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Tuning of a PID controller using improved chaotic Krill Herd algorithm

Q1 Saber Yaghoobi, Hamed Mojallali*

Electrical Engineering Department, Faculty of Engineering, University of Guilan, Rasht, Iran

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ABSTRACT

Purpose of this paper is to design coefficients of a PID controller to achieve a controller to optimize system's behavior. Krill Herd algorithm is a new evolutionary algorithm in swarm intelligence field. In this paper we've tried to determine optimized controller coefficients using combination of Chaos theory and Improved Krill Herd algorithm (ICKH). In this work, the goal will be minimizing the weighted sum of maximum overshoot, reverse distance of nearest pole from imaginary axis and settling time of a closed-loop system. Indeed the cost function has been formulated as a single-objective problem. Later some illustrative examples will be presented. Simulation results compare proposed algorithm (ICKH) with other algorithms e.g. GA and PSO etc.

1. Introduction

Mankind has always been looking for best possible solution of problems in environment around himself. According to the situation these answers could be whether smallest or biggest answer possible. Nowadays in engineering problems, achieving the answer with the lowest possible price, is one of the designers' disturbances. Former methods to solve optimization problems require enormous computational efforts, which tend to fail as the problem size increases. This is the motivation for employing bio-inspired stochastic optimization algorithms as computationally efficient alternatives to deterministic approach [1]. Observation in nature for inspiration was one the old interests of scientists to achieve optimization methods. Therefore many optimization algorithms such as ACO [2], PSO [3], GA [4] etc. have been designed with nature-inspired and animals' behavior.

Recently a new evolutionary algorithm had been proposed by Alavi and Gandomi, inspired by herding behavior of Antarctic krill [5]. Antarctic krill is one of the best-studied species of marine animal. The Krill Herds are aggregations with no parallel orientation existing on time scales of hours to days and space scales of 10 s to 100 s of meters. One of the main characteristics of this specie is its ability to form large swarms [6,7]. Conceptual models have been proposed to explain the observed formation of the Krill Herds [8].

The time-dependent position of an individual krill in 2D surface is governed by the following three main actions [9]:

(i) Movement induced by other krill individuals;
(ii) Foraging activity; and
(iii) Random diffusion

Krill Herd algorithm nevertheless its powerful skill in solving optimization enigmas it has the problem of trapping in local optima. Confronting this problem using Chaos theory and its mappings would be suggested in Krill Herd algorithm. Chaos can be described as a bounded nonlinear system with deterministic dynamic behavior that has stochastic properties [10]. In what is called the "butterfly effect", small variations of an initial variable will result in huge differences in the solutions after some iteration. Mathematically, chaos is random and unpredictable, yet it also possesses an element of regularity [11]. Chaotic Krill Herd algorithm using logistic mapping is suggested to solve the problem of trapping in local optima. Later, proposed algorithm will be used to determine coefficients of a PID controller and then simulation results will be compared with other optimization algorithms.

Proportional–Integral–Derivative controller or in extenuating words; PID are the most widely used and the most popular controllers in industrial means in order of ease of design and low cost [12,13]. But the problem is there is no (neither) precise and (nor) optimized method for tuning coefficients of a PID controller. In various books and papers evolutionary algorithms such as PSO [14], DE [15], GA [16] etc. has been used to appoint PID's coefficients.

Surveying the operation of a closed-loop system parameters which generally would be considered are maximum overshoot, settling time, steady state error, and rise time. In this paper

* Corresponding author. Postal address: Electrical Engineering Department, Faculty of Engineering, University of Guilan, PO Box: 3756-41635, Rasht, Iran. Tel.: +98 13 33690270; fax: +98 13 33690271.
E-mail addresses: saber.yaghoobi@gmail.com (S. Yaghoobi), mojallali@guilan.ac.ir (H. Mojallali).

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maximum overshoot, distance of nearest pole from imaginary axis, and settling time would be examined as indicators of system's performance. Desirable systems have short settling time, distance of their nearest pole from imaginary axis is approximately long and their maximum overshoot is about zero. Thus the goal is to minimize the weighted sum of maximum overshoot, settling time and distance of nearest pole from imaginary axis.

2. Krill Herd algorithm

Krill Herd (KH) algorithm is a new bio-inspired swarm intelligence, that is based on herding conduct of krill and formulating their swarm movement. The herding of the krill individuals is a multi-objective process including two main goals: (I) increasing krill density, and (II) reaching food. In the present study, this process is taken into account to propose a new metaheuristic algorithm for solving global optimization problems. Density-dependent attraction of krill (increasing density) and finding food (areas of high food concentration) are used as objectives which can finally lead the krill to herd around the global minima. In this process, an individual krill moves toward the best solution when it searches for the highest density and food [5].

