A Novel Approach for Image Denoising Based on Evolutionary Game Theory

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***Abstract*—In this paper, we study the image denoising from the game theoretic perspective and formulate the image denoising problem as an evolutionary game. In the latter, pixels in image are modeled as autonomous players that seek to maximize a payoff function using one of a set of different *strategies*. The set of strategies of the players are the neighbors. By regarding the non-negative weights of the neighboring pixels as the probabilities of selecting the strategies, the problem of estimating the value of pixels becomes finding the evolutionarily stable strategies for the evolutionary game. Experimental results show that the filtering performance of the proposed approach is very satisfactory and can achieve better performance than the median filter in terms of PSNR, MSSIM and visual quality.**

***Keywords*—Image denoising, Salt and pepper noise, Game theory, Evolutionary game theory, Mixed strategies.**

I. INTRODUCTION

The sensors are generally contaminating noise on image. The main sources of noise are imperfect instruments, problems with the data acquisition process, interfering natural phenomena, transmission and compression. These can all degrade the data of interest [1]. The noise reduction in digital image is an important step on image processing; it serves to enhance the quality of image before any type of processing. The added noise will be of various kinds like additive random noise (Gaussian noise), impulse noise (salt and pepper noise) and multiplicative noise (speckle noise).

Depending on the type of the noise, the degradation of the image will vary. The traditional methods of noise removal include mean filter, Wiener filter, median filter, in addition to other different transforms like wavelet transform, curvelet transform. The majority of these denoising algorithms till now are based on the type of noise which is introduced. Median filter performs better with salt and pepper noise [2], [3] and [4]. Wavelet Transform performs better in removing Gaussian noise and Speckle noise than other filters because of its unique signal analysis technique [5]. Recently [6] and [7] proposed a coalition game model for denoising image with Gaussian noise.

In this work we propose an image denoising method based on evolutionary game theory for filtering images noised by salt and pepper noise. Thus, we will have to explore the idea of emergent optimization, where denoising image emerges as a result of local autonomous decisions taken by the players. Pixels in image are modeled as autonomous players that seek to maximize a payoff function using one of a set of different *strategies*. The set of strategies of the players are the neighbors.

The performance of the proposed filter is compared with Median filter based on Peak Signal to Noise Ratio (PSNR) and Mean Structural Similarity (MSSIM). The experimental results demonstrate that the proposed filter can remove noise automatically and effectively.

The paper is organized as follows: We give an introduction about both of evolutionary game theory and evolutionary game model in section 2 and in section 3 respectively. Then in section 4, we describe in details how to formulate the image denoising problem as an evolutionary game and how to define the payoff function. Experimental results are shown in section 5. Finally, conclusions and future work are drawn in section 6.

II. EVOLUTIONARY GAME THEORY

The expression "Game Theory" was used by John von Neumann and Oskar Morgenstern in 1944 in their book Theory of Games and Economic Behavior [8]. It provides useful mathematical tools to understand the possible strategies that individuals may follow when competing or collaborating in games. Since then, the applications of Game Theory are found in political science, economics, international relations, biology, philosophy, engineering, and computer science.

A particular branch of game theory, developed by some of biologists from the 1970s, is bringing out from the initial theory: this is the *evolutionary game theory* (sometimes referred to as *evolutionary games*). The evolutionary game theory has the advantage of avoiding the major problem of the classical game theory: the characterization of rational behavior and the need to anticipate the actions of other players. In fact, in the beginning of the classical game theory is proposed to reflect on the rational behavior of players in interactions, i.e. aware of the situations and thinking to increase their payoff. Faced of insurmountable difficulties introduced into the game by the notions of belief, information level and subjectivity of the players, the concept of rationality can never be identified. So, in evolutionary games any idea of strategic choice and anticipation, that is means rationality, is abandoned.

III. EVOLUTIONARY GAME MODEL

An evolutionary game is a game that studies the evolution of the interaction dependent strategy in populations. One key concept in evolutionary games is the evolutionarily stable strategy (ESS), which is a strategy with the following property: a population in which all members play this strategy is resistant to invasion by a small group of mutants who play an alternative strategy.

