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# Data mining based noise diagnosis and fuzzy filter design for image processing ☆

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ABSTRACT

In image processing, both diagnosis of noise types and filter design are critical. Conventional filtering techniques for image restoration such as median filter and mean filter are not effective in many cases, such as the case lacking the information of noise types or the case having mixed noise in images. This paper develops a data mining approach for noise type diagnosis, and proposes a fuzzy filter design for enhancing the quality of noise corrupted images. The experimental results demonstrate that the proposed technique outperforms the conventional filters, particularly for dealing with the images corrupted by mixed noise with additive Gaussian noise and impulse noise.



## 1. Introduction

Digital Images play a vital role in daily life applications such as satellite television, medical devices and also in scientific research, such as geometrical information systems, astronomy and mathematical morphology. However, digital images are often corrupted by noise generated in data sampling, quantification, acquisition and transmission. The noise as useless information can substantially reduce the image quality. There are two kinds of popular noise: one is the additive Gaussian noise, such as the channel noise in image transmission, the other is impulse noise, such as salt-and-pepper noise [1].

In image processing, the most common denoising methods are median filter and mean filter [2–4], which are the representatives of nonlinear filters and linear filters respectively. Mean filter is suitable to restrain Gaussian noise, and median filter is suitable to restrain impulse noise. Many other approaches developed for image restoration are also aimed at removing either Gaussian or uniform impulsive noise. The existing noise type diagnosis methods can only identify Gaussian noise or salt-and-pepper noise. However, in many applications, an image is often contaminated by more than one type of noise. The mixed noise corruption often happens in many complex environment. It is important to develop approaches to remove mixed noise in an image processing without blurring the image details or edges.

Generally, image denoising or noise filtering technique consists of two main steps: noise detection and noise removal. In noise filtering, the original image features, such as edges, size and shape, should be kept unchanged. In image processing, the noise type in the corrupted image should be effectively identified, so as to allow the filtering algorithm to match with the noise types of the image.

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Fuzzy logic has been successfully developed for modeling the vagueness and ambiguity in complex systems, and has been extended to image processing [5–7]. In many image processing applications, such as object recognition and scene analysis, expert knowledge must be used. Fuzzy logic provide powerful tools to represent and process expert knowledge in form of fuzzy if-then rules. Many difficulties in image processing lie in the uncertainty in data, tasks and results. This uncertainty can be attributed the randomness on to the inherent ambiguity and vagueness of image data. In this paper, an approach is proposed to obtain the fuzzy rules from image data and use these fuzzy rules to identify noise type. Then, a novel fuzzy median-mean filter is proposed. The proposed approach is capable of removing large amounts of mixed Gaussian and impulsive noise, while preserving edge information. The simulation results demonstrate that the proposed approach is much better than the well-known median filters and mean filter for the processing of image corrupted by mixed noise.

The outline of this paper is as follows. In Section 2, we simply analyze mixed noise model. Then, a noise classifier to identify different types of noise is presented in Section 3. In Section 4, a fuzzy median-mean filter for denoising is introduced. The experiments and results are shown in Section 5. Conclusion is given in Section 6.

## 2. Mixed noise model

There are certain different types of mixed noise models commonly considered in the literature, such as, blur and Gaussian (or impulse) noise [8–11]; Poisson plus Gaussian noise [12] and Gaussian plus impulse noise [13–19].

As noise can be regarded as stochastic and uncertain error, probability density function (PDF) has been used to describe noise. Gaussian noise is the most common noise type in nature, whose PDF is expressed as

$$p(r) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(r-\varphi)^2/2\sigma^2}, \tag{1}$$

where $\varphi$ is the mean value, $\sigma$ is the standard deviation of Gaussian noise.

For 8 bit gray image with 256 gray grades, the PDF of salt-and-pepper noise can be described as

$$p(r) = \begin{cases} p_a, & r = 0, \\ p_b, & r = 255, \\ 0, & otherwise. \end{cases} \tag{2}$$

In general, impulse noise means the noise pixel of image with the two kinds of extremeness gray values, with gray value 0 representing white and gray value 255 representing black. When $p_a$ and $p_b$ are non-zero and approximately equal, the impulse noise looks like the pepper-and-salt particulates distributing in image random.

Many images, such as those from radiography, contain noise that satisfies a Poisson distribution. The PDF of the Poisson distribution is given as:

$$p(s) = \frac{m^s}{s!} e^{-m}, \tag{3}$$

where $s$ is the number of occurrences of an event and $m$ is the expected number of occurrences during a given interval. It has been shown that a Poisson distribution can be approximated by a Gaussian distribution, the following equation can be obtained:

$$p(s) \simeq e^{-(s-m)^2/2m} \Big/ \sqrt{2\pi m}. \tag{4}$$

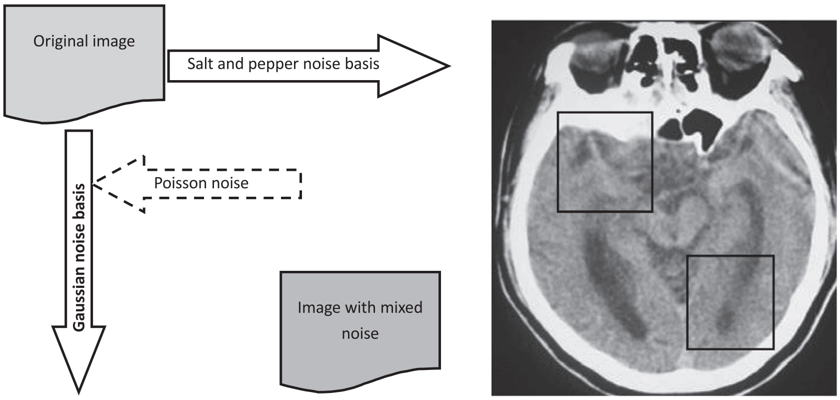
In the literature [20,21], several studies presented and discussed that the Poisson distribution approaches a Gaussian density function in the case of high number of counts. Moreover, Miller in [20] showed that the Gaussian approximation is surprisingly accurate, even for a fairly small number of counts.