The position of a krill in a 2D surface is determines by the following three actions [9–1]:

(i) Movement induced by other krill individuals;
(ii) Foraging activity; and
(iii) Random diffusion

Therefore, the following Lagrangian model is generalized to an n dimensional decision space:

$$\frac{\mathrm{d}X_i}{\mathrm{d}t} = N_i + F_i + D_i$$

where N_i is the movement induced by other krill, F_i the is foraging movement and D_i is the physical diffusion of ith krill.

Direction of N_i which is called α_i, is affected by local density and position of the best krill. N_i can be – with following formula:

$$N_i^{\text{new}} = N^{\max}\alpha_i + \omega_n N_i^{\text{old}}$$

where

$$\alpha_i = \alpha_i^{\text{local}} + \alpha_i^{\text{target}}$$

and $N^{\max}$ is the maximum induced speed, ω_n is the inertia weight of the motion induced in the range [0,1], N_i^{old} is the last motion induced, α_i^{local} is the local effect provided by the neighbors and α_i^{target} is the target direction effect provided by the best krill individual. According to the measurements the maximum induced speed [9], it is taken 0.01 (ms^{-1}).

In KH algorithm the effect of neighbors (${\alpha_i}^{\text{local}}$) is formulated as follows:

$$\alpha_i^{\text{local}} = \sum_{j=1}^{\text{NN}} \hat{K}_{i,j}\hat{X}_{i,j}$$

$$\hat{X}_{i,j} = \frac{X_i - X_j}{||X_i - X_j|| + \varepsilon}$$

$$\hat{K}_{i,j} = \frac{K_j - K_j}{K_{\text{worst}} - K_{\text{best}}}$$

where K_{best} and K_{worst} are the best and the worst fitness values of the krill individuals so far; K_i represents the fitness or the objective function value of the ith krill individual; K_j is th fitness of jth (j = 1,2,. . .,NN) neighbor; X represents the related positions; and NN is the number of the neighbors.

For choosing neighbors at the first hand the sensing distance of each krill must be calculated with following formula:

$$d_{s,i} = \frac{1}{5N}\sum_{j=1}^{N} ||X_i - X_j||$$

If the distance of two krill individuals is less than the defined sensing distance, then they would be neighbors [5].

The best krill effects on others by ${\alpha_i}^{\text{target}}$ which is given by:

$$\alpha_i^{\text{target}} = C^{\text{best}}\hat{K}_{i,\text{best}}\hat{X}_{i,\text{best}}$$

and

$$C^{\text{best}} = 2\left(\text{rand} + \frac{I}{I_{\max}}\right)$$

The foraging motion (F_i) is estimated by the two main components. One is the food location and the other would be the prior knowledge about the food location. For the ith krill individual, this motion can be approximately formulated as follows: [17]

$$F_i = V_f\beta_i + \omega_f F_i^{\text{old}}$$

where

$$\beta_i = \beta_i^{\text{food}} + \beta_i^{\text{best}}$$

V_f is the foraging speed, ω_f is the inertia weight of the foraging motion and it's a number in range [0,1], ${F_i}^{\text{old}}$ is the last foraging motion. In this paper we set V_f to 0.02 [18].

The physical diffusion of the krill individuals is considered to be a random process. It can be formulated as follows:

$$D_i = D^{\max}\delta$$

where $D^{\max}$ is the maximum diffusion speed, and δ is the random directional vector, and its arrays are random values in range of [−1,1]. The better the position of the krill is, the less random the motion is. Furthermore, another term would be added to the physical diffusion formula to consider this effect. This term linearly decreases the random speed with the time (iterations):

$$D_i = D^{\max}\left(1 - \frac{I}{I_{\max}}\right)\delta$$

According to three main actions mentioned above, velocity of each krill can be calculated. The new position of each krill from t to $t + \Delta t$ is formulated as below:

$$X_i(t + \Delta t) = X_i(t) + \Delta t\frac{\mathrm{d}X_i}{\mathrm{d}t}$$

Δt is a very important parameter which determines the effect of velocity on the new position of krill. This parameter is extremely affected by search space, so that it can be as the following equation:

$$\Delta t = C_t\sum_{j=1}^{\text{NV}}\left(\text{UB}_j - \text{LB}_j\right)$$

where NV is the number variables and LB_j and UB_j are lower and upper limits of the jth variable. C_t's variability would be about [0,2]. It is obvious small values of C_t results precisely search in the search space.