We focus here on games played by a single population (i.e., games in which all agents play equivalent roles). We suppose that there is a unit mass of agents, each of whom chooses a pure strategy from the set $S=\{1,\cdots,n\}$. In many instances, it is natural to allow players to choose mixed (or randomized) strategies. When a player chooses mixed strategy from the simplex $P=\left\{p\in R_+^n : \sum_{i=1}^n p_i=1\right\}$, his behavior is stochastic: he commits to playing pure strategy $i\in S$ with probability p_i.

The aggregate behavior of these agents is described by a *population state* $p\in P$. We identify a *population game* with a continuous vector-valued payoff function $F: P\to R^n$. The scalar $F_i(p)$ represents the payoff to strategy i when the population state is

p [9].

There is no evolutionarily stable strategy if individuals are restricted to pure strategies, although there is an evolutionarily stable strategy if players may use mixed strategies. It is mentioned that in a mixed strategy, a player assigns a probability to each pure strategy, and chooses which strategy to play using a randomization device. For calculate these probabilities we adopt the Wright-Fisher model [10], which is by far the most popular stochastic model for reproduction in population genetics. It is based on the assumption that the probability of an individual adopting a certain strategy is proportional to the expected payoff of the population using that strategy. Therefore, the strategy spreading equation can be written as:

$$p_i^{t+1} = \frac{p_i^t F_i(p)}{\sum_{i=1}^{n} p_i^t F_i(p)} \quad (1)$$

Where the numerator $p_i^t F_i(p)$ is the expected payoff of a player using strategy i, and the denominator $\sum_{i=1}^{n} p_i^t F_i(p)$ is the total expected payoff of a player using different strategies, which is the normalization term that ensures $\sum_{i=1}^{n} p_i^{t+1} = 1$.

IV. IMAGE DENOISING AS AN EVOLUTIONARY GAME

As in many research papers, we assume that the noisy image is directly result from applying noise on the original image. The noise model was computer simulated. The essential problem of image denoising is to restore the original value of pixels from the noisy image. To restore the original image from noisy one, we need to use some prior information such as the correlations among spatial neighboring pixels. One possible approach is to use the spatially adaptive linear filtering approach.

For the pixel located at k position, we find the estimate $\hat{I}(k)$ using the weighted average of the set of spatially neighboring pixels, i.e.

$$\hat{I}(k) = \frac{\sum_{l \in S(k)} w_{k,l} \cdot I(l)}{\sum_{l \in S(k)} w_{k,l}} \quad (2)$$

Where $S(k)$ is the set that contains the spatially neighboring pixels for k, and $w_{k,l}$ is the weight for pixel $I(l)$.

A. Game theoretic formulation

Now, let us formulate the image denoising problem as an evolutionary game. We first match the variables in (2) to the components of an evolutionary game as follows.

- Players: each pixel in the image
- Pure strategies: neighbors of the concerned pixel
- Pure strategy set: set of spatially neighboring pixels
- Mixed strategy: the estimate $\hat{I}$
- Probabilities in the mixed strategy: the non-negative normalized weights w

Based on the above variable matching, the problem of finding the estimate of the pixels becomes the problem of finding a good mixed strategy of the evolutionary game, which is the evolutionarily stable strategy (ESS).

B. Payoff function

After choosing the pure strategy set, we now discuss how to define the payoff function. The payoff function $F_i(p)$ measure the player's payoff of taking strategy i when population state is p. Since the objective of the image denoising problem is to recover the original value of image's pixels, the strategy that can lead to better value should have a higher payoff. With such an intuition, we define the payoff function as follows:

$$F_i(p) = \exp\left(-\frac{(I(k) - I(i))^2}{\sigma^2}\right) \quad (3)$$

Where $I(k)$ is value of target pixel, $I(i)$ is value of neighbor, and σ is a parameter.

C. Parametric description

The performance parameters are most important criteria to justify results through evaluation. The parameters considered here are Peak Signal to Noise Ratio (PSNR) and Mean Structural Similarity (MSSIM).