Although different kinds of mixed noise models are available, it is quite challenging to tell exactly which specific type of mixed noise model fits specific noisy images. Theoretically a mixed noise model may be modeled by summing up independent types of basis noise models as shown in Fig. 1(a). In order to obtain better denoising performance, specific filter is needed for a specific mixed noise model.

In the following, we will focus on the specific image with noise composed of Gaussian and salt-and-pepper noise. Image restoration is an important task in image processing. The general idea is to estimate an ideal image $I_{org}$ from the observed noisy image $I_{imp}$. Here, we assume that the observed noisy image is obtained by using the mixed noise model:

$$I_{imp} = \mathsf{F}_{image}(I_{org} + G), \tag{5}$$

where $G$ is an additive Gaussian noise with mean value $\varphi$ and standard variance $\sigma$, $\mathsf{F}_{image}$ denotes the image degradation by salt-and-pepper noise.




(a) Mixed noise and noise basis (b) Sampling window

**Fig. 1.** Mixed noise and sampling window.

## 3. Identifying mixed noise using data mining

Generally, fuzzy rule extraction is the process of analyzing data from different perspectives and summarizing it into useful information [22–24]. In the following, the data mining method is extended to identify mixed noise in a given noisy image.

Let $I_{imp}$ denote an image corrupted by mixed noise. Pixel $x_{m,n}$ is located at position $(m, n)$, where the coordinate origin is assumed to be the left upper corner of the image. Then $x_{m,n}$ can be written as $x_{m,n} = s_{m,n} + n_{m,n}$ where $s_{m,n}$ denotes the original image signal and $n_{m,n}$ denotes the noise. Next, we define the local area or neighborhood as a sampling window of pixels of size $(2N + 1) \times (2N + 1)$.

We calculate the mean value of the sampled window as follows

$$x = \frac{1}{(2N+1) \times (2N+1)} \sum_{i=-N}^{N} \sum_{j=-N}^{N} x_{i,j}. \tag{6}$$

The gray level histogram is a function showing, for each gray level, the number of pixels in the image that have that gray level. Define the sampling window image $x_{i,j}$ composed of $v$ discrete gray levels, denoted by $\{r_0, \ldots, r_k, \ldots, r_v\}$. The PDF of $r_k$ is defined as

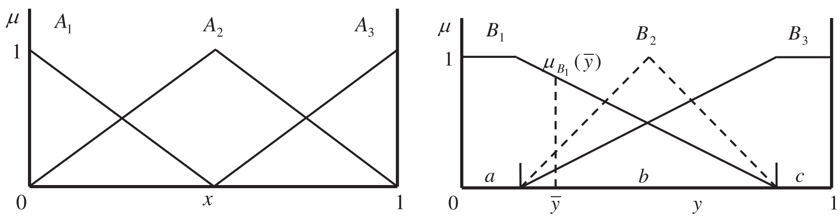
$$p(r_k) = \frac{n_k}{n}, \tag{7}$$

where $k = 0, 1, \ldots, v$; $n_k$ is number of pixels that have the value of $k$, and $n$ is the total number of pixel for sampling window. Based on the PDF, the feature variable $y$ using identification of the mixed noise is defined as

$$y = \frac{\mathrm{Min}\{p(r_0), p(r_{255})\}}{\mathrm{Max}\{(p(r_0), p(r_{255})\}}. \tag{8}$$

As an example of brain CT image shown in Fig. 1(b), a set of $(x, y)$ data pairs is given by moving sampling window randomly image:

$$(x_p, y_p), \;\; p = 1, 2, \ldots, P \tag{9}$$




(a) Fuzzy sets of $x$ variable (b) Fuzzy sets of $y$ variable

**Fig. 2.** Fuzzy sets and membership functions.

where $x$ is the mean value of the sampling window, $y$ is the feature parameter and $P$ is sampling times. The task here is to generate a set of fuzzy rules from the sampling input-output pairs of (9), and use these fuzzy rules to determine a noise type of image. The proposed approach consists of the following steps:

