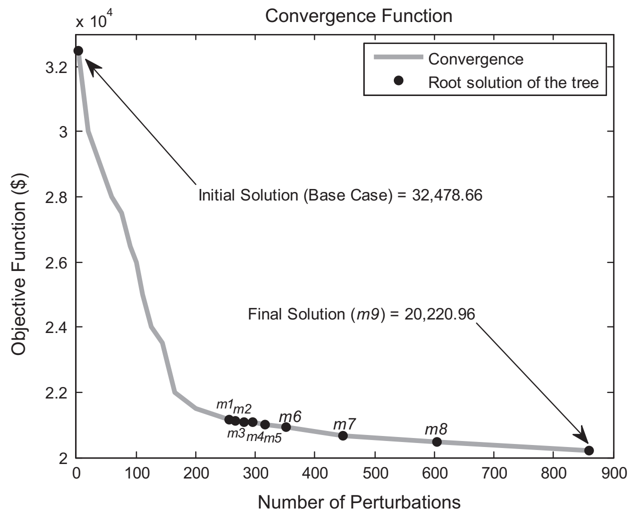

Fig. 4. Tutorial case: convergence of the proposed MMS algorithm.

cumulated number of perturbations until the final solution of corresponding *mi* in the convergence curve is obtained.

It can be observed in Table 5 and in Fig. 4 that as the optimization process evolves, i.e., as the index *i* of tree *mi* increases, a larger number of perturbations is required to obtain a solution better than its root and then the number of evaluated solutions of mi increases with the index *i*. This behavior occurs because as the algorithm evolves, better solutions are obtained and it becomes more difficult to overcome the current best solution.

Table 6 presents the results associated with the best solution found by the proposed algorithm after its convergence or after the convergence of tree *m*9, as well as the main aspects of this algorithm for the tutorial case as the computational time. The second line presents the numbers of the busbars that received capacitor allocation with the corresponding amount of reactive support in parentheses. As the size of each capacitor bank is 200 kVAr, the algorithm established the allocation of one (1) bank to each of busbars 4, 6, 7, 12 and 15.

From Table 6, the total cost of the best solution found by the MMS algorithm is $ 20,220.96, which represents a reduction of 37.74% related to the base case. After evaluating 351 candidate solutions, or after searching trees *m*1 to *m*6, 301 solutions had been included in the adaptive memory, of which 100 established five capacitors banks. As the parameter that controls the intensification process (*nperi*) is equal to 100, the proposed MMS begun to maintain the total number of banks of all candidate solutions as 5 from this point until the convergence of the algorithm (search in trees *m*7 to *m*9). Thereby, from tree *m*7 the MMS algorithm evaluated 508 more solutions setting the total number of banks to 5 until the convergence totaling 859 evaluated solutions. Table 7 presents the numbers of solutions that had been included in the adaptive memory until the beginning of tree *m*7 with the corresponding number of capacitor banks, where it can be noticed that 100 solutions defined five banks until this point, as highlighted in the table.

Table 6
Tutorial case: results and general aspects of MMS.

Total loss (kW h)	270,349.33
Optimal locations and sizes (kVAr)	4(200), 6(200), 7(200), 12(200), 15(200)
Total (kVAr)	1000
Costs of losses ($)	16,220.96
Costs of allocation ($)	4000.00
Total cost ($)	20,220.96
Cost reduction (%)	37.74
Minimum voltage (pu)	0.9674
Processing time (seconds)	2.82
Number of covered trees	9
Total number of evaluated solutions	859

Case 1

The 33-busbar system [29], nominal voltage 1266 kV, base power 100 MVA, has a power loss of 202.68 kW and is pictured in Fig. 5. This case study considers an operation during one year or 8760 h and a unique load level (1.0 pu). The energy cost for losses is 300 $/kW h, the size of each capacitor bank is 300 kVAr, the maximum number of capacitor banks per busbar is three, and all load busbars are candidate for capacitors allocation with an investment cost of 25,000 $/kVAr [33]. Without capacitor allocation, the total cost corresponds to the loss cost and is equal to $ 5,326,354.85. Table 8 presents the results obtained by the proposed MMS approach and by other methods, considering the objective function previously modeled in Eq. (1). The solution of the proposed approach is highlighted in Table 8. The financial percentage return relates to the total cost.