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3. Improved chaotic Krill Herd optimization (ICKH)

3.1. Applying limits for velocity and position

To control and improve the Krill Herd algorithm, their velocity and range of movement could be limited. In order to apply these limits, velocity of each krill could be limited as the following equation:

$$-\left(\frac{dX_i}{dt}\right)_{\max} < \left|\frac{dX_i}{dt}\right| < \left(\frac{dX_i}{dt}\right)_{\max}$$

$$\left(\frac{dX_i}{dt}\right)_{\max} = \alpha\left(UB_j - LB_j\right)$$

$$\frac{dX_i}{dt} = \min\left\{\max\left(\frac{dX_i}{dt}, -\left(\frac{dX_i}{dt}\right)_{\max}\right), \left(\frac{dX_i}{dt}\right)_{\max}\right\}$$

where α's value has been empirically estimated about 0.1.

Sometimes according to the calculated velocity, new position of the krill could be somewhere out of the search space. To avoid exiting krill form the search space their position could be limited with equation below:

$$X_i = \min\left\{\max\left(X_i, LB_j\right), UB_j\right\}$$

3.2. Chaotic Krill Herd

Chaos theory studies the behavior of systems that follow deterministic laws but appear random and unpredictable or we can say a dynamical system that has a sensitive dependence on its initial conditions; small changes in those conditions can lead to quite different outcomes [19]. The chaotic system can be described by a phenomenon, in which a small change in the initial condition will lead to nonlinear change in future behavior, besides the system exhibits distinct behaviors under different phases, e.g. stable fixed points, periodic oscillations, bifurcations, and periodicity [20]. Due to these characteristics, chaos theory can be applied in optimization. In KH algorithm D_i and N_i parameters which contain random numbers can be modified with chaos mappings. Our suggestion would be using the successions made by the logistic mapping instead of above-mentioned random numbers. Sequences generated by the logistic mapping are formulated as below:

$$x(t+1) = 4 \times x(t) \times (1 - x(t))$$

In Eq. (?) $x(0)$ is created randomly between 0 and 1 for every iteration. Notice that $x(0)$ should not be 0, 0.25, 0.5, 0.75 or 1. Fig. 1 shows the chaotic $x(t)$ value using a logistic map for 50 iterations where $x(0) = 0.2$. So flowchart of ICKH is like Fig. 3.

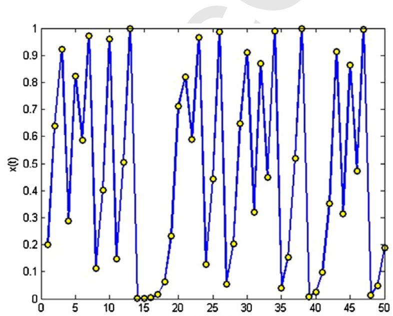

Fig. 1. Chaotic $x(t)$ using logistic mapping.

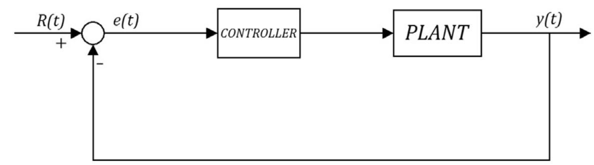

Fig. 2. Block diagram of a simple controlled feedback system.

4. PID controller

PID is an acronym for *Proportional–Integral–Derivative*, referring to the three terms operating on the error signal to produce a control signal. PID control is one of the earlier control strategies [21]. Its early implementation was in pneumatic devices, followed by vacuum and solid state analog electronics, before arriving at today's digital implementation of microprocessors. Since many process plants controlled by PID controllers have similar dynamics, it has been found possible to set satisfactory controller parameters from less plant information than a complete mathematical model [21]. Consider the following unity feedback system:

In PID controlling strategy the signal (u) which just past the controller is equal to the proportional gain (K_p) times the magnitude of the error plus the integral gain (K_i) times the integral of