**Step 1.** Divide the input and output spaces into fuzzy regions
Assume that the domain intervals of $x$ and $y$ are $[x_-, x_+]$ and $[y_-, y_+]$, respectively, where the domain interval of a variable means that most probably, this variable will lie in this range. The sets of linguistic labels are denoted by $A(x) = \{A_1, A_2, A_3\}$ and $B(y) = \{B_1, B_2, B_3\}$, where each linguistic label is associated with a fuzzy membership function. The fuzzy sets and membership functions as shown in Fig. 2 are used.
The values of $a$, $b$ and $c$ in the fuzzy sets of $y$ variable will influence the estimation of noise type directly. Deviation will generate if improper assignment of value $a$ or $c$ are made. For example, the result of identification could be Gaussian noise even though it is mixed noise in reality, if an overlarge value is chosen. In order to justify the identification processing between the elements of Gaussian noise and salt-and-pepper noise and make the range of identification as large as possible, we select $b$ equals to 0.5, and both $a$ and $c$ equal to 0.25.
**Step 2.** Convert ordinary records into fuzzy records
The input-output data pairs obtained are stored in relational databases. The data in relational databases are stored in a table, where each row is a record and each column represents one of the attributes of the records. Let $L_1 = \{x, y\}$ be a set of attributes, $t_p$ be the $p$-th record with certain attribute values. A set of records associated to attribute $L_1$ is denoted by $T_{L_1}$, that is,

$$T_{L_1} = \{t_1, \cdots, t_p, \cdots, t_P\}. \tag{10}$$

The fuzzified $t_p$ is written as $\mu(t_p)$, associated with membership values of the set of attributes $L_2 = \{A(x) \cup B(y)\}$. A set of fuzzified records associated to attribute $L_2$ is denoted by $T_{L_2}$, that is,

$$T_{L_2} = \{\mu(t_1), \ldots, \mu(t_p), \cdots, \mu(t_P)\}. \tag{11}$$

**Step 3.** Calculate degree of support
From a data mining perspective, the degree of support is the percentage of records where the rule holds. If a fuzzy rule has practical meaning, it must have a large enough degree of support from sample data. Therefore, the degree of support for a specific fuzzy input space is a good indicator for extracting fuzzy rules from numerical data. The degree of support for a fuzzy rule is defined as follows:

$$Supp(x \Rightarrow y) = \frac{\sum_{p=1}^{P} \mu_{(B_l)^p}(y)\mu_{(A_j)^p}(x)}{\sum_{p=1}^{P} \mu_{(A_j)^p}(x)}, \tag{12}$$

where $\mu_{(A_j)^p}(x)$ and $\mu_{(B_l)^p}(y)$ are values of membership functions for the $p$-th record respectively, $P$ is the total number of records in $T_{L_2}, l, j \in \{1, 2, 3\}$.
For simplicity, the following formula is used to calculate the degree of support,

$$Supp(x \Rightarrow y) = \frac{1}{P}\sum_{p=1}^{P} \mu_{(B_l)^p}(y)\mu_{(A_j)^p}(x). \tag{13}$$

**Step 4.** Define degree of confidence
The degree of confidence (DOC) will be used to weight the degree of support calculated from (12) or (13). When the clean pixels of sampling window are closed to white or black, i.e., $x$ belongs to fuzzy set of $A_1$ or $A_3$, the values of some pixels in sampling window may be changed to 0 or 255 due to Gaussian noise. Based on above consideration, it is necessary to revise the degree of support by DOC coefficient in Table 1, where values of $a$, $b$, $c$ are shown in Fig. 2(b). Then, the final degree of support is obtained by means of the following weighted relation

$$Supp(x \Rightarrow y) = \text{DOC} \times Supp(x \Rightarrow y). \tag{14}$$

**Step 5.** Generate a fuzzy rulebase

**Table 1**
Degree of confidence.

| DOC | Fuzzy set of variable | | $y$ | | |
|---|---|---|---|---|---|
| | | | $B_1$ | $B_2$ | $B_3$ |
| 1 | $x$ | $A_1$ | $a$ | $b$ | $c$ |
| 2 | | $A_2$ | 1 | 1 | 1 |
| 3 | | $A_3$ | $a$ | $b$ | $c$ |

Details of the data mining algorithm with pseudo code:
**Input**: The inputs comprise the following specifications given by the user,
Image data with noise.
**Output**: Generate a fuzzy rulebase
The detailed description of the algorithm is given as follows:
**P-1**: Initialization
Define global variables $i$, Max-sup and Max-num.
**P-2**: Obtain fuzzy rules based on maximum degree of support

```
for(i = 1; 3; i++) {
Call P-3;
Obtain rule[j] based on Max-num:
}
end
```

**P-3**: Scan all fuzzy sets and calculate membership values for $x$

```
for(l = 1; 3; l++) {
Calculate μ_{A_l}(x);
Call P-4;
}
return
```

**P-4**: Scan all fuzzy sets for $y$ and calculate degree of support

```
for (j = 1; 3; j++) {
Calculate μ_{B_j}(y);
Use (16) to calculate:
E[j] = Supp(ẋ ⇒ y);
If (E[j] > E[0]){
   Max-sup = E[j];
   Max-num = j;
   E[0] = E[j];
}
}
return
```

The process for a fuzzy rulebase generation is given as follows. Assume that $A = \{A_1, A_2, A_3\}$ is a set of linguistic labels for attribute $x$, and $B = \{B_1, B_2, B_3\}$ is another set of linguistic labels for attribute $y$. For $\{A_1\}$, we first calculate the respective degrees of support for pairs $\{A_1, B_1\}$, $\{A_1, B_2\}$ and $\{A_1, B_3\}$. Then, we select a fuzzy subspace with the maximum degree of support for this column. Repeating this process for $\{A_2\}$ and $\{A_3\}$, we can obtain one of the following fuzzy rules $R^{(1)}$, $R^{(2)}$ and $R^{(3)}$ in Step 6.