From Table 8, it can be observed that the proposed MMS algorithm led to the result with the smallest total cost. Besides, the minimum voltage obtained with the MMS approach is the highest among the evaluated methods. Moreover, it is observed in Table 8 that references [22,34] considered other sizes for the capacitor banks. The CPU time for reference [33] was not given.

Case 2

The topology of the 34-busbar system [30] is shown in Fig. 6. This case considers the same conditions of reference [35] such as the operation time (one year or 8760 h), a unique load level (1.0 pu), and commercially available capacitors sizes with the corresponding costs in $/kVAr. To enable a proper comparison with [35] and other references, in this case the objective function is composed of two terms [35]: (i) the investment cost in reactive power support from capacitors, $/kVAr as in Table 9, and (ii) the annual cost of peak power loss, where *kp* is 168.00 $/kW-year. Moreover, the total energy loss cost is not considered as in [35], and all load busbars are candidate for capacitor allocation.

The results obtained by the proposed MMS algorithm and by other approaches are given in Table 10. The net saving and percentage return relates to the total cost. It can be observed from Table 10 that references [22,35,39] consider continuous sizes for the capacitors, because they did not determine only multiple values of the sizes in Table 9. However, in the present work, the reactive powers from capacitors were modeled as discrete and multiple values of the size of a bank because it is more practical for fixed capacitors banks. Moreover, this modeling leads to better results, total cost and minimum voltage, than [22,35–39] for this case, as well as if compared to the developed MS, GA and SA algorithms.

Case 3

The 85-busbar [28] has a distribution feeder with voltage level of 11 kV. This system is shown in Fig. 7. This case considers the same load levels, unitary costs for peak power loss and capacitors, sizes for capacitors, candidate busbars, operation time, and objective function described for Case 2 and obtained from [35]. Table 11 presents the results of the MMS algorithm and of other methods for this network. For this case, references [35,36,39] considered continuous sizes for the capacitors, as described in the previous case, instead of discrete banks as the proposed approach, which gave the best total cost and minimum voltage.

Table 7
Tutorial case: number of solutions with the corresponding number of capacitors banks until tree *m*7.

Capacitors banks	1	2	3	4	**5**	6	7	8	9	10	11	12	13	14	15
Number of solutions	7	27	42	65	**100**	46	13	1	0	0	0	0	0	0	0

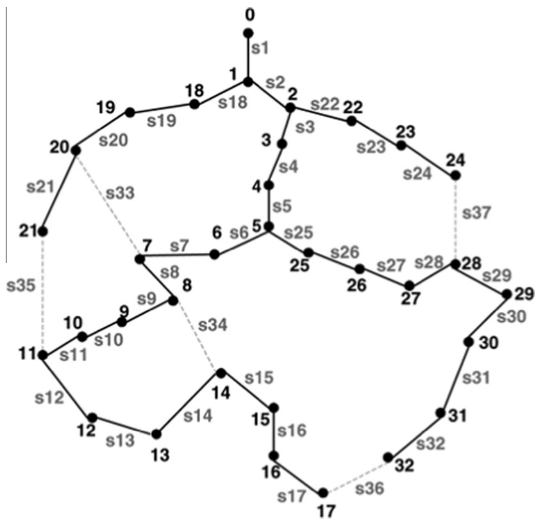

Fig. 5. 33-Busbar system.

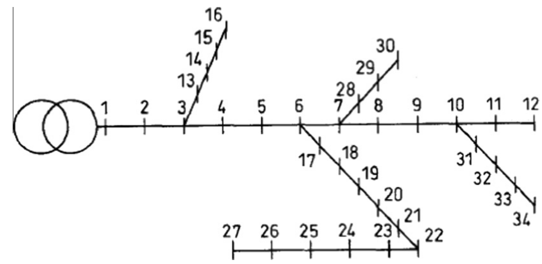

Fig. 6. 34-Busbar system.