The objective quality of the reconstructed image is measured by:

$$PSNR = 10\log_{10}\left(\frac{R^2}{MSE}\right) \quad (4)$$

Where R is maximum value of the pixel present in an image and MSE is mean square error between the original and denoised image with size $M \times N$.

$$MSSIM(X,Y) = \frac{1}{M}\sum_{j=1}^{M} SSIM(x_j, y_j) \quad (5)$$

Where X and Y are the reference and the distorted images, respectively; x_j and y_j are the image contents at the j^{th} local window; and M is the number of local windows of the image. And SSIM is defined as:

$$SSIM(x_j, y_j) = \frac{(2\mu_{x_j}\mu_{y_j} + C_1)(2\sigma_{x_j y_j} + C_2)}{(\mu_{x_j}^2 + \mu_{y_j}^2 + C_1)(\sigma_{x_j}^2 + \sigma_{y_j}^2 + C_2)} \quad (6)$$

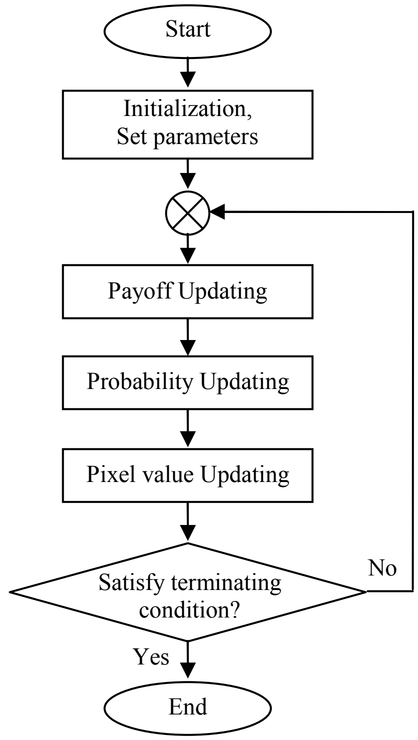
where, μ_{x_j} and μ_{y_j} are the mean intensity of the image contents at the j^{th} local window; σ_{x_j} and σ_{y_j} are the standard deviation of the image contents at the j^{th} local window; $\sigma_{x_j y_j}$ is the correlation coefficient between x_j and y_j; C_1 and C_2 are constants.

D. The proposed method

The proposed method starts with associating a set of strategies to each agent using Get_Neighborhood function. The Evaluation function calculates the payoff of each strategy for each agent. Then, the probability of each strategy is updated using the Update function. Finally, the new value of each agent is calculated. This process is repeated until stability. The algorithm and flowchart of the proposed method are shown on Fig. 1.

```
Algorithm. Pseudocode for the proposed method
// parameters setting
while ! StopCondition() do
  for Agent ←1 to imageSize do
    n_list←Get Neighborhood(Agent.position);
      for Strategy ←1 to n_listeSize do
        Evaluation (strategy.payoff);
        Update (strategy.Prob);
      end for
    Update (Agent.value);
  end for
end while
return denoised image
```

(a)



(b)

Fig. 1. The proposed method: (a) The Pseudocode; (b) The Flowchart.

V. EXPERIMENTAL RESULTS

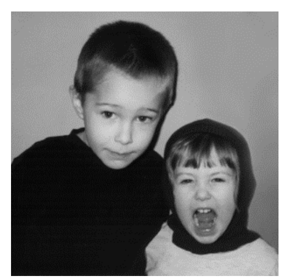
This section presents the simulation results illustrating the performance evaluation of our filtering algorithm. In our experiments we followed the steps of the flow chart depicted in Fig. 1. The proposed filter is tested on different images with different sizes. The salt and pepper noise is added into the images with different noise densities. Our algorithm is compared to the median filter because it performs well with salt and pepper noise [2], [5] and [11].

The performance of our algorithm is evaluated using quantitative performance measures such as Peak Signal to Noise Ratio (PSNR), Mean Structural Similarity (MSSIM) [12], as well as in terms of visual quality of the images. The Figure 2 presents a set of tested images for measuring the performance of our algorithm.