**Step 6.** Decide noise type using fuzzy rulebase
Focusing on the $R^{(1)}$ rulebase, i.e. the group of rules having $B_1$ as the consequent, we conclude that the noise type is Gaussian noise. Similarly, if we obtain the $R^{(2)}$ rulebase, i.e. the group of rules having $B_3$ as the consequent, the noise type is generally considered as salt and pepper noise.

$$R^{(1)} : IF \quad (x, A_i) \quad THEN \quad (y, B_1), \quad i = 1, 2, 3. \tag{15}$$

$$R^{(2)} : IF \quad (x, A_i) \quad THEN \quad (y, B_3), \quad i = 1, 2, 3. \tag{16}$$

When the consequents of the fuzzy rules are not identical, i.e., various combinations of $B_1$, $B_2$ and $B_3$ (except $R^{(1)}$ and $R^{(2)}$) appear in the THEN part, we define the type of noise as a mixed one. In such a case, the proposed fuzzy mean-median filter will be applied to remove noises from images.

$$R^{(3)} : \begin{cases} IF \quad (x, A_1) \quad THEN \quad (y, B_i) \\ IF \quad (x, A_2) \quad THEN \quad (y, B_j), \; i, j, k = 1, 2, 3. \\ IF \quad (x, A_3) \quad THEN \quad (y, B_k) \end{cases} \tag{17}$$

## 4. Filter design

In image processing, the most common methods are median filter and mean filter. In a mixed noise environment, where both Gaussian and impulsive noise are present. In the following, we propose an approach to combine both mean filter and median filter together with an adjustment parameter. This adjustment parameter is just equal to the fuzzy membership function $B1$ in Section 3. The flow chart of the image denoising process is given in Fig. 3.

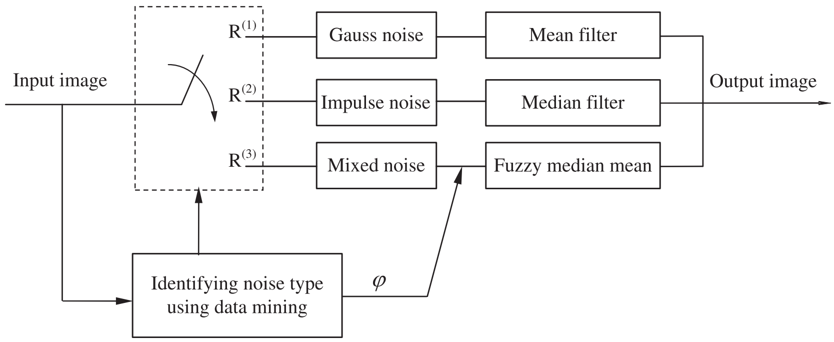



**Fig. 3.** The flow chart of noise removal.

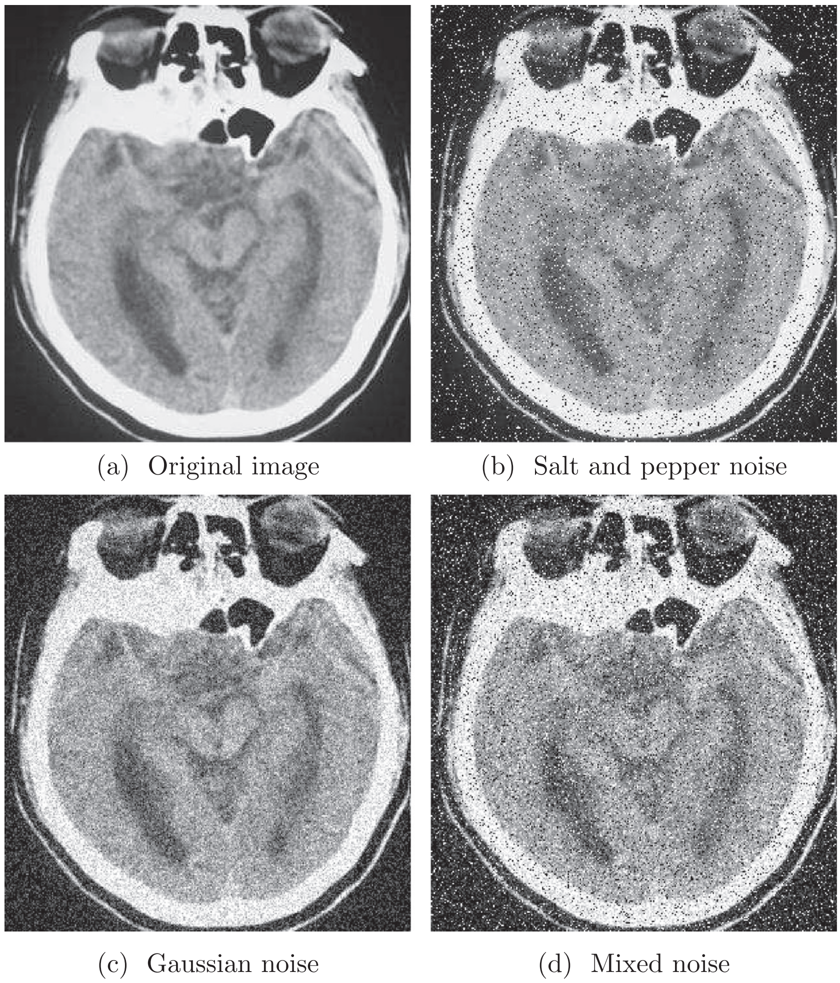
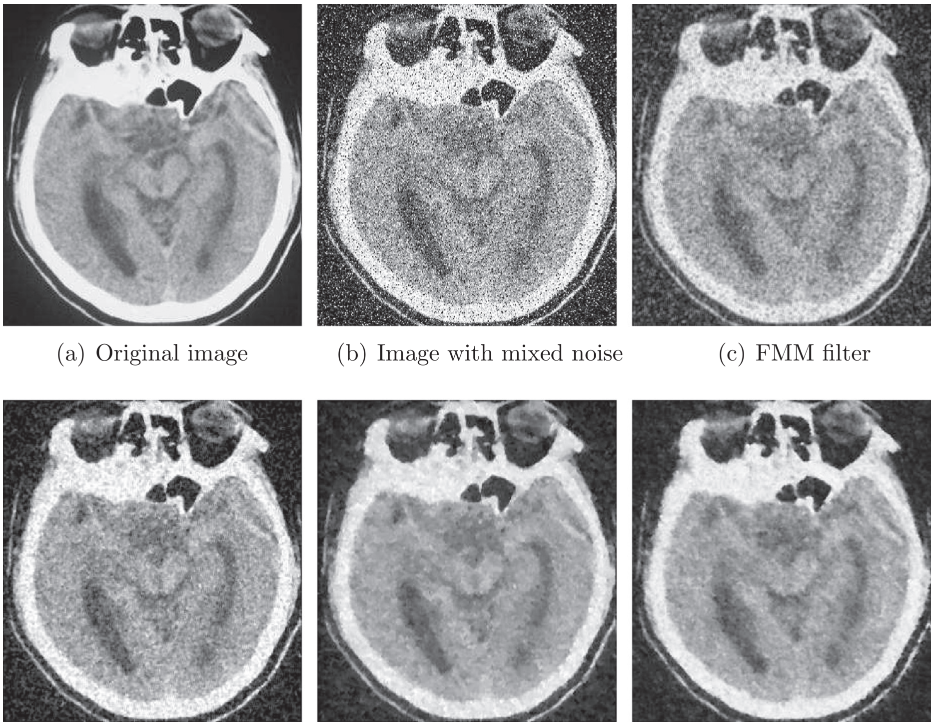