Case 4

The 69-busbar system [31], whose voltage level is 12.66 kV, has 1 substation, 69 busbars, 74 distribution branches and the configuration of Fig. 8. This case comprises three load levels: light (0.5 pu during 1000 h), medium (1.0 pu during 6760 h) and heavy load (2.45 pu during 1000 h). Therefore, the total operation time is 8760 h or one year. The costs for the energy loss and capacitors are 0.06 $/kW h e 4.00 $/kVAr, respectively. The capacitors size is 200 kVAr and the maximum number of banks per busbar is three. These same operation conditions are given in reference [5], which also considers the following set of busbars candidate for capacitor allocation: [7 8 9 10 11 12 14 15 16 17 18 21 24 26 27 49 50 51 54 55 59 61 62 64 65 66 67 68 69]. This same set is considered by the proposed MMS in this case for comparison purposes. The objective function is given by Eq. (1) as in Case 1.

For this case, two analyses were done: *Case 4a* – without the constraints of nodal voltage limits for comparison purposes, and *Case 4b* – with the lower voltage limit constraints (0.9 pu at all busbars). By considering the voltage limits, *Case 4b*, the power flow does not converge for the heavy load level and a load cut is required for the system operation. The present paper did not use a load cut model, but the maximum load level to operate the system with the minimum voltage of 0.9 pu was determined empirically (1.3 pu). Therefore, *Case 4a* considers 2.45 pu for the heavy load level and *Case 4b* considers 1.3 pu.

The results obtained by the MMS approach and by others are shown in Table 12 (*Case 4a*) and Table 13 (*Case 4b*), where the three values for columns "Minimum Voltage", "Total Loss" and "Loss Cost" consist of the results for the light, medium and heavy load levels.

As previously described, Refs. [22,40] did not consider discrete values for the capacitor banks and the proposed approach also determined for this case the best total cost among the methods used for comparison. From Tables 12 and 13, it can be pointed out that in *Case 4b* all the minimum voltages are above the limit of 0.9 pu, whereas in *Case 4a*, which does not consider the voltage limit constraint, some voltages present unsuitable for the system operation. By considering the voltage limits constraints, the proposed MMS approach gave the smallest total cost compared with the developed MS, GA and SA algorithms.

Case 5

This case handles a real medium-scale (13.8 kV) system with 476 busbars [32] that comprises two distribution feeders. This

Table 8
Case 1: comparison of the proposed method results with previous publications.

Methodology	Optimal locations and size (kVAr)	Minimum voltage (pu)	Total loss (kW h)	Loss cost ($)	Capacitor cost ($)	Total cost ($)	CPU time (s)
Without capacitor	–	0.9131	1,775,451.62	5,326,354.85	0.00	5,326,354.85	N/A
Proposed MMS	9(300), 13(300), 29(900)	0.9416	1,189,333.11	3,567,999.33	375,000.00	3,942,999.33	5.22
Developed MS	6(300), 13(300), 27(600), 30(600)	0.9409	1,188,481.79	3,565,445.38	450,000.00	4,015,445.38	16.79
Developed GA	9(300), 10(300), 26(300), 28(300), 30(600)	0.9403	1,196,063.11	3,588,189.33	450,000.00	4,038,189.33	23.39
Developed AS	6(300), 8(300), 10(300), 26(300), 29(900)	0.9401	1,181,154.34	3,543,463.03	525,000.00	4,068,463.03	126.78
[33]	8(300), 15(300), 20(300), 21(300), 24(300), 26(300), 27(600), 28(300)	0.9349	1,186,980.00	3,560,940.00	675,000.00	4,235,940.00	N/A
[22]	7(850), 29(25), 30(900)	0.9341	1,196,712.39	3,590,137.16	443,750.00	4,033,887.16	2.94
[34]	6(1200), 28(760), 29(200)	0.9390	1,186,104.00	3,558,312.00	540,000.00	4,098,312.00	5.13

Table 9
Case 2: possible sizes of capacitors and costs in $/kVAr.

kVAr	150	300	450	600	750	900	1050	1200	1350	1500	1650	1800	1950	2100
$/kVAr	0.500	0.350	0.253	0.220	0.276	0.183	0.228	0.170	0.207	0.201	0.193	0.187	0.211	0.176
kVAr	2250	2400	2550	2700	2850	3000	3150	3300	3450	3600	3750	3900	4050	N/A
$/kVAr	0.197	0.170	0.189	0.187	0.183	0.180	0.195	0.174	0.188	0.170	0.183	0.182	0.179	N/A

Table 10
Case 2: comparison of the proposed method results with previous publications.