Cameraman

Boat

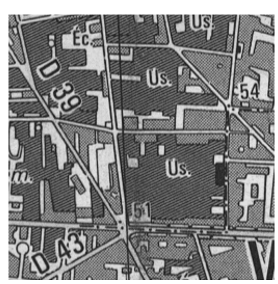
Lena



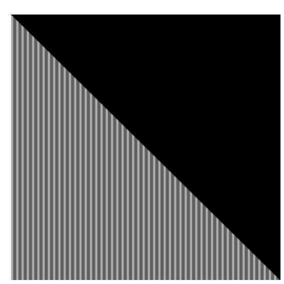
Roar

Eight

Montreuil



Map



Stripes

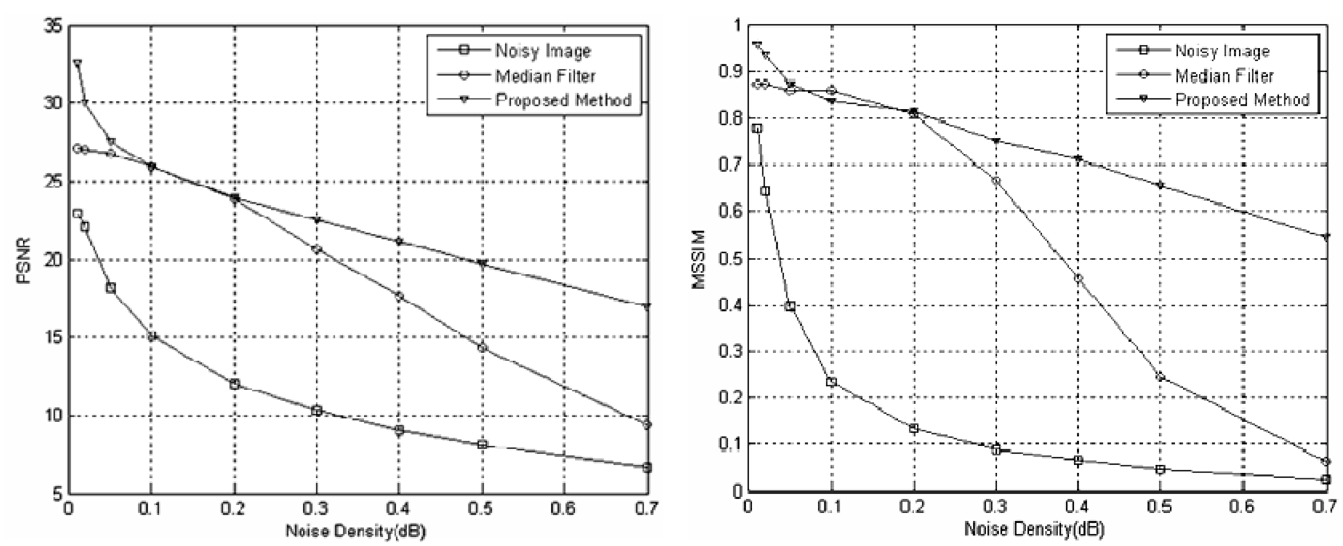
Fig. 2. The set of tested images.

The Table 1 shows PSNR and MSSIM values for the set of tested images, when the density of noise varies from smaller values to larger values. Not all researchers use high value of density to test the performance of the algorithm when the noise is comparable to the signal strength.

From the Table 1, we note that when the density of impulse noise is increasing the PSNR is decreasing and the same for MSSIM. The PSNR and MSSIM are at maximum for the better results. We check from the below data that PSNR and MSSIM are increasing when the impulse noise density is decreasing. Also, this table shows that the proposed method is more efficient for removing salt and pepper noise than median filter. But here also Median filter can sometimes denoise image better than the proposed method. The Figure 3 shows that the proposed method is more stable than median filter when the densities values of noise varying from low values to high values.