**Fig. 4.** Original image plus different type noise

This novel filter, fuzzy media-mean (FMM) filter, is aimed to remove the mixed noise. The fuzzy media-mean filter operates on a filtering window. For simplicity, let us consider this window of size $3 \times 3$ to be operated on a digitized image having $L$ gray levels, where $I(i,j)$ is located at the center of the filtering window. This window matrix will scan the whole image matrix from top to bottom and left to right. The proposed method operates as follows. First, weighted average fuzzy membership function in Fig. 2(b) is defined according to the following relation:

**Table 2**
Identification of mixed noise.

| Results | $\varphi,\ \sigma$ | Noise ratio: salt and pepper (%) | | | |
|---|---|---|---|---|---|
| | | 5 | 10 | 15 | 20 |
| Gaussian | 0.0, 0.01 | ✓ | ✓ | ✓ | ✓ |
| | 0.0, 0.03 | ✓ | ✓ | ✓ | ✓ |
| | 0.0, 0.05 | ✓ | ✓ | ✓ | ✓ |
| Gaussian | 0.02, 0.01 | ✓ | ✓ | ✓ | ✓ |
| | 0.02, 0.03 | ✓ | ✓ | ✓ | ✓ |
| | 0.02, 0.05 | ✓ | ✓ | ✓ | ✓ |
| Gaussian | 0.04, 0.01 | ✓ | ✓ | ✓ | ✓ |
| | 0.04, 0.03 | ✓ | ✓ | ✓ | ✓ |
| | 0.04, 0.05 | ✓ | ✓ | ✓ | ✓ |



(a) Original image (b) Image with mixed noise (c) FMM filter

(d) Mean filter (e) Median filter (f) Fuzzy filter

**Fig. 5.** Noise reduction with fuzzy median-mean filter.

$$\mu_{B_1}(\bar{y}) = \frac{\sum_{p=1}^{P} y_p \times \mu_{B_1}(y_p)}{\sum_{p=1}^{P} y_p},\ p = 1, \ldots, P \tag{18}$$

then, the final fuzzy media-mean filter is obtained by means of the following relation:

$$\begin{cases} \widehat{I}(i,j) = \varphi m_1 + (1-\varphi) m_2, \\ \varphi = \mu_{B_1}(\bar{y}), \end{cases} \tag{19}$$

where $m_1$ is the mean value, $m_2$ is the median value of the filtering window. Adjustment parameter $\varphi$ equals to fuzzy membership function $(\mu_{B_1}(\bar{y}))$, representing the portion of mean filter. The fuzzy membership function $(\mu_{B_1}(\bar{y}))$ is a presentation of occupancy of the Gaussian noise portion in the mixed noise. $\widehat{I}(i,j)$ is the estimation of $I(i,j)$.