Methodology	Optimal locations and size (kVAr)	Minimum voltage (pu)	Peak loss (kW)	Peak loss Cost ($)	Capacitor costs ($)	Total cost ($)	CPU time (s)
Without capacitor	–	0.9417	221.68	37,241.00	0.00	37,241.00	–
Proposed MMS	8(150), 12(450), 18(450), 21(600), 24(150), 25(450), 28(300)	0.9506	159.04	26,719.44	728.55	27,447.99	2.90
Developed MS	4(450), 9(600), 17(450), 20(300), 23(900)	0.9501	160.09	26,894.99	629.40	27,524.39	16.83
Developed GA	6(300), 12(600), 15(300), 18(750), 20(450), 24(450)	0.9500	160.53	26,968.75	776.70	27,745.45	19.65
Developed AS	5(900), 20(150), 22(150), 24(900), 29(150), 31(150), 33(300)	0.9504	160.41	26,949.36	734.40	27,683.76	106.31
[35]	4(300), 10(600), 14(100), 18(500), 22(300), 27(,1000)	0.9515	161.78	27,178.49	788.40	27,966.89	N/A
[22]	8(25), 18(2150), 25(875)	0.9494	163.47	27,462.96	1551.03	29,013.99	2.99
[36]	19(1,200), 20(200), 22(639)	0.9492	161.07	27,059.76	1424.24	28,484.00	11.00
[37]	4(250), 11(750), 17(300), 26(1400)	0.9470	168.47	28,302.96	1689.30	33,182.00	N/A
[38]	19(683), 20(145), 21(144), 22(143), 23(143), 24(143), 25(228)	0.9491	168.95	28,383.60	687.00	29,070.60	N/A
[39]	19(781), 20(479), 22(803)	0.9496	168.80	28,358.40	1577.60	29,936.00	N/A

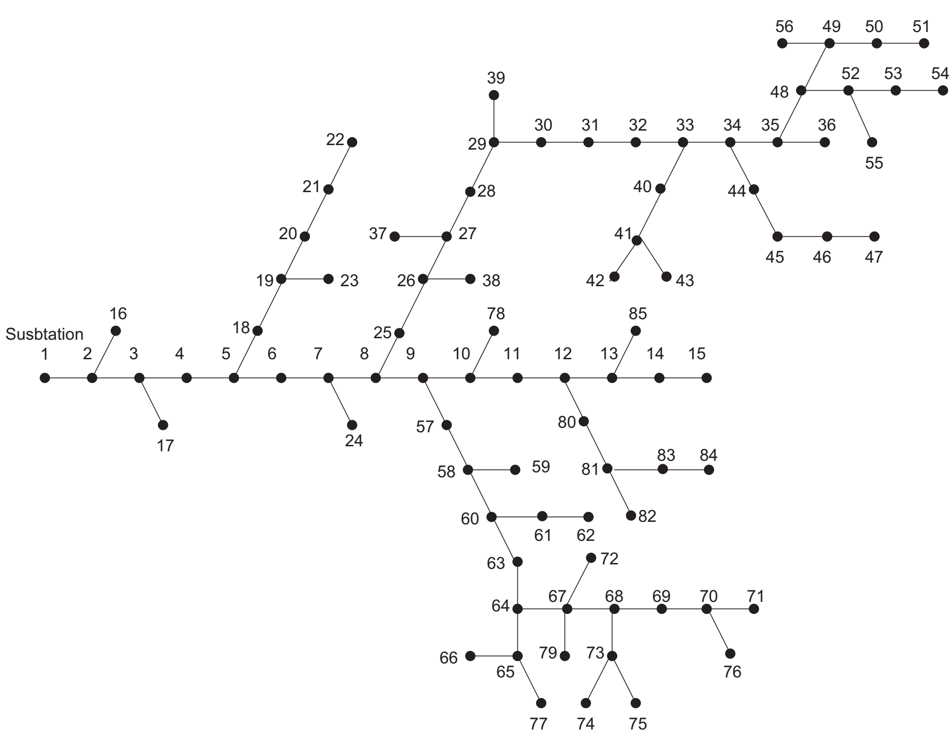

Fig. 7. 85-Busbar system.

study involves three load levels: light (0.5 pu during 1000 h), medium (1.0 pu during 6760 h) and heavy load (2.45 pu during 1000 h), totaling one year of operation. The unitary cost of energy loss is given by $ce_u = 0.06$ $/kW h for the light and medium loads and $ce_u = 0.108$ $/kW h for the heavy load. The capacitors size is 200 kVAr, the maximum number of capacitor banks per busbar is three and their cost is given by 4 $/kVAr [5].