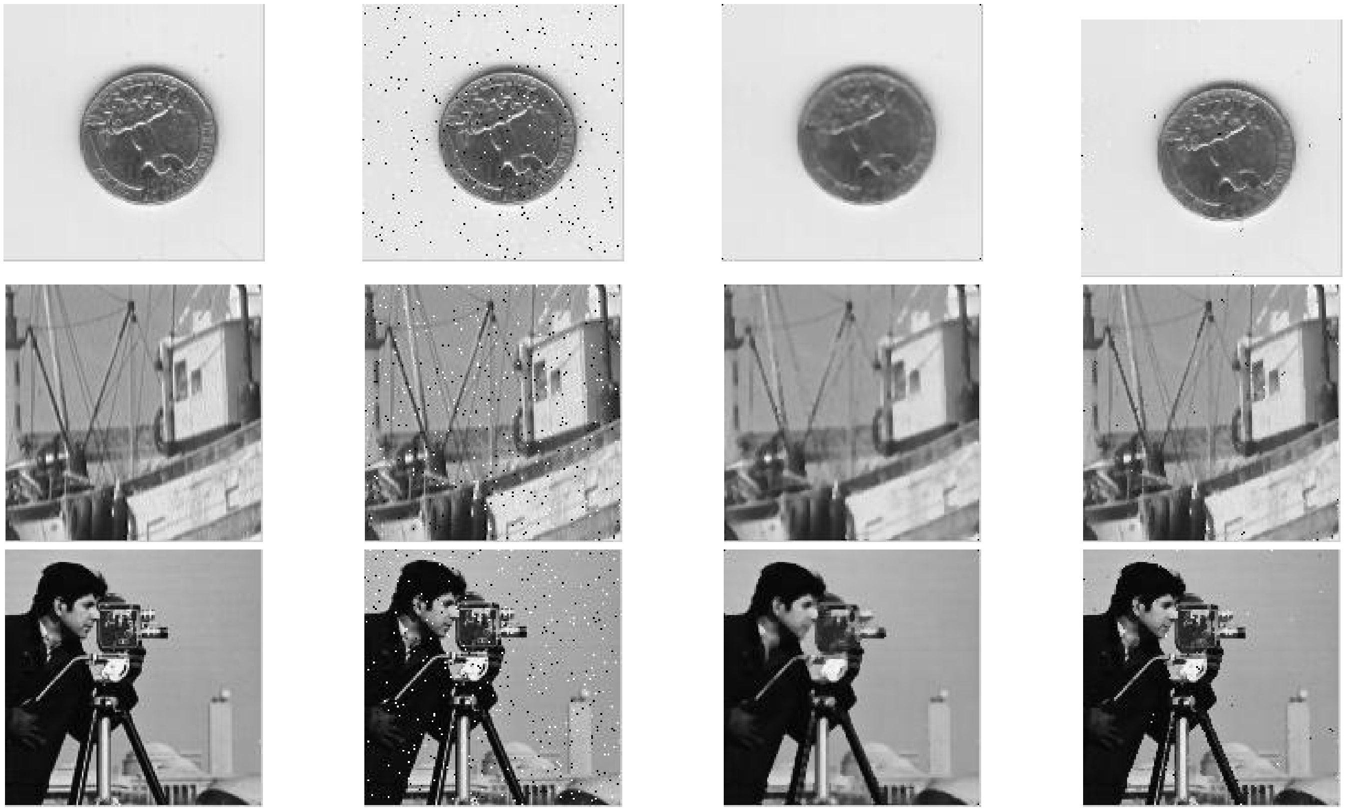
Table 1. Table of performance measures.