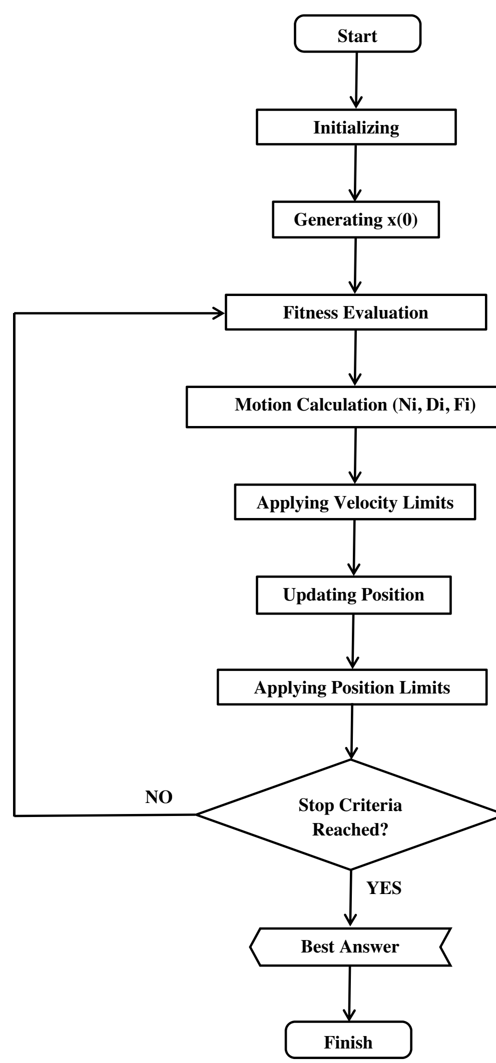

Fig. 3. Flowchart of Improved Chaotic Krill Herd algorithm.

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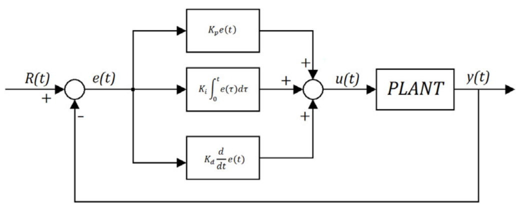

Fig. 4. A system controlled with regular PID controller.

Table 1
Effect of each PID coefficient on system's performance indicators.

	Rise time	Overshoot	Settling time
K_p	Decrease	Increase	Small change
K_i	Decrease	Increase	Increase
K_d	Small change	Decrease	Decrease

the error plus the derivative gain (K_d) times the derivative of the error.

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\mathrm{d}\tau + K_d \frac{\mathrm{d}}{\mathrm{d}t} e(t)$$

So Fig. 2 can be edited like Fig. 4.

The effect of each part (proportional, integral and derivative) in step respond of the system is shown in Table 1.

To optimize system's behavior the cost function which is going to be minimized is given by following equation:

$$f = T_s + 2\mathrm{MP} + \mathrm{SI}$$

where T_s would be settling time of the step response, MP would be maximum overshoot, and SI is stability index, which is defined as:

$$\mathrm{SI} = \frac{-1}{\min(\max(\mathrm{real}(\mathrm{poles}(G))), 0)}$$

and G is the closed-loop system's transfer function. In order to this equation the big real pole part causes small SI.

5. Illustrative examples

In this section two examples are given to illustrate the proposed algorithm. Later we're going to compare this algorithm with other algorithms such as KH, GA, PSO, CPSO and Ziegler–Nichols tuning method [22]. The lower and upper bounds of variables (coefficients) are: K_p: [0,100], K_i: [0,100], K_d: [0,10].

Example 1. Consider the following third order system (plant of a DC motor):

$$G(s) = \frac{1}{s^3 + 9s^2 + 23s + 15}$$

Utilizing the optimization method presented in this paper, the three parameters of PID controller are tuned as follows:

$$K_p = 39.709, \quad K_i = 27.03, \quad K_d = 10$$

With population of 20 and 30 iterations the step response indicators of PIDs designed by each algorithm are shown in Table 2.

Fig. 5 shows the step response of the plant, controlled with designed PIDs by each algorithm and uncontrolled system.

Table 2
Performance indicators of each algorithm with step response for the system of Example 2.