**Table 3**
Comparison of MSE, RMSE, PSNR, and NMSE for different filters.

| Results | Filter | Gauss: $\varphi = 0.00,\ \sigma = 0.03$; salt and pepper (%) | | | |
|---|---|---|---|---|---|
| | | 5 | 10 | 15 | 20 |
| MSE | Mean | 431.985 | 621.266 | 818.646 | 1058.70 |
| | Median | 398.134 | 452.107 | 526.117 | 649.401 |
| | Fuzzy | 325.961 | 362.484 | 396.344 | 445.375 |
| | FMM | 274.109 | 324.471 | 360.515 | 412.705 |
| RMSE | Mean | 20.7840 | 24.9240 | 28.6120 | 32.5370 |
| | Median | 19.9530 | 21.2630 | 22.9370 | 25.4830 |
| | Fuzzy | 18.0544 | 19.0390 | 19.9084 | 21.1039 |
| | FMM | 16.5560 | 18.0130 | 18.9870 | 20.3150 |
| PSNR | Mean | 21.7760 | 20.1980 | 18.9660 | 17.8490 |
| | Median | 22.1310 | 21.5780 | 20.9200 | 20.0060 |
| | Fuzzy | 22.9308 | 22.5038 | 21.9781 | 19.9084 |
| | FMM | 23.7170 | 22.9850 | 22.5270 | 21.9400 |
| NMSE | Mean | 0.0200 | 0.0290 | 0.0390 | 0.0500 |
| | Median | 0.0190 | 0.0210 | 0.0250 | 0.0310 |
| | Fuzzy | 0.0154 | 0.0171 | 0.0187 | 0.0211 |
| | FMM | 0.0130 | 0.0150 | 0.0170 | 0.0200 |

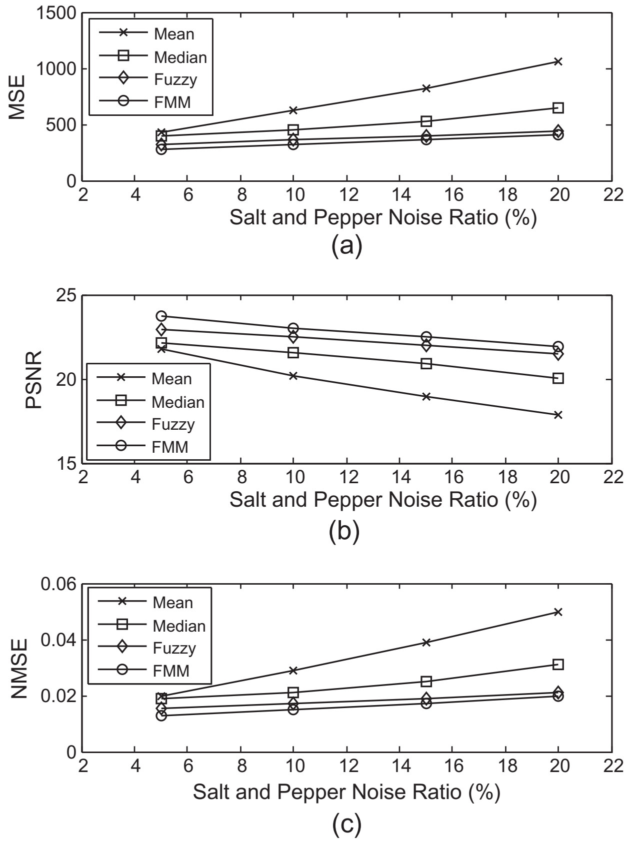



**Fig. 6.** Comparison of FMM: (a) MSE, (b) PSNR, (c) NMSE values prepared for CT image.

## 5. Results and discussion

The reliability of the images can be evaluated by mean square error (MSE), root mean square error (RMSE), normalized mean square error (NMSE) and peak signal-to-noise ratio (PSNR) criteria. The MSE, RMSE, PSNR, and NMSE are given in Eqs. (20)–(23), respectively.

$$\mathrm{MSE} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\|F(i,j) - G(i,j)\|^2}{NM}, \tag{20}$$

$$\mathrm{RMSE} = \sqrt{MSE}, \tag{21}$$

$$\mathrm{PSNR} = 20log\frac{255}{RMSE}, \tag{22}$$

$$\mathrm{NMSE} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\|F(i,j) - G(i,j)\|^2}{\sum_{i=1}^{N}\sum_{j=1}^{M}\|F(i,j)\|^2}, \tag{23}$$