Image / **Noise**		0.02		0.05		0.2		0.3		0.4	
		PSNR	MSSIM	PSNR	MSSIM	PSNR	MSSIM	PSNR	MSSIM	PSNR	MSSIM
Cameraman	Initial	22.09	0.643	18.18	0.396	12.01	0.134	10.32	0.090	9.06	0.066
	Median	27.05	0.875	26.76	0.860	23.92	0.808	20.67	0.667	17.65	0.456
	Proposed	**29.98**	**0.936**	**27.53**	**0.873**	**23.96**	**0.813**	**22.51**	**0.752**	**21.16**	**0.711**
Boat	Initial	22.50	0.761	18.50	0.599	12.49	0.315	10.71	0.232	9.50	0.174
	Median	30.76	0.967	30.35	0.964	26.67	0.915	22.73	0.777	18.63	0.542
	Proposed	**32.35**	**0.978**	**30.53**	**0.968**	**27.15**	**0.928**	**25.38**	**0.883**	**21.75**	**0.685**
Lena	Initial	22.35	0.729	18.52	0.545	12.45	0.270	10.69	0.199	9.45	0.150
	Median	35.09	0.985	**34.46**	**0.982**	28.68	0.933	23.51	0.779	18.96	0.529
	Proposed	**35.17**	**0.983**	33.09	0.981	**29.32**	**0.946**	**27.29**	**0.909**	**25.01**	**0.862**
Roar	Initial	21.81	0.603	17.78	0.389	11.78	0.170	10.01	0.122	8.74	0.093
	Median	**44.36**	**0.997**	**41.95**	**0.997**	30.52	0.948	23.76	0.752	18.52	0.444
	Proposed	40.19	0.997	37.17	0.994	**31.65**	**0.975**	**29.52**	**0.952**	**27.25**	**0.927**
Eight	Initial	21.17	0.592	17.57	0.332	11.38	0.082	9.55	0.051	8.33	0.037
	Median	30.21	0.927	29.43	0.918	24.99	0.873	20.73	0.717	17.20	0.480
	Proposed	**31.86**	**0.946**	**29.68**	**0.920**	**26.18**	**0.874**	**24.20**	**0.842**	**23.28**	**0.813**
Montreuil	Initial	22.43	0.831	18.50	0.68	12.56	0.387	10.81	0.292	9.56	0.222
	Median	31.47	0.965	**30.98**	0.962	26.70	0.915	22.81	0.809	0.794	0.605
	Proposed	**32.55**	**0.979**	30.84	**0.968**	**27.00**	**0.916**	**24.98**	**0.869**	**22.88**	**18.54**
Map	Initial	21.93	0.883	18.14	0.758	12.24	0.425	10.50	0.308	9.27	0.231
	Median	19.12	0.536	18.93	0.529	17.49	0.483	16.13	0.432	14.46	0.365
	Proposed	**23.70**	**0.872**	**21.85**	**0.815**	**18.11**	**0.529**	**16.76**	**0.471**	**15.36**	**0.388**
Stripes	Initial	21.04	0.721	17.14	0.518	11.17	0.231	9.34	0.164	8.21	0.121
	Median	34.71	0.997	31.63	0.991	22.94	0.907	19.24	0.757	16.28	0.568
	Proposed	**36.50**	**0.998**	**32.55**	**0.995**	**24.08**	**0.925**	**20.96**	**0.811**	**18.05**	**0.638**