	MP (%)	Rise time (s)	Settling time (s)	Peak time (s)	SI	Cost function
ICKH	0.15	0.68	1.01	1.39	0.84	2.15
IKH	0.06	0.9	1.32	2.02	1	2.44
GA	0	0.95	1.39	2.1	1.05	3.49
PSO	0	1.14	1.67	11.59	1.52	2.66
ZN	66.2	0.26	5.1	0.75	1.86	139.36
Uncontrolled system	0	2.24	4.12	7.34	1.62	5.74

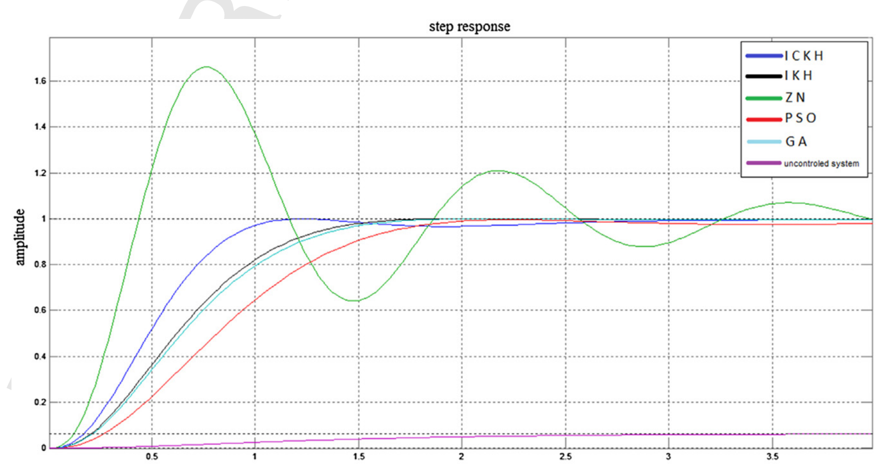

Fig. 5. Step response of the system 1.

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Table 3
Performance indicators of each algorithm with step response for the system of Example 2.

	MP (%)	Rise time (s)	Settling time (s)	Peak time (s)	SI	Cost function
ICKH	$8 \times 10^{-4} \approx 0$	0.15	0.21	0.28	0.36	0.57
IKH	0.01	0.206	0.28	0.39	0.55	0.85
GA	0	0.24	0.34	1.19	0.47	0.81
PSO	0	0.15	0.53	0.28	0.53	1.06
ZN	62	0.06	2.02	0.15	0.72	126.74
Uncontrolled system	0	1.01	1.84	3.34	0.48	2.32

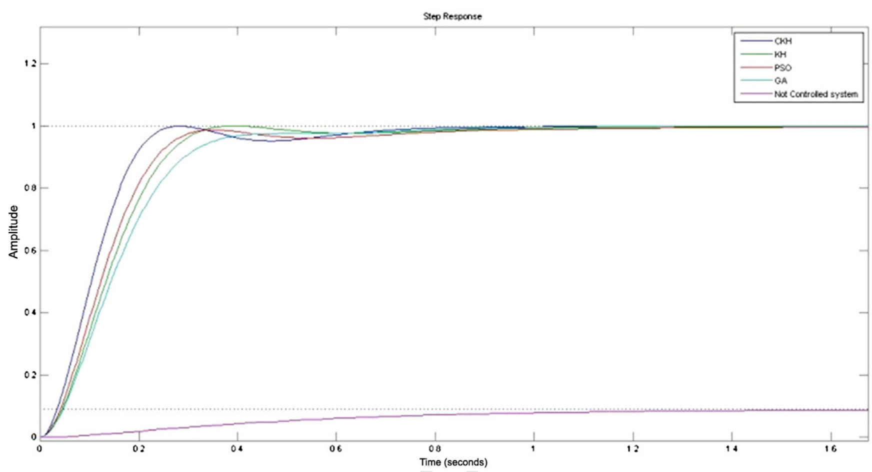

Fig. 6. Step response of the system 2.

Example 2. As the second experiment, consider a second order system with transfer function as below (this is a DC motor plant too):

$$G(s) = \frac{0.01}{(0.01s + 0.1)(0.5s + 1)(0.01)^2}$$

Using the ICKH optimization algorithm, the three parameters of PID controller are calculated as follows:

$$K_p = 47.48, \quad K_i = 81.86, \quad K_d = 4.06$$

and Table 3 shows the controlled system's indicators in comparison.

Fig. 6 shows the step response of the uncontrolled system and system with PIDs designed by various algorithms.

6. Conclusions and future work

A recently developed bio-inspired meta-heuristic algorithm called Krill Herd has been developed with Chaos theory and some other improvements and became Improved Chaotic Krill Herd (ICKH). Proposed algorithm (ICKH) applied for the design of PID controllers, harvesting better performance in indicators such as maximum overshoot, rise time, settling time, peak time, and stability index than other algorithms.

Our future purpose is to delineate parameters of a Fractional Order PID with presented algorithm and demonstrate that the controller designed with this algorithm has less cost and more benefits against other algorithms.

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