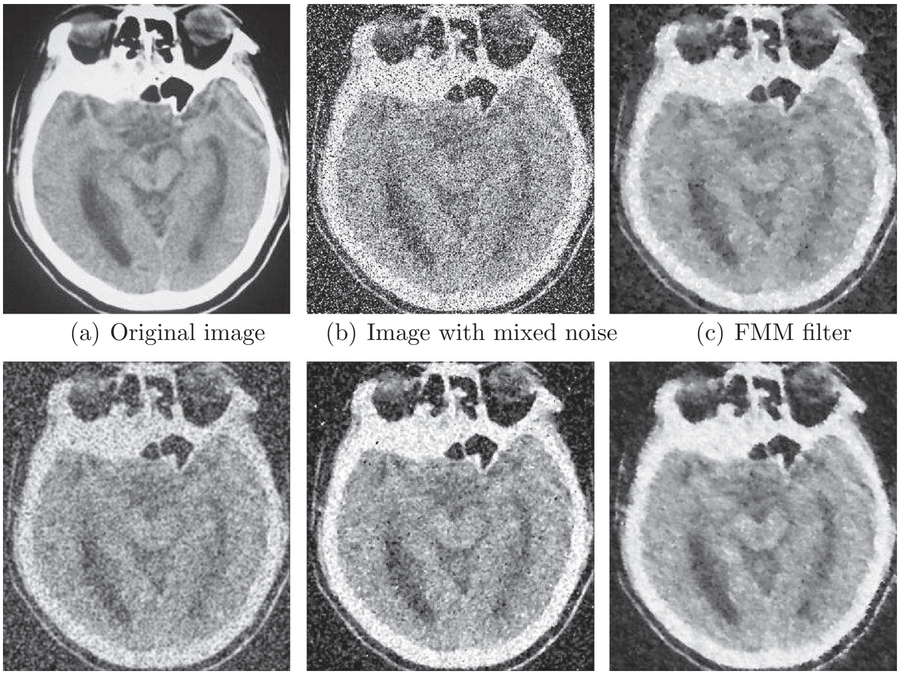
where $F(i,j)$ and $G(i,j)$ denote the pixel values of the restored image and original image respectively.

PSNR value gives the information on the quality of the image. MSE, RMSE and NMSE values demonstrate the error values of the same images compared with the original one. The gray levels of the pixels in the image are taken as data. If the signal ratio of an image is high and the noise ratio is low, the image is considered to have good quality. Therefore, in the analysis of an image, PSNR value must be examined. Since the magnitude of this ratio is proportional to the quality of the image, the PSNR of the original image and the image containing noise is determined. The parameters used in Fig. 2(b) and Table 1 for the following experiments are specified as follows: $a = 0.25,\ b = 0.50$ and $c = 0.25$.

### 5.1. Experiment 1

To illustrate the different types of noises prior to experiment, Fig. 4 shows the original brain CT image and images with Gaussian noise, salt-and-pepper noise and mixed noise respectively. In the following experiment, the range of identification will be tested. For this purpose, salt-and-pepper noise in different ratios (from 5% to 20%) and Gaussian noise in different mean values (from 0 to 0.04) and standard deviations (from 0.01 to 0.05) are added to the original CT image. Then the CT images with different degree of mixed noise were tested by arithmetic of noise type identification based on data mining. The results of the experiment were summarized in Table 2.

Table 2 shows that all kinds of mixed noise are able to be identified. It is concluded that salt-and-pepper noise in different ratios from 5% to 20% mixed with Gaussian noise in mean values from 0 to 0.04 and standard deviations from 0.01 to 0.05 can be identified by the proposed method.



(a) Original image (b) Image with mixed noise (c) FMM filter

(d) Mean filter (e) Median filter (f) Fuzzy filter

**Fig. 7.** Noise reduction with fuzzy median-mean filter.

**Table 4**
Comparison of MSE, RMSE, PSNR, and NMSE for different filters.

| Results | Filter | Salt and pepper: 20%; Gaussian | | | |
|---|---|---|---|---|---|
| | | $\varphi = 0.00$ $\sigma = 0.01$ | $\varphi = 0.00$ $\sigma = 0.02$ | $\varphi = 0.00$ $\sigma = 0.03$ | $\varphi = 0.00$ $\sigma = 0.04$ |
| MSE | Mean | 850.435 | 963.537 | 1042.30 | 1132.40 |
| | Median | 301.882 | 464.689 | 637.542 | 788.550 |
| | Fuzzy | 266.492 | 397.695 | 443.924 | 524.501 |
| | FMM | 244.586 | 352.537 | 409.987 | 495.903 |
| RMSE | Mean | 29.1620 | 31.0410 | 32.2850 | 33.6520 |
| | Median | 17.3750 | 21.5570 | 25.2500 | 28.0810 |
| | Fuzzy | 16.3246 | 19.9423 | 21.0695 | 22.9020 |
| | FMM | 15.6390 | 18.7760 | 20.2480 | 22.2690 |
| PSNR | Mean | 18.7320 | 18.2920 | 17.9510 | 17.5910 |
| | Median | 23.3320 | 21.4590 | 20.0860 | 19.1630 |
| | Fuzzy | 23.8056 | 22.1038 | 21.4857 | 20.8992 |
| | FMM | 24.2120 | 22.6250 | 21.9690 | 21.1430 |
| NMSE | Mean | 0.0400 | 0.0460 | 0.0490 | 0.0540 |
| | Median | 0.0140 | 0.0220 | 0.0300 | 0.0370 |
| | Fuzzy | 0.0126 | 0.0188 | 0.0197 | 0.0248 |
| | FMM | 0.0120 | 0.0170 | 0.0190 | 0.0240 |