In this case, the sensitivity index (*BS*) proposed in [5] for selecting the candidate busbars for capacitor allocation was used for comparison purpose, which is defined for each busbar *i* as:

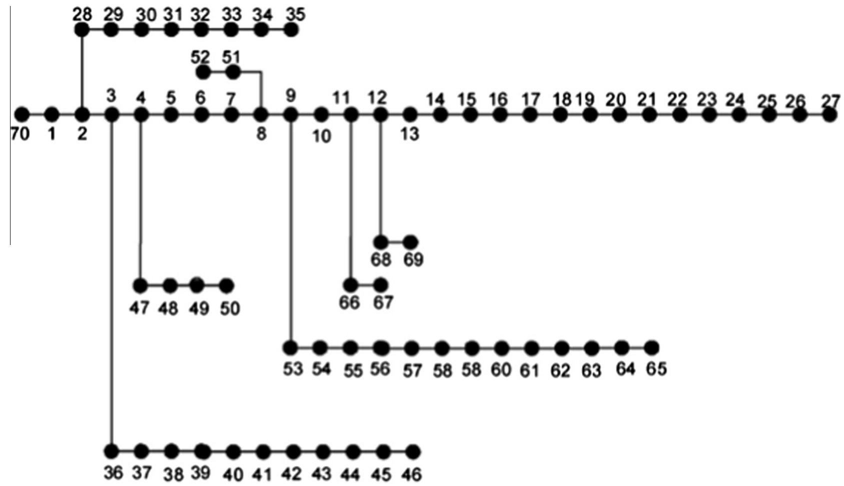

Fig. 8. 69-Busbar system.

Table 11
Case 3: comparison of the proposed method results with previous publications.

Methodology	Optimal locations and size (kVAr)	Minimum voltage (pu)	Peak loss (kW)	Peak loss cost ($)	Capacitor costs ($)	Total cost ($)	CPU time (s)
Without capacitor	–	0.8709	315.71	53,039.98	0.00	53,039.98	N/A
Proposed MMS	10–17–28–32–45–68–73–84(150), 14–20–26–49(300) 60(450)	0.9263	145.10	24,377.04	2133.85	26,510.89	12.70
Developed MS	41–60–69(150), 23–77–81(300), 36(450), 4(600), 26(750)	0.9256	146.93	24,684.54	1992.85	26,677.39	46.04
Developed GA	6–17–18(150), 3–49–84(300), 9(450), 30–67(600)	0.9250	147.64	24,803.55	1917.85	26,721.40	86.21
Developed SA	5–24–30–35–47–50–58–61–62–70–72–75–78–81(150)	0.9149	149.24	25,072.96	2050.00	27,122.96	295.15
[35]	7(300), 8(700), 29(900), 58(500)	0.9174	159.87	26,858.16	788.84	27,647.00	N/A
[22]	9(1200), 26(1200)	0.9155	172.06	28,905.37	1408.00	30,313.37	5.94
[36]	7(200), 8(1200), 58(908)	0.9088	161.40	27,115.20	1469.80	28,585.00	N/A
[39]	7(324), 8(796), 27(901), 58(453)	0.9154	163.32	27,437.76	1613.24	29,051.00	N/A

Table 12
Case 4a: comparison of the proposed method results with previous publications.