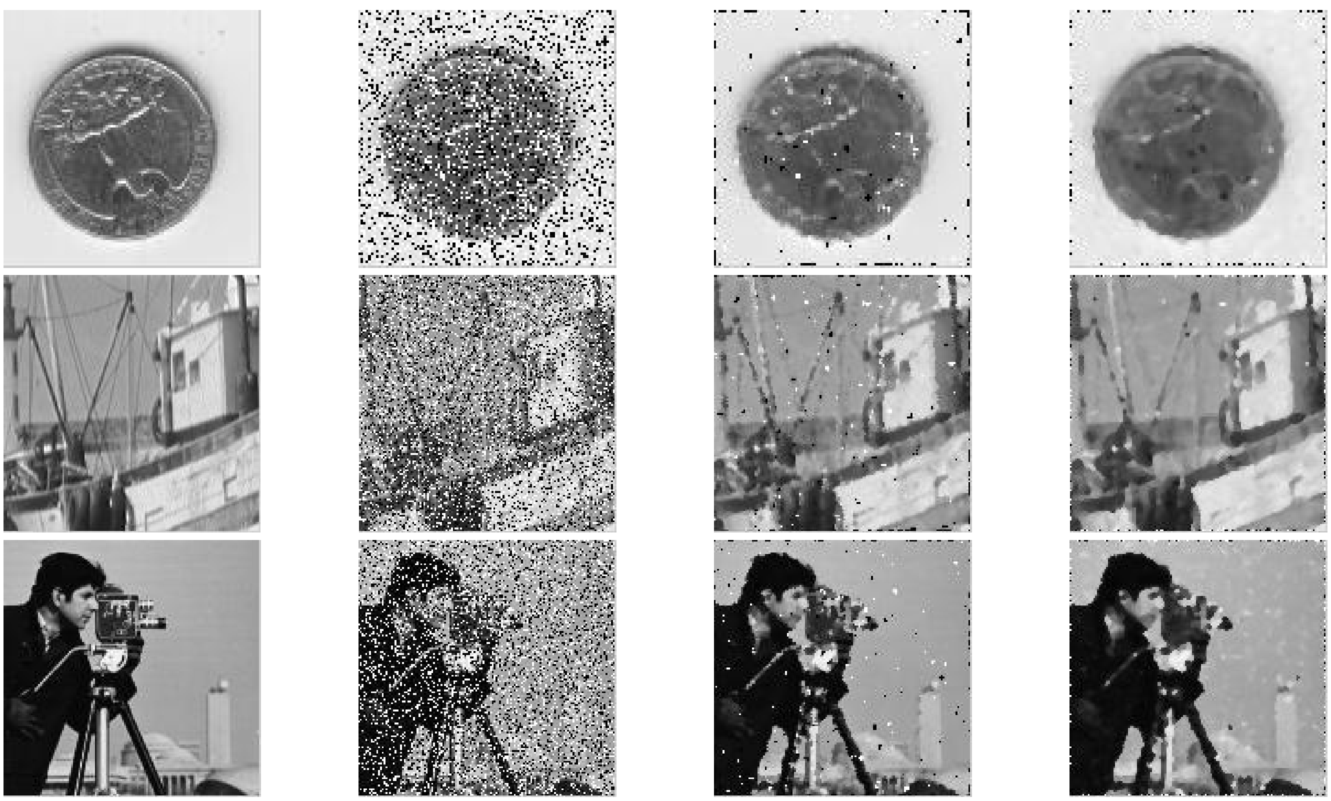
(a) (b)

Fig. 3. Performance measures for Cameraman image: (a) PSNR, (b) MSSIM.



(a) (b) (c) (d)

Fig. 4. The performance comparison for various images: (a) Original images; (b) Images with salt and pepper noise with density value set to 0.02; (c) Denoising results by median filter; (d) Denoising results by the proposed method.



(a) (b) (c) (d)

Fig. 5. The performance comparison for various images: (a) original images; (b) noisy images by salt and pepper noise with 0.3 value of density; (c) denoising noisy images by median filter; (d) denoising noisy images by the proposed method.

The visual quality comparison is shown in Fig. 4 and Fig. 5 with density value of noise set to 0.02 and 0.3 respectively. To better compare the details of the denoising among the proposed method and the Median filter, we only illustrate the results of sub-images.

The Figure 4 shows that the proposed method, when the noise density is low, preserves edges better than Median filter. It shows also that the performance of the proposed method for denoising images is better than Median filter. The Figure 5 shows that the proposed method clears noise better than Median filter. But, it introduces some distortion.

From the two Figures 4 and 5 we can conclude that the image details are recovered back exactly when the noise percentage is low.

VI. CONCLUSION

In this paper we proposed a noise filtering algorithm for removal of salt and pepper noise from the angle of evolutionary game theory. We treated each pixel in image as an autonomous agent where it seeks to maximize its payoff, and formulate the image denoising problem as an evolutionary game where denoised image emerges as a result of local autonomous decisions taken by the agents. From such a perspective, the problem of restoring the value of pixels becomes finding the evolutionarily stable strategies for the evolutionary game.

The proposed method provides better results in terms of image quality and similarity measures compared to median filter for salt and pepper noise. It was carried out on various noisy images to analyze its performance taking into account measures such as peak signal to noise ratio and mean structure similarity. As a future work, the use of other payoff functions may certainly improve the quality of the results. Moreover, the proposed method can be easily extended to handle other types of noise such as speckle, Gaussian, Poisson, etc.

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