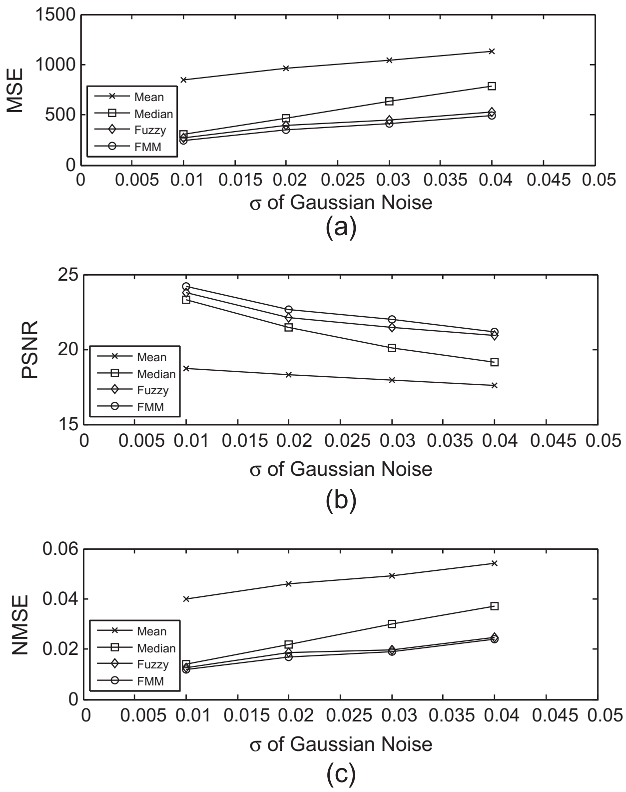



**Fig. 8.** Comparison of FMM: (a) MSE, (b) PSNR, (c) NMSE values for CT image.

## *5.2. Experiment 2*

In order to illustrate the performance of the proposed fuzzy median-mean filter, we consider a brain CT image shown in Fig. 5(a). Fig. 5(b) shows the image corrupted by mixed noise, which has Gaussian noise ($\varphi = 0, \sigma = 0.03$) and 10% salt-and-pepper noise. Fig. 5(c) is the result by using proposed FMM. The results of the applications of mean filter and median filer are shown in Fig. 5(d) and (e), respectively, and the results of fuzzy filer [6] are shown in Fig. 5(f). It can be seen that compared with the results yielded by traditional filters the new filter (FMM) achieves a better effect as shown in Fig. 5(c). Results from FMM have clearer picture and have less black and white dots than mean filter and median filter do.

**Table 5**
Fuzzy membership function and filtering result.

| | *a* | *b* | *c* | MSE | RMSE | PSNR | NMSE |
|---|---|---|---|---|---|---|---|
| 1 | 0.15 | 0.7 | 0.15 | 326.015 | 18.056 | 22.964 | 0.0154 |
| 2 | 0.25 | 0.5 | 0.25 | 324.874 | 18.024 | 22.980 | 0.0154 |
| 3 | 0.35 | 0.3 | 0.35 | 329.554 | 18.154 | 22.917 | 0.0156 |

Gaussian noise with zero mean value and 0.03 standard deviations and salt-and-pepper noise in different ratios (from 5% to 20%) are added to the original CT image in this experiment. To make a further comparison, MSE, RMSE, PSNR and NMSE values of the filters as summarized in Table 3.

Fig. 6 shows graphics of the MSE, PSNR and NMSE values of mean filter, median filter, fuzzy filter and FMM. Median filter has lower values in MSE and NMSE than mean filter while FMM is the lowest. FMM has the highest PSNR value than the rest filters. It can be seen clearly that FMM filter has better performance in the sense of statistics than the rest filters.

### 5.3. Experiment 3

This experiment presents some simulation results using the proposed FMM filer and some traditional filters dealing with fixed salt-and-pepper noise and different Gaussian noises. Fig. 7(a) and (b) show the original image and image corrupted by mixed noise, which have Gaussian noise($\varphi = 0, \sigma = 0.04$) and salt-and-pepper noise in rate 20%. The results of mean filter and median filer are respectively represented in Fig. 7(d) and (e), and the results of fuzzy filer are represented in Fig. 7(f). Comparing the results yielded by traditional filters with new filter (FMM) shown in Fig. 7(c), it can be found that FMM also has a better performance than the others. The filtered image using mean filter and median have much more dots as shown in Fig. 7(d) and (e), however, FMM filtered image does not change so much.

Salt-and-pepper noise fixed in 20% ratio and Gaussian noise with zero mean value and different standard deviations (from 0.01 to 0.04) are added to the original CT image in this experiment. A further comparison of MSE, RMSE, PSNR, and NMSE values of the filters is shown in Table 4.

Fig. 8 shows graphics of the MSE, PSNR and NMSE values of mean filter, median filter and FMM. FMM has the lowest MSE and NMSE values and the highest PSNR value than mean filter and median filter. It can be seen that FMM filter also has a better performance in all statistics index than the rest filters.

In addition, we conduct another experiment to how the changes of fuzzy membership function slope affects the filtering performance. To compare with the original parameter of fuzzy membership function ($a = 0.25, b = 0.5, c = 0.25$) and its filtering performance, two more sets of parameters and the filtering results are also shown in Table 5. The results in Table 5 show that the changes of fuzzy membership function slope in a certain extent have little effect on the filtering performance.

## 6. Conclusions

In this paper, a novel noise classifier using data mining techniques and fuzzy median-mean filter for removing complex noise from corrupted images are proposed. The fuzzy median-mean filter combining a median-type filter with a mean filter is able to effectively recover images corrupted by Gaussian plus salt-and-pepper noise. The performance of the proposed approach is evaluated and compared with the classical median filter, mean filter and fuzzy filter algorithms for both salt-and-pepper noise and Gaussian noise removal tasks. The quantitative and qualitative results on test images demonstrate that the proposed approach can remove the noise effectively while preserving the image local features, even at a very high impulse noise and Gaussian noise level.

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