Methodology	Optimal locations and size (kVAr)	Minimum voltage (pu)	Total loss (kW h)	Loss cost ($)	Total loss cost ($)	Capacitor cost ($)	Total cost ($)	CPU time (s)
Without capacitor	–	0.9567 0.9092 0.7268	51,599.64 1,521,018.69 1,970,390.18	3095.98 91,261.12 118,223.41	212,580.51	0.00	212,580.51	N/A
Proposed MMS	12(200), 21(200), 61(600), 62(600), 64(200)	0.9789 0.9341 0.7696	58,109.37 988,941.09 1,374,396.67	3486.56 59,336.47 82,463.80	145,286.83	7200.00	152,486.83	6.54
Developed MS	12(200), 24(200), 61(600), 62(600), 64(200)	0.9788 0.9340 0.7695	58,218.42 989,543.68 1,374,400.17	3493.11 59,372.62 82,464.01	145,329.74	7200.00	152,529.74	38.55
Developed GA	14(200), 21(200), 59(200), 61(400), 62(400), 64(200), 65(200)	0.9790 0.9345 0.7701	57,988.37 991,194.80 1,380,040.97	3479.30 59,471.69 82,802.46	145,753.45	7200.00	152,953.45	55.52
Developed SA	12(200), 18(200), 61(600), 62(600), 64(200)	0.9788 0.9340 0.7695	58,022.66 989,365.92 1,375,115.97	3481.36 59,361.96 82,506.96	145,350.27	7200.00	152,550.27	250.92
[5]	12(200), 21(200), 59(200), 61(600), 62(200), 64(400)	0.9788 0.9340 0.7692	56,627.84 986,299.31 1,382,101.95	3397.67 59,177.96 82,926.12	145,501.75	7200.00	152,701.75	42.00
[22]	19(225), 62(900), 63(225)	0.9742 0.9289 0.7613	44,309.02 1,006,784.41 1,469,030.80	2658.54 60,407.06 88,141.85	151,207.45	5400.00	156,607.45	N/A
[40]	61(1123), 64(207)	0.9765 0.9314 0.7656	51,832.97 1,025,535.42 1,435,245.53	3109.98 61,532.13 86,114.73	150,756.84	5320.00	156,076.84	N/A

$$BS_i = \lambda Q_i \frac{QI_i}{V_i} \quad (5)$$

From this index application, the list of candidate busbars is given by:

Table 13
Case 4b: results from the proposed and developed methods.

Methodology	Optimal locations and size (kVAr)	Minimum voltage (pu)	Total loss (kW h)	Loss cost ($)	Total loss cost ($)	Capacitor cost ($)	Total cost ($)	CPU time (s)
Without capacitor	–	0.9567 0.9092 0.8781	51,599.64 1,521,018.69 403,289.70	3095.98 91,261.12 24,197.38	118,554.48	0.00	118,554.48	N/A
Proposed MMS	12(200), 21(200), 59(200), 61(600), 62(200), 64(200)	0.9756 0.9304 0.9010	46,543.31 984,477.81 268,662.11	2792.60 59,068.67 16,119.73	77,980.99	6400.00	84,380.99	7.20
Developed MS	21(200), 61(600), 62(400), 64(200), 66(200)	0.9761 0.9309 0.9016	47,807.16 986,005.68 267,995.70	2868.43 59,160.34 16,079.74	78,108.51	6400.00	84,508.51	45.34
Developed GA	10(200), 21(200), 59(400), 61(400), 62(200), 64(200),	0.9751 0.9298 0.9004	45,490.45 990,671.74 271,103.97	2729.43 59,440.30 16,266.24	78,435.97	6400.00	84,835.97	60.24
Developed SA	9(200), 16(200), 18(200), 54(200), 61(400), 62(400), 64(200)	0.9748 0.9295 0.9000	45,578.52 993,011.40 271,825.33	2734.71 59,580.68 16,309.52	78,624.92	7200.00	85,824.92	267.50

Table 14
Case 5a: comparison of the proposed method results with previous publications.

Methodology	Optimal locations and size (kVAr)	Minimum voltage (pu)	Total losses (kW h)	Loss cost ($)	Total loss cost ($)	Capacitor cost ($)	Total cost ($)	CPU time (s)
Without capacitor	–	0.9738 0.9460 0.8536	48,540.00 1,370,454.80 1,417,020.00	2912.39 82,227.39 153,038.15	238,178.18	0.00	238,178.18	–
Proposed MMS	33–41–43–56–62–64–70–100–125–145–160(200), 161(400)	0.9942 0.9698 0.8813	50,590.00 1,175,293.60 1,242,760.00	3035.55 70,515.79 134,217.81	207,769.15	10,400.00	218,169.15	537.70
Developed MS	15–20–24–41–43–62–66–135–156–161(200), 82–145(400)	0.9938 0.9695 0.8809	51,220.00 1,174,076.80 1,239,190.00	3073.33 70,445.18 133,832.55	207,351.06	11,200.00	218,551.06	821.80
Developed GA	13–16–38–43–49–54–70–82–109–112–119–129(200), 116(400)	0.9941 0.9697 0.8812	51,490.00 1,176,037.20 1,239,400.00	3089.33 70,561.04 133,854.99	207,505.36	11,200.00	218,705.36	5921.60
Developed SA	11–23–24–49–55–59–69–75–101–109–128–133–141–156(200)	0.9940 0.9696 0.8810	51,310.00 1,175,969.60 1,240,240.00	3078.47 70,559.67 133,945.53	207,583.68	11,200.00	218,783.68	10,099.80
[5]	16–20(200), 31–59–66–70(400), 160(600)	0.9930 0.9664 0.8780	47,540.00 1,175,766.80 1,257,940.00	2852.37 70,547.47 135,857.61	209,257.46	10,400.00	219,657.46	1920.00

Table 15
Case 5b: results from the proposed and developed methods.

Methodology	Optimal locations and size (kVAr)	Minimum voltage (pu)	Total losses (kW h)	Loss cost ($)	Total loss cost ($)	Capacitor cost ($)	Total cost ($)	CPU time (s)
Proposed MMS	66–70–82–102(200), 20–109(400), 59–118–161–214(600)	0.9941 0.9855 0.9005	82,384.43 1,282,111.87 1,184,380.39	4943.07 76,926.71 127,913.08	209,782.86	16,000.00	225,782.86	679.56
Developed MS	11–33–40–62–77–81–82–100–105–107–132–133–134–156–159–229(200), 109–160(400)	0.9941 0.9852 0.9001	85,211.45 1,303,709.72 1,188,996.22	5112.69 78,222.58 128,411.59	211,746.86	16,000.00	227,746.86	1240.27
Developed GA	13–26–49–70–82–100–102–108–112–118–123–127–136–145–161–256(200), 150–159(400)	0.9941 0.9847 0.9000	83,939.02 1,306,419.98 1,196,984.63	5036.34 78,385.20 129,274.34	212,695.88	15,200.00	227,895.88	6787.12
Developed SA	22–36–38–75–87–115–125–134–135–160–229(200), 27–156–159–256(400)	0.9941 0.9854 0.9004	86,578.50 1,315,263.28 1,192,433.23	5194.71 78,915.80 128,782.79	212,893.30	16,000.00	228,893.30	12,047.33

[9 10 11 13 14 15 16 19 20 22 23 24 25 26 27 29 31 33 35 36 37 38 40 41 42 43 44 46 48 49 52 54 55 56 59 60 62 63 64 66 69 70 72 73 74 75 77 79 81 82 83 87 90 93 96 97 100 101 102 105 107 108 109 111 112 113 114 115 116 118 119 122 123 125 127 128 129 132 133 134 135 136 140 141 145 150 155 156 159 160 161 228 229 230 256 267 268 289]

The same analyses carried out in Case 4 were done in Case 5: *Case* 5*a* – without the constraints of nodal voltage limits for comparison purposes, and *Case* 5*b* – with the lower voltage limit constraints (0.9 pu). In both cases, the same aforementioned load levels from [5] were considered and the objective function is given as in Eq. (1). The results for *Cases* 5*a* and 5*b* are presented in Tables 14 and 15.

It can be observed that the total cost increases from *Case* 5*a* to *Case* 5*b*, because *Case* 5*b* is more constrained and practical due to the voltage limits and then it requires more capacitors support. Even so, *Case* 5*b* leads to a reduction of 5.20% in relation to the base case (without capacitor allocation). It can be notice that the solutions of the proposed approach are highlighted in Tables 10–15.

Conclusions

This paper presented an algorithm based on a Modified Monkey Search optimization technique for capacitor allocation in distribution systems to minimize the total cost involving the system losses and investment in capacitors. Therefore, modifications were proposed to the Monkey Search algorithm of the literature consisted of technical improvements to adequately represent the characteristics and constraints of the capacitor allocation problem. These modifications allowed the suitable application of the Monkey Search based approach to the practical problem of capacitor allocation in electrical distribution networks, which reinforce the research lines that investigate the application of combinatorial optimization methods to practical engineering problems. Important constraints, such as different system load levels, are considered by the proposed methodology. From the results obtained, the proposed algorithm seemed to be robust and computationally efficient for the systems tested, thereby combining quality and reduced processing times